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Empirical Analysis of SPX Options Volatility Arbitrage Strategies: Relative Differences in Historical and Implied Volatility

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Abstract: Contemporarily, volatility arbitrage strategy is one of the common strategies in quantitative finance. This study evaluates the effectiveness of volatility arbitrage strategies within the SPX options market by analyzing the relative differences between daily historical volatility and daily implied volatility. Using the Black-Scholes-Merton (BSM) model and Newton's method, the research finds that the strategy performs particularly well during periods of heightened market volatility. Specifically, in the final 60 days of the study period, when implied volatility increased significantly, the strategy achieved a maximum single trade profit of $162.5, contributing to a total gain of $326.1. These findings highlight the strategy's potential for substantial profits and demonstrate its effectiveness in capitalizing on volatility discrepancies. The study enhances theoretical models by showing how increased volatility impacts arbitrage opportunities and fills a gap in the literature regarding the empirical application of these strategies, offering valuable insights for options traders and risk managers. However, limitations related to the BSM model's assumptions and the omission of market shocks and macroeconomic factors suggest that future research should refine models, incorporate broader market factors, and use more comprehensive datasets to better capture real-world dynamics.

Keywords: Volatility arbitrage strategy, Historical volatility, Implied volatility, Relative difference.

1. Introduction

SPX options, or S&P 500 Index options, are European-style options based on the S&P 500 Index, widely used for risk management and speculation. Their importance in financial markets lies in providing hedging tools and measuring market volatility expectations [1]. Volatility arbitrage profits by exploiting the differences between historical volatility and implied volatility [2]. This strategy is widely applied in options markets due to the high sensitivity of option prices to volatility. By buying or selling options with relatively overestimated or underestimated volatility, investors can achieve profits when volatility reverts to its mean.

The objective of this study is to evaluate the effectiveness of volatility arbitrage strategies in the SPX options market by calculating the relative differences between daily historical volatility and the daily implied volatility of call option prices. Specifically, this study derives the daily relative

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difference from the daily historical and implied volatilities, then applies the corresponding option volatility arbitrage strategy based on this relative difference. One computes the final profit over the period from February 1, 2023, to January 18, 2024, to validate the feasibility and performance of the strategy. Despite extensive research on the relationship between historical volatility and implied volatility, there is limited work on systematically utilizing this relative difference for volatility arbitrage [3]. Moreover, most studies focus on static models and theoretical analysis, lacking empirical validation with real market data. This study aims to fill this gap by applying the relative difference indicator in an empirical analysis to evaluate the effectiveness of volatility arbitrage strategies under actual market conditions. The remaining portion of the paper is organized as follows: Section 2 presents the data, models, and results analysis. Section 3 discusses some limitations. Section 4 concludes the study.

2. Model Formulation

2.1. Application of 66 Trailing Days in Calculating Daily Historical Return

This study employs a volatility arbitrage trading strategy by calculating the daily historical volatility of the underlying mid price (the average price of the bid and ask prices) over a 66-trailing-day window, which corresponds to approximately three months. This period is selected to align with the 5-workday week convention commonly used in financial markets. The daily historical volatility, denoted as $\sigma_h(t)$, is computed without including the current day's data to avoid bias from future information that would not be available in real-time trading decisions. This methodology ensures that only past data is utilized, maintaining the integrity of the trading strategy by preventing the incorporation of yet-to-be-known market movements. The formula for calculating daily historical volatility is as follows:

$$\sigma_h(\mathrm{t}) = \sqrt{\frac{1}{66}\sum_{i=\mathrm{t}-67}^{i=t-1}(R_i - \bar{R})} \tag{1}$$

Here, $\sigma_h(\mathrm{t})$ denotes the daily historical return of the day t, R_i denotes the return of the ith day, $\bar{R}$ denotes the arithmetic average return for the 66 days preceding day t. Due to the inability to calculate the daily historical volatility for the first 67 days using a 66-day trailing window, this study reduced the trailing days from 66 to 2 to compute the daily historical volatility for days 4 through 67. The daily historical volatility can only be calculated starting from day 4 because the log return for the first day cannot be computed, and the volatility for day 4 requires the log returns from days 2 and 3. Although this approach may introduce some error, it allows for the necessary calculations.

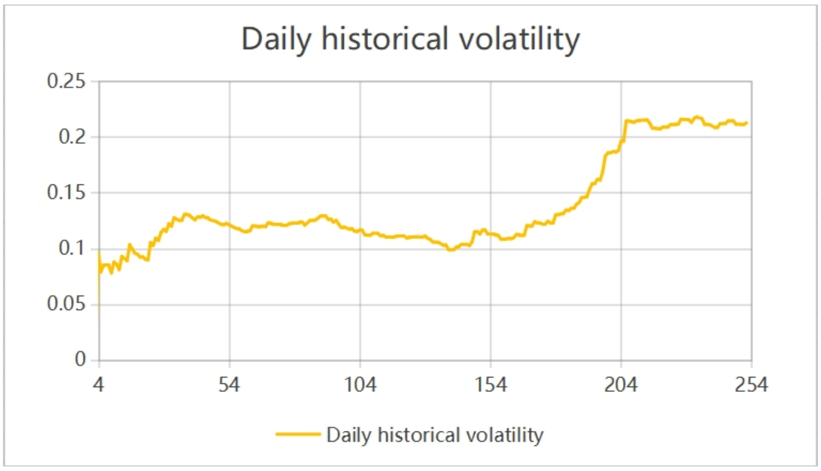

Figure 1: Daily historical return (Photo/Picture credit: Original).

