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The Impact of Bitcoin and Gold in the Portfolio

—A Research Based on Copula

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Abstract: Gold and cryptocurrencies play an important role in portfolios, especially in risk management. Due to the special nature of these financial products, people usually add a small amount of gold or cryptocurrencies to the origin portfolio to balance return and risk. This article takes Bitcoin as the representative of cryptocurrencies to analyze the different impacts of Bitcoin and gold in the portfolio. This article employs copula functions to fit the Value-at-Risk, Conditional Value-at-Risk, mean return, and Sharpe ratio. Value-at-Risk and Conditional Value-at-Risk are used to measure the portfolio's risk. In addition, mean return and Sharpe ratio are used to measure the returns. Empirical results demonstrate that gold and Bitcoin can both serve as hedging assets; Bitcoin can enhance portfolio returns, while gold might lead to a decrease in portfolio returns. This result offers a reference on the asset allocation to investors. Adding an appropriate proportion of gold and Bitcoin can optimize the portfolio's risk-return profile.

Keywords: Gold, Bitcoin, Value-at-Risk, Conditional Value-at-Risk, Sharpe ratio.

1. Introduction

In modern investment theory, optimizing the portfolio to balance risk and return is a vital concern for investors. Traditional assets including bonds, stocks, and funds, are the focus of investment in the past. However, with the development of the era, cryptocurrencies represented by Bitcoin gradually gained more attention. Bitcoin is known as "digital gold" because it shares many characteristics similar to gold. Both gold and Bitcoin have a low correlation with traditional assets. The price of gold is less affected by price fluctuations during market volatility thus it is preferred for hedging. Although Bitcoin is highly volatile, its potential for high returns also makes it attractive for investors to include in their portfolios.

In recent years, more and more people focus on studying the potential of adding gold and Bitcoin to portfolios. Ciner, using multiple regression analysis and other statistical models, empirically demonstrates that an appropriate allocation of gold can significantly reduce the overall risk of a portfolio and improve its risk-adjusted returns [1]. Baur, using models related to modern portfolio theory, shows Bitcoin's role as an investment tool and its diversification benefits in an investment portfolio, finding that Bitcoin has high return potential and low market correlation, which helps diversify risk in traditional asset portfolios [2]. Adams stresses the importance of balancing risk and return in portfolio management. By analyzing the performance of different investment strategies,

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Adams points out that neglecting risk management may lead to increased uncertainty in portfolio returns, thus affecting investors' long-term returns [3]. Patton introduces the application of dynamic copula models in financial market risk management, focusing on capturing the time-varying characteristics of dependencies between assets to enhance risk prediction accuracy [4]. Smith reviews the latest developments and applications of the Value-at-Risk method in financial market risk management, introducing various improved Value-at-Risk models and their effectiveness in practical risk assessment [5]. Bilbao-Terol shows that Conditional Value-at-Risk can account for losses beyond the Value-at-Risk threshold, providing a more comprehensive risk assessment, which is more significant for studying potential losses in extreme market conditions [6]. Liu conducts a comparative study on the role of the Sharpe ratio in risk management of traditional and alternative investments, pointing out the applicability and limitations of the Sharpe ratio in evaluating the risk-adjusted returns of different types of investment products [7]. In conclusion, researchers make significant progress in enhancing the diversification benefits and reducing the overall risk of investment portfolios by appropriately configuring assets such as gold and Bitcoin. However, most of these studies analyze the role of gold or Bitcoin individually, rather than directly comparing the specific effects of both in an investment portfolio. Based on the research of predecessors, this article will compare the different effects of gold and Bitcoin adding to portfolio in two perspectives, which is respectively risk and return, to make the conclusion more comprehensive and intuitive.

This article introduces copula functions to measure the impact of adding small amounts of gold and Bitcoin to a classic investment portfolio on Value-at-Risk and Conditional Value-at-Risk to evaluate portfolio risk. Also, this article uses mean return and Sharpe ratio to measure returns, quantifying the hedging effects of gold and Bitcoin in investments. Former researchers often use Monte Carlo simulations to fit Value-at-Risk and Conditional Value-at-Risk values, but this article uses t-copula. Combining emerging and traditional assets in a portfolio is a new attempt aimed at providing investors with investment strategic recommendations.

2. Model Introduction

2.1. Copula Function

According to Jiang, copula used in capturing the dependence structure between default times of different firms leads to more accurate credit risk assessments [8].

A copula is a multivariate distribution function that connects continuous distributions of multiple random variables to form a joint distribution.

