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A Study of Option Pricing Models with Market Price Adjustments: Empirical Analysis Beyond the Black-Scholes Model

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Abstract: In 1973, Fischer Black and Myron Scholes unveiled the Black-Scholes option pricing model, a groundbreaking contribution that profoundly influenced the domain of option pricing theory. The introduction of the Black-Scholes pricing formula has garnered substantial acclaim across both academic and industrial spheres, leading to its widespread dissemination and application. This formula not only underscores its vital significance but also exemplifies its unique position as a cornerstone of financial theory, reshaping how options are valued and traded in markets worldwide. However, in the real financial market, the Black-Scholes option pricing model has a serious deviation from empirical research in option pricing, which reduces its practicality and accuracy. This paper first briefly introduces the basic knowledge of options, covering both option-related concepts and option pricing theories, gives the definition of Black-Scholes option pricing deviation, and explains the volatility smile theory in detail. Starting from the probability of positive returns and the beliefs of traders, the probability of call option returns is obtained from historical trading data, and then decisions are made from these probabilities to overcome the deviations caused by Black-Scholes European option pricing and find an option pricing model that is more consistent with the market price of options. Through comprehensive simulation studies utilizing synthesized data, we conduct rigorous empirical tests to compare this theoretical model with the Black-Scholes option pricing model. The market prices of call options are derived from investor sentiments, allowing us to validate all three types of deviations from the Black-Scholes pricing formula within this numerical framework. The results reveal that the growth rates of stock returns can effectively serve as a substitute for the volatility smile, thereby facilitating their exclusion from risk-neutral analyses. These insights significantly enhance our understanding of option pricing dynamics in real-world scenarios.

Keywords: Black-Scholes model, trader beliefs, probability of positive returns, geometric Brownian motion.

1. Introduction

The options market has a very long history, with the over-the-counter options market being the first to emerge, dating back several centuries. In the 1630s, the well-known “Dutch tulip fever” occurred. In the early 1920s, the London Stock Exchange began to trade stock options. As time goes by, the

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world's economy is developing rapidly, and politics is also undergoing tremendous changes. Foreign exchange and interest rate derivatives are beginning to emerge and continue to play new roles. The modern on-exchange options market has seen significant progress and expansion. After entering the new century, China has carried out a comprehensive and thorough cleanup and rectification of the futures market, and various trading products have also emerged. The momentum of vigorous development. So far, the research on options and futures has gradually increased and become a hot topic of concern.

This article provides a comprehensive overview of option pricing theory prior to the advent of the Black-Scholes framework, followed by an in-depth examination of Black-Scholes option pricing theory, its subsequent extensions, and the various deviations associated with it. Initially, the concept of deviation within the context of the Black-Scholes option pricing model is defined, followed by a detailed exploration of the volatility smile phenomenon. This discussion elucidates how these foundational elements have shaped contemporary approaches to option pricing and the ongoing evolution of the theory.

The pricing of options starts with the probability of positive returns and the beliefs of traders, and the probability of return of call options is obtained from historical trading data. Then, decisions are made based on these probabilities, overcoming the deviations brought by the Black-Scholes European option pricing formula, and obtaining an option pricing model that is more consistent with the market price of options, which has stronger practicality.

2. The Definition of the Black-Scholes Model

In 1952, Harry Markowitz published *Theory of Portfolio Selection*. In 1973, Black and Scholes published their paper and obtained the famous Black-Scholes (BS) option pricing formula, which was derived by applying the theory of stochastic differential formula.

The BS option pricing formula can only be established under certain premises and assumptions.

The “ideal conditions” of the Black-Scholes model are as follows:

(1) The option is European and cannot have the right to exercise the option at any time before the expiration date as enjoyed by American options;

(2) During the validity period of the option, the risk-free interest rate and the financial asset return variable are relatively constant, which means that during the validity period of the option, the risk-free rate of return will always be a constant;

(3) The value of the underlying asset is random and the underlying asset does not pay dividends and bonuses;

(4) The market is free of taxes and transaction costs, and short selling is allowed;

(5) The return on financial assets follows a log-normal distribution.