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Fig. 1 illustrates the trend of daily historical volatility from Day 4 to Day 254. It is evident that from Day 4 to Day 154, the daily historical volatility remained relatively low and stable. However, starting around Day 200, there was a significant increase in historical volatility, indicating heightened market fluctuations. Overall, this rise in volatility suggests that the market became increasingly unstable in the later period.

2.2. Determination the Theoretical Value Based on Black-Scholes-Merton Model

The Black-Scholes-Merton (BSM) model, developed by Fischer Black, Myron Scholes, and Robert Merton in 1973, is one of the most widely used models for pricing European-style options [4]. The BSM model assumes that the underlying asset prices follow a geometric Brownian motion, the market is frictionless with continuous trading, volatility remains constant, and there are no arbitrage opportunities [5]. While these assumptions provide theoretical elegance and simplicity to the model, they may not hold true in real-world markets. The model provides a theoretical estimate of the price of a call or put option. The formulas for BSM model are as follows:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{2}$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{4}$$

Here, C denotes the price of a European call option under the BSM model, S_0 denotes the current price of the underlying asset(the bid or ask price of the European call option in market), K denotes the strike price of the option, T denotes the time to expiration (in years), r denotes the risk-free interest rate, σ denotes the volatility of the underlying asset, $N(\,.\,)$ denotes the cumulative distribution function of the standard normal distribution. This study primarily employs the Black-Scholes-Merton (BSM) model to calculate the theoretical option prices. The results obtained serve as the foundation for implementing Newton’s method [6].

2.3. Calculation the Daily Implied Volatility Using Newton’s Method

Newton's Method, also known as the Newton-Raphson method, is an iterative numerical technique primarily used for finding roots of a real-valued function [7]. Given a function $f(x)$ that is twice continuously differentiable, the method aims to find the value of x that satisfies $f(x) = 0$. The fundamental idea is to start with an initial guess x_0 and then refine this guess iteratively using the following formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f^{'}(x_n)} \tag{5}$$

Here, $f^{'}(x_n)$ denotes the derivative of the function at x_n. Based on the option's theoretical price calculated using the Black-Scholes-Merton (BSM) model, this study employs Newton's method to iteratively adjust the volatility until the theoretical price aligns with the market price, thereby determining the implied volatility σ_i [8].

Seen from Figure 2, one can observe that the daily implied volatilities of the bid price, ask price, and mid price (IV_ Mid is the arithmetic average price of IV_ Bid and IV_ Ask) follow a similar pattern throughout Days 4 to 251. The implied volatilities are relatively stable between Days 1 and 190, with only minor fluctuations. However, starting around Day 200, there is a noticeable increase in volatility, especially in the bid and ask prices, leading to a significant spike in all three volatility measures. This suggests that market expectations of future volatility significantly increased during the latter part of the period.

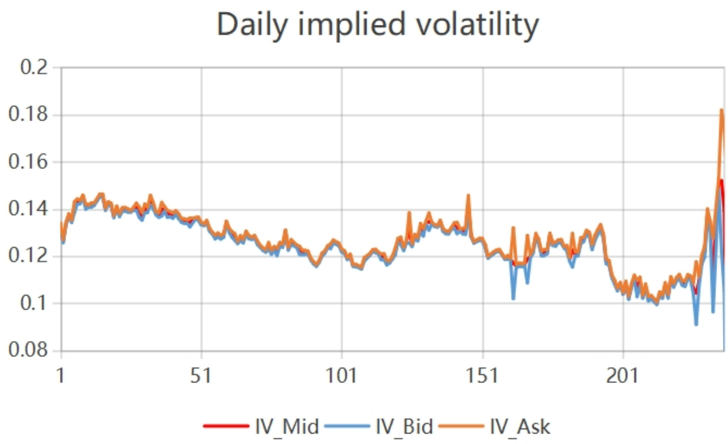

Figure 2: Daily implied return of IV_ Mid, IV_ Bid and IV_ Ask (Photo/Picture credit: Original).

2.4. Using Relative Difference to Conduct Volatility Arbitrage

The relative difference is crucial for implementing and evaluating volatility arbitrage strategies, as it helps identify potential inefficiencies between expected and realized market conditions [9]. The formula for relative difference is as follows:

$$i(t) = \frac{\sigma_i(t) - \sigma_h(t)}{\sigma_h(t)} \tag{6}$$

Here, $i(t)$ denotes the relative difference of day t. After calculating the daily relative difference, this study investigates the profitability of a volatility arbitrage strategy based on the following approach. Assuming that only one call option is traded, with no transaction costs, the strategy utilizes bid and ask prices: always buy at the ask price and sell at the bid price. The entry strategy is as follows: if the relative difference indicator $i(t) > 0.25$, a short position in the call option is taken at the end of the day; if $i(t) < 0.25$, a long position in the call option is initiated at the end of the day. The exit strategy involves closing the position at the end of the day if a non-zero position exists and the indicator has changed its sign over the last two days. Fig. 3 illustrates the daily and cumulative profit and loss of the volatility arbitrage strategy.