Definition of an N-dimensional copula function:

(1) Domain:$[0,1]^p$

(2) $\mathrm{C}(x_1, x_2, \ldots, x_n)$ is monotonically increasing

(3) The marginal distribution (CDF) of $\mathrm{C}(x_1, x_2, \ldots, x_n)$ satisfies: $C_n(x_n) = C(1, \ldots, 1, x_n, 1, \ldots, 1)$(where n is a positive interger)

Sklar's Theorem: For any random vector $(x_1, x_2, \ldots, x_n)$ with marginal distribution functions $F_1, F_2, \ldots, F_n$,there exists a copula function such that $F(x_1, x_2, \ldots, x_n)$=C$(F_1, F_2, \ldots, F_n)$.This theorem describes the relationship between the joint distribution function, its marginal distribution functions, and the copula function.

Common copulas include the Gaussian copula, t-copula, Clayton copula and so on. According to Nguyen, a comprehensive review of commonly used copula models in financial data was conducted, categorizing different types of copulas and their applicable scenarios. Nguyen analyzes the advantages and disadvantages of the Gaussian copula, t-copula, and Archimedean copula in capturing dependency structures between assets [9]. Therefore, this article adopts the t-copula method.

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2.2. T-Copula

This formula defines the cumulative distribution function of a multivariate t-distribution with v degrees of freedom and a correlation matrix.

$$C_{v,R}\left(x_1,\ldots,x_n\right)=\int_{-\infty}^{t_v^{-1}\left(u_1\right)}\ldots\int_{-\infty}^{t_v^{-1}(u_n)}\frac{\Gamma[(v+\mathrm{n})/2]}{\Gamma(v/2)(v\pi)^{\mathrm{n}/2}}|R|^{-1/2}\left[1+\frac{1}{v}\mathrm{X}^TR^{-1}\mathrm{X}\right]^{-(v+\mathrm{n})/2}dx_1\ldots dx_n \quad (1)$$

Where t_v^{-1} denotes the inverse CDF function of a standard univariate t_v distribution, v refers to the degrees of freedom parameter,$\mathrm{X}=(x_1,\ldots,x_n)^T$,$\mathrm{R}=(\rho_{ij})$ is a correlation matrix.

According to Molina, t-copula is based on the t-distribution and has the property of heavy tails, making it more suitable for capturing extreme events in financial markets [10].

2.3. Value-at-Risk

Quantile: Suppose $X\sim F(x)$,for any $0<q<1$, the qth quantile of X is defined as $Q_q(X)=max\{x:F(x)\leq q\}$.

Value-at-Risk (VaR): If X is the net or log return of an asset, then $-Q_q(X)$ is called the $100q\%$ VaR.

$$VaR_q(X)=-Q_q(X) \quad (2)$$

Conditional Value-at-Risk (CVaR): The CVaR at $100q\%$ level is the average loss of the asset in the worst $100q\%$ of the cases.

$$CVaR_q(X)=\frac{1}{q}\int_0^q VaR_\alpha(X)d\alpha \quad (3)$$

2.4. Sharpe Ratio

The Sharpe ratio is used to calculate the excess return per unit of risk.

$$Sharpe\ Ratio=\frac{R_p-R_f}{\sigma_p} \quad (4)$$

Where R_p is the average return of the portfolio, R_f is the risk-free rate, σ_p is the standard deviation of the portfolio's returns.

According to Smith, a high Sharpe ratio means that the portfolio has achieved higher returns relative to its risk (volatility), indicating that the portfolio has performed well on a risk-adjusted basis [11].

3. Research Design

3.1. Data Selection

The closing prices of SPY (SPDR S&P 500 ETF Trust), AGG (Bloomberg Barclays Aggregate Bond Index), Bitcoin, and gold of 248 valid trading days from 1st January 2023 to 1st January 2024 come from Investing.com.

The website is a globally renowned financial information provider platform, widely used by investors, analysts, and research institutions.

3.2. Data Processing

This formula turns the price of the portfolio into a log return at time t.

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$$r_t = ln(\frac{P_t}{P_{t-1}}) = lnP_t - lnP_{t-1} \quad (5)$$

Where r_t is the log return of the portfolio at time t, P_t is the price of the portfolio at time t, P_{t-1} is the price of the portfolio at time t-1.

According to Johnson, log returns have significant advantages in calculating multi-period returns, risk management, and portfolio optimization [12]. Therefore, this study processes the data into log returns.

3.3. Variable Control

This article sets up 4 groups of portfolios:

(1) 60% stocks, 40% bonds

(2) Based on (1), adding 5%-10% gold, proportionally reducing the amounts of stocks and bonds

(3) Based on (1), adding 5%-10% Bitcoin, proportionally reducing the amounts of stocks and bonds

According to Sharpe, the advantages of the 60/40 portfolio as a standardized testing model are discussed, and its continued relevance as a benchmark model in investment research is analyzed [13].

