

Relationship Between Product Sales and Advertising Investment Based on Linear Regression Analysis and Discussion on the Best Advertising Strategy

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Abstract: In today's competitive business landscape, companies seek to optimize marketing strategies, with advertising investment significantly impacting product sales. However, identifying the most effective advertising plan is complex. This paper examines the relationship between product sales and advertising investment using linear regression analysis. By collecting data and establishing a linear regression model, it aims to identify the optimal advertising strategy for maximizing product sales. This paper mainly discusses the relationship between product sales and advertising investment by using linear regression analysis method. Through the data collection, the linear regression model establishment and analysis, it aims to determine the best advertising scheme to achieve the maximum sales of products. The research thoroughly details the principles of linear regression analysis, covering data collection, model development, and validation. It discusses advertising strategies based on model results, using case studies for analysis. The text presents relevant data and variables, with visuals to demonstrate practical applications of linear regression equations and their derivations, laying a foundation for enterprise advertising recommendations.

Keywords: Linear regression analysis, Product sales, Advertising investment, Advertising strategy, Electronic enterprise.

1. Introduction

In the contemporary marketplace, enterprises allocate substantial financial resources to advertising with the aim of boosting product sales. However, the key challenge lies not in the quantity of advertising but in devising the most effective advertising strategy to maximize sales. Linear regression analysis, a straightforward and transparent statistical technique, effectively elucidates the relationship between product sales and advertising expenditure, offering a solid foundation for enterprises to develop advertising strategies. This study delves into the impact of various factors—such as advertising channels, frequencies, and durations—within the scope of advertising investment on product sales volume and profit margins through linear regression. The objective is to construct a precise linear regression model grounded in comprehensive data collection and analysis. This model serves to simulate and forecast sales outcomes across different advertising scenarios,

thereby providing a scientific basis for enterprises to optimize advertising strategies, allocate resources judiciously, and ultimately enhance overall marketing efficacy and economic returns.

2. Principles of Linear Regression Analysis

2.1. Basic Concept of Linear Regression

Linear regression is a quantitative analytical technique employed to determine a linear association between a dependent variable and one or more independent variables.[1]. In analyzing the relationship between product sales and advertising investment, product sales serve as the dependent variable while advertising investment is the independent variable. A linear regression model can predict product sales based on varying advertising investments.

2.2. Form of Linear Regression Model

The form of the simple linear regression model is $y = \beta_0 + \beta_1 x + \varepsilon$, where y represents product sales, x represents advertising investment, β_0 is intercept, β_1 is slope representing the change in product sales when advertising investment increases by one unit, and ε is the random error.

2.3. Parameter Estimation of Linear Regression

Parameter estimation is realized by least square method. The objective of least squares analysis is to reduce the aggregate of squared deviations between the observed values and the predicted values produced by the model.[2]. By analyzing the equations, the estimates of the sum of parameters can be obtained.

3. Data Collection and Processing

3.1. Data Source

In order to analyze the linear regression relationship between product sales and advertising spend, the relevant data needs to be searched. Data can come from corporate records, reports from market research companies, and industry databases. When gathering data, it is imperative to guarantee that the information is precise, dependable, and comprehensive.[2].

Using an electronic product company as a case study, it analyzed the monthly advertising expenditure (in 10,000 yuan) and product sales volume (in thousands) over the past three years. It also gathered data on product pricing (in yuan), competitors' advertising investments (in 10,000 yuan), seasonal factors (1-4 for spring to winter), and other relevant control variables. The specific data is presented in the following table:

Table 1: Specific data

Month	Advertising expenditure	Product sales volume	Product price	Competitors' advertising investment	Seasonal factors
1	20	15	2000	15	1
2	22	16	2100	16	1
3	25	18	2200	18	2
...

3.2. Data Variable Selection and Processing

Besides advertising expenditure as an independent variable, other factors that may influence product sales can also be considered in this category, such as product pricing, competitors' advertising spend, and market demand, among others. These factors can be used as control variables to control and improve the accuracy of the model in regression analysis.

When processing collected company data, remove the one when it is abnormal[3]. Standardized treatment of variables such as advertising investment and product sales aims to make them comparable.

4. Establishment and Verification of Linear Regression Model

4.1. Model Building

It can use the collected data and statistical software to build linear regression models of product sales and advertising spend. These can subsequently be utilized to create scatterplots and compute the correlation coefficient, along with other methodologies to ascertain the presence of a linear relationship between product sales and advertising expenditures. A linear regression model is developed using data from selected electronic product companies, analyzing the relationship between product sales and variables such as advertising input, product price, competitor advertising, and seasonal factors.