The strength of the BS option pricing model lies in the fact that the variable in the formula is a model that can be “observed.” This allows the model to be extended to price other financial derivatives as well [1]

The assumptions about the underlying assets are:

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{1}$$

Where μ is the instantaneous expected return on common stock, σ is the instantaneous standard deviation of returns.

2.1. Call option model

The price of an option is represented by $c(S, t)$, which is determined by the stock price and expiration date of the uncertainty source. Assuming that in a time interval Δt, a combination of options,

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underlying assets and risk-free securities can be formed. The weight of the combination can be determined according to the risks of market-related aspects.

According to the BS option pricing model, a hedging portfolio can be constructed as $[\partial c(S,t)/\partial S]^{-1}$. First, it is assumed that the change in stock price is ΔS, and the corresponding change in option price would then be $[\partial c(S,t)/\partial S]\Delta S$. Therefore, the change in the value of the long position in the stock can be approximately offset by the change in the price of option $[\partial c(S,t)/\partial S]^{-1}$.

The above hedging process can be carried out continuously, and eventually the returns of the hedge portfolio will have nothing to do with the changes in the underlying assets. By then, the returns on the hedge positions will have reached a relatively stable state.

2.2. Call-Put Parity Relationship

The call-put parity relationship is derived as follows:

First, a portfolio A is taken, which consists of a call option with an expiration date of t^* and a discount bond that pays on the option expiration date.

From the above two combinations of values expressed on the expiration date, it is clear that they have the same value on the expiration date. Therefore, they must have the same initial value at time t, otherwise there will be arbitrage opportunities[2].

3. Improvements to the Black-Scholes model

The Black-Scholes option pricing model is of far-reaching significance. It not only brings convenience to investors, but also facilitates scholars who conduct research in this area. In such a long process of option pricing research, the conclusion of this theory has an epoch-making role, and it has made an indelible contribution to the continuous development and rapid progress of the entire financial derivative securities pricing theory. However, it is also important to recognize the limitations of the Black-Scholes formula. Research under various ideal conditions will inevitably weaken its practicality. Since then, many scholars have derived new option pricing formulas through various explorations and analyses, relaxing conditions, setting new assumptions, etc., so as to truly integrate theory with practice and achieve more efficient, accurate and practical results.

Since the Black-Scholes model is predominantly designed for pricing European options, it does not fully account for the unique features of American options, which grant holders the flexibility to exercise their options at any point during their validity period. To address this distinction, Fischer Black once proposed an innovative approach akin to that used for European options. Specifically, he suggested employing the Black-Scholes formula to calculate the value of the European option both at expiration and prior to expiration, ultimately selecting the greater of the two values as the price of the American option. Subsequently, this concept has been further explored by other scholars, who have conducted research leading to additional insights and results in this area. For example, Bakshi, G., Kapadia, N., and Madan, D. gave a way to deal with the pricing problem of American options and made very good progress [3].

The Black-Scholes option pricing model is predicated on the assumption of idealized market conditions, notably that stock price movements follow a geometric Brownian motion. This implies that stock price changes occur as a continuous random process. However, real-world scenarios often deviate from this assumption, prompting extensive research by numerous scholars in the field. In a significant advancement, Robert Merton introduced the jump diffusion process into the realm of option pricing, effectively broadening the stock price movement model from a purely continuous geometric Brownian motion to a more nuanced discontinuous jump diffusion process. This extension acknowledges the inherent complexities of market dynamics and enhances the model's applicability to actual trading environments.

In 1976, Cox and Ross proposed the risk-neutral pricing theory. In 1979, Harrison and Kreps proposed the concept of equivalent martingale, that is, by using the martingale method to characterize an arbitrage-free market and an incomplete market respectively. In 1981, Harrison and Pliska built on these findings to develop further conclusions.

For modern financial theory, the introduction of martingale theory is very important. Assuming that the financial market is an efficient market, the price of the underlying asset is equivalent to a martingale stochastic process, as proposed by Carr, P. and Wu, L. [4]. The martingale method they advocated can achieve the purpose of studying the pricing problem of undetermined equity by utilizing the equivalent martingale measure. Ultimately, the results obtained can not only reveal the operating laws of the financial market, but also propose a set of effective algorithms, facilitating the resolution of complex undetermined equity pricing and risk management problems.