4. Empirical Analysis

Table 1 presents the results of different investment portfolios, showing various combinations of SPY, AGG, gold, and Bitcoin. The first line is several financial metrics: Mean Return, VaR, CVaR, and Sharpe Ratio, and the constitution of the portfolio. Other lines are the specific data, which keeps six decimal places.

Table 1: The Result of Different Portfolios

Portfolio (SPY/AGG/gold/Bitcoin)	Mean Return	Value-at-Risk	Conditional-Value-at-Risk	Sharpe Ratio
60%/40%/0/0	0.000570	0.008985	0.008985	0.078056
57%/38%/5%/0	0.000559	0.008536	0.015366	0.080304
56.4%/37.6%/6%/0	0.000554	0.008386	0.015106	0.080728
55.8%/37.2%/7%/0	0.000555	0.008356	0.015042	0.081132
55.2%/36.8%/8%/0	0.000553	0.008267	0.014880	0.081525
54.6%/36.4%/9%/0	0.000550	0.008177	0.014719	0.081904
54%/36%/10%/0	0.000548	0.008087	0.014557	0.082266
57%/38%/0/5%	0.000666	0.008536	0.015366	0.094582
56.4%/37.6%/0/6%	0.000686	0.008643	0.015243	0.097312
55.8%/37.2%/0/7%	0.000705	0.008606	0.015183	0.099803
55.2%/36.8%/0/8%	0.000724	0.008628	0.015172	0.102043
54.6%/36.4%/0/9%	0.000743	0.009229	0.015202	0.104026
54%/36%/0/10%	0.000763	0.009884	0.015271	0.105753

Through comparison, it is found that adding 5%-10% of gold or 5%-9% of Bitcoin reduces the VaR, but increases the CVaR. This indicates that gold has strong hedging properties, while Bitcoin's hedging ability is weaker than gold's, but still effective when added in small amounts. However, the significant increase in CVaR suggests that during poor market performance, so they will both lead to higher potential losses.

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From Table 1, it can be seen that after adding gold, the average return and Sharpe ratio decrease. After adding Bitcoin, both the average return and Sharpe ratio significantly increase. This indicates that Bitcoin can significantly enhance portfolio returns, while gold performs moderately in terms of boosting portfolio returns. What's more, both assets can improve risk-adjusted returns.

5. Discussion

In stable market conditions, gold's price stability makes it an excellent defensive asset, helping to reduce overall portfolio risk. Conservative investors might increase their allocation to gold to protect against potential downturns. Because gold added to the portfolio causes the decline of the Value-at-Risk. In times of market uncertainty, gold's safe-haven properties become more valuable, providing a buffer against market volatility. Bitcoin, despite its higher volatility, can also act as a hedge due to its low correlation with traditional assets. A small allocation to Bitcoin might enhance portfolio diversification. Because Bitcoin adding to the portfolio decreases the risk and increases the return. During extreme market conditions, the contrasting characteristics of gold and Bitcoin can offer balanced protection and potential growth. Gold maintains stability, while Bitcoin's potential for high returns can offset losses from other assets.

In this case, those with lower risk tolerance should increase their allocation to gold due to its stability and lower volatility. A minimal addition of Bitcoin can still be beneficial for diversification without significantly increasing risk. Investors aiming for higher returns might consider a larger, yet still cautious, allocation to Bitcoin. Its high return potential can significantly enhance portfolio performance, albeit with higher risk.

6. Conclusion

This study finds that gold adding to the portfolio decreases the average return, Sharpe ratio, and VaR, but increases the CVaR. After adding Bitcoin, the average return, Sharpe ratio, VaR and CVaR all elevate. It can conclude that gold and Bitcoin can reduce the risk; Bitcoin can raise portfolio returns, while gold takes a chance to lead to a decrease in portfolio returns. The result is consistent with those former researches, this might be on account of the reasonable methods used in the research. This article provides a more meticulous reference on asset allocation for investors.

There are still some limitations in this article: (1) Only a classic investment portfolio was selected. Including more portfolios could make the results more generalizable. (2) Market changes were not considered, only the static market situation was analyzed. (3) The paper did not examine every combination from 5% to 10% but instead selected changes with a step size of 1%.

This study may contribute to future research: (1) The analysis in this paper can be applied to more financial products, such as foreign exchange and funds. (2) Traditional asset allocation methods usually assume fixed asset weights. Future research can explore dynamic asset allocation methods using machine learning.

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