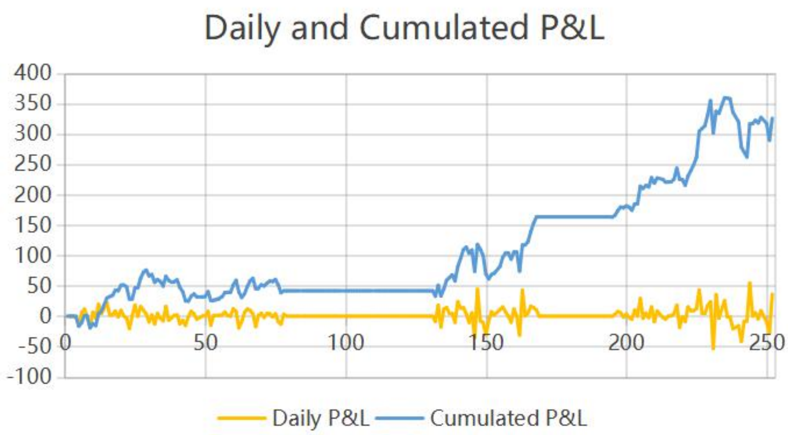

Figure 3: Daily and cumulated profit and loss (Photo/Picture credit: Original).

Based on the results shown in Table 1 and the tabulated data, the volatility arbitrage strategy implemented in this study has proven to be profitable across all three trades. The strategy involved entering short positions in the call options on dates 4 and 132, and a long position on date 195, with each position yielding positive returns upon exit. The cumulative gains from these trades, 41.9 dollars, 121.7 dollars, and 162.5 dollars respectively, demonstrate the effectiveness of the strategy in exploiting market inefficiencies. These findings suggest that the volatility arbitrage strategy, as formulated, is a viable approach for generating profits in options trading. From the Fig. 3 and Table 1, it can be observed that during the first 200 days, when daily implied volatility remained relatively stable, the strategy yielded modest profits on each occasion. However, after day 200, with a sharp increase in daily implied volatility, the strategy achieved a maximum single profit of 162.5. This indicates that the volatility arbitrage strategy can capture more substantial profit opportunities when the underlying asset's daily implied volatility significantly increases. Therefore, this volatility arbitrage strategy can serve as an effective tool in options trading, offering traders opportunities for profit.

Table 1: The entry and exit points of the strategy.

Entry Date	Exit Date	Position	Entry Price	Exit Price	P&L
4	78	Short call	121.2	79.3	41.9
132	168	Short call	168.9	47.2	121.7
195	252	Long call	8.1	170.6	162.5

3. Limitations

Due to various assumptions inherent in the BSM model, such as a frictionless market and continuous trading, as well as a constant and known risk-free interest rate, the model's predictions may not fully align with real-world conditions, leading to potential discrepancies between the results of this paper and actual market outcomes [10]. Furthermore, inaccuracies may arise from variations in trailing days when calculating daily historical volatility, as well as from neglecting market shocks, unusual volatility, and macroeconomic factors, all of which can affect the effectiveness of the proposed strategy.

4. Conclusion

To sum up, this study provides a thorough evaluation of volatility arbitrage strategies within the SPX options market by analyzing the relative differences between daily historical and implied volatilities. The analysis reveals that the volatility arbitrage strategy was particularly successful during periods of increased volatility. During the final 60 days when daily implied volatility significantly increased, the strategy achieved a maximum single trade profit of $162.5. This further demonstrates the strategy's profit potential during periods of heightened volatility. Specifically, the study found that while the strategy produced modest gains when implied volatility remained stable, it achieved significant profits during times of heightened market fluctuations. The strategy's ability to generate total gain of $326.1 demonstrates its effectiveness in exploiting volatility inefficiencies and capturing profit opportunities. The findings highlight the strategy's effectiveness in capitalizing on volatility discrepancies and underscore its potential for generating profits in varying market conditions. This study enhances the theoretical understanding of volatility arbitrage strategies. It improves existing models by showing how increased volatility affects arbitrage opportunities and addresses gaps in the literature regarding the real-world applicability of these strategies in current market conditions. The practical implications of this research are significant for options traders and risk managers. The impressive total gain of $326.1 during high market volatility underscores its potential for significant profits and offers key insights for enhancing trading and risk management practices. However, due to the BSM model's assumptions of a frictionless market, constant risk-free rate, and the omission of

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market shocks and macroeconomic factors, these results may not fully reflect real-world conditions, leading to potential discrepancies. Future research should aim to enhance volatility arbitrage strategies by using more refined models, incorporating additional market factors, and employing comprehensive datasets to better capture real-world dynamics and uncertainties.

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