Heston, S. L.[5], Cugnon, J. and Vandermeulen, J.[6], Barndorff-Nielsen, O. E. and Shephard, N.[7] employed the underlying assets with jumps to price the options, but the values obtained in this way are still not equal to the real values. In the real market, there are other sources of deviations, such as random dividends on stock returns, transaction taxes, and transaction costs. These factors may also cause a series of deviations between the price of options obtained by the Black-Scholes option pricing model and the real value. At the same time, traders will predict the expected value of volatility and the mean dividend as the deviation of the volatility smile [8].

Given the limitations of traders in knowledge and foresight, heterogeneous beliefs, and learning mechanisms, the different impacts on option pricing have caused the BS option pricing formula to produce a series of deviations in empirical research. These problems have not been truly solved in previous literature, limiting improvements in the accuracy and practicality of option pricing. This paper starts from these aspects and then explores the deviation of the BS option pricing formula in empirical research. Considering that traders' beliefs are based on the probability of positive returns, traders can obtain the probability of return of call options from historical trading data and then make decisions based on these probabilities. Conduct empirical analysis to provide a reference for the reasonable avoidance of deviations in the future application of the Black-Scholes option pricing formula[9].

Investors can learn about the probability of positive returns of stocks through historical data. In other words, they can calculate the probability of positive returns under specific market conditions. In order to obtain a certain profit, an option writer aims to sell an option whose probability of positive returns is below their expected level, while an option holder seeks to buy an option with a probability of positive returns exceeding their expectations [10].

Given the limitations of traders in knowledge and foresight, heterogeneous beliefs and learning mechanisms, the different impacts on option pricing have led to various deviations in the Black-Scholes option pricing formula in actual operations [11]. Starting from the root and focusing on solving practical problems, the deviations of positive return probability, investor beliefs, and the BS option pricing formula are demonstrated thus to obtain the revised model.

From the modified model, it is evident that these biases can be mitigated. Considering that the trader's belief is based on the probability of positive returns, the trader obtains the return probability of the call option from the historical trading data and then makes decisions from these probabilities. When the stock price follows the geometric Brownian motion and the option price is generated by the trader's belief, the deviation of the Black-Scholes option pricing formula can be observed from different market conditions [12]. In the numerical study, by using the improved model, the price of the option obtained by the model is consistent with the market price of the option, thereby improving the accuracy and practicality of option pricing [13].

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4. Conclusion

When the market price of a call option is generated from the traders' beliefs, in other words, based on the traders' limitations in knowledge and foresight, heterogeneous beliefs and lack of learning mechanisms, and summarizing and applying relevant experience from historical data, these will bring certain deviations to our reasonable option pricing. This is more intuitively illustrated in the two figures above, which truly illustrate the deviation between the Black-Scholes option pricing formula and the market price in actual operation.

The preceding analysis reveals that the deviations observed in the Black-Scholes option pricing model indicate a consistent pattern: for at-the-money options, the model often values them higher than their corresponding market prices, whereas out-of-the-money options tend to be overvalued. Conversely, in-the-money options generally show model valuations that fall below their market prices. These deviations are closely associated with the option's expiration date, with model valuations approaching expiration often being lower than the prevailing market prices. Furthermore, the extent of these deviations is influenced by volatility; specifically, when the variance estimate of the underlying asset is elevated, the model's valuation frequently exceeds the market price of the option. This nuanced understanding underscores the complexities inherent in option pricing dynamics.

To minimize these deviations and provide a more practical model for empirical research, it is important to account for these factors. By balancing the relevant characteristics of options and conducting empirical tests from multiple perspectives, the efficiency and practicality of the simulation model can be validated.

Furthermore, an additional finding reveals that the growth rate of stock returns can replace the volatility smile and may be excluded from the risk-neutral analysis. This conclusion has a very positive effect on reducing the deviation of option pricing and provides valuable insights for empirical research.

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