4.2. Model Parameter Estimation

The least square method uses β_0 and β_1 to estimate the sum of parameters of the linear regression model[3]. The forecasted output of the model is derived from the parameter estimates, and the actual observed value is juxtaposed with the predicted output to assess the model's goodness of fit.[4].

Assume product sales are y , advertising input is x_1 , product price is x_2 , competitor advertising input is x_3 , and seasonal factor is x_4 , then the linear regression model can be expressed as[4]:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon \quad (1)$$

The process of solving parameter estimates by least square method is as follows:

- Define the Sum of Squared Error: $S(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)$

Where n is the number of samples, y_i is the actual value of product sales in the i^{th} sample, X_1 , X_2 , X_3 and X_4 are respectively the values of advertising input, product price, competitor advertising input and seasonal factors in the i^{th} sample.

- Take the partial derivative of $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ respectively, and make the partial derivative zero
- 3) Solve the above equations and get the parameter estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4$ [4].

4.3. Model Checking

The linear regression model's test mainly includes fit test, significance test, and residual analysis [5]. the first one evaluates the degree of fit of the model by calculating the determination coefficient R^2 . The better the fit of the graph, the closer R^2 is to 1[5].

where \hat{y}_i is the model prediction value of the x^{th} sample and \bar{y} is the average product sales of the sample volume .

The second one includes t test and F test, testing whether a single parameter is significant is not a zero, and testing whether the whole model is significant, respectively.

where $\hat{\beta}_j$ is the parameter estimate and $SE\hat{\beta}_j$ is the standard error of the parameter estimate[5].

where SSR is the sum of regression squares, SSE is the sum of residuals squares, k is the number of independent variables, and n is the number of samples[5].

The third test analyzes the residual of the model to check whether it satisfies the assumptions of linear regression, such as independence, normality and equal variance. If the residual does not meet the assumed conditions, the model needs to be revised[5].

5. Result Analysis and Discussion

5.1. Interpretation of Model Results

The correlation between product sales and advertising expenditures can be assessed using parameter estimation in a linear regression model. A positive slope indicates a direct relationship between advertising spend and sales, while a negative slope suggests an inverse relationship. The intercept reflects sales volume when advertising spend is zero.

Taking the results of the electronic products company model as an example, assume that the resulting parameter estimates are $\hat{\beta}_0 = 10$, $\hat{\beta}_1 = 0.5$, $\hat{\beta}_2 = -0.2$, $\hat{\beta}_3 = -0.3$, $\hat{\beta}_4 = 1$ (seasonal factors are based on spring, and the difference between other seasons and spring). Then the linear regression equation is:

$$y = 10 + 0.5x_1 - 0.2x_2 - 0.3x_3 + x_4 \quad (2)$$

An increase of 10,000 yuan in advertising is expected to boost product sales by about 500 units, while a 1 yuan price hike may reduce sales by approximately 200 units. If competitors raise their advertising by 10,000 yuan, a decline of around 300 units in sales is anticipated. Additionally, autumn and winter product sales are projected to surpass spring sales by 1,000 and 2,000 units, respectively, with a 2-unit variance between winter and spring.

5.2. Analysis of Influencing Factors

In addition to the allocation of advertising resources, it is imperative to evaluate the impact of additional control variables on product sales.[6]. Which factors have a greater impact on product sales and which factors have a lesser impact can be compared by the parameter estimates and significance levels of different variables.

In an electronics firm, parameter estimations show that marketing spending boosts product sales, while competitor advertising negatively impacts them. Seasonal fluctuations also significantly influence sales performance. A significant result indicates a notable effect of advertising investment on product sales.

5.3. Model Limitation Analysis

While linear regression can illustrate the relationship between product sales and advertising investment, it has limitations. It assumes a linear relationship, which may not reflect reality, and relies on specific assumptions such as independence, normality, and homoscedasticity.[5], but these assumptions may not be valid in the actual data.

In addressing the limitations of the electronic product company's linear regression model, nonlinear relationships can be accommodated by incorporating quadratic or interaction terms. If residuals violate assumptions, data transformation or alternative statistical methods can be employed for correction.

5.4. Graphic Analysis

The correlation between product sales and advertising investment can be effectively illustrated through scatter plots and trend lines. We will present a scatter plot linking sales to advertising expenditure, enhanced with a trend line for clarity. Furthermore, we will examine seasonal effects on this relationship by generating distinct scatter plots for each season to enable comparative analysis.

Based on the Specific data, linear equation can be modeled as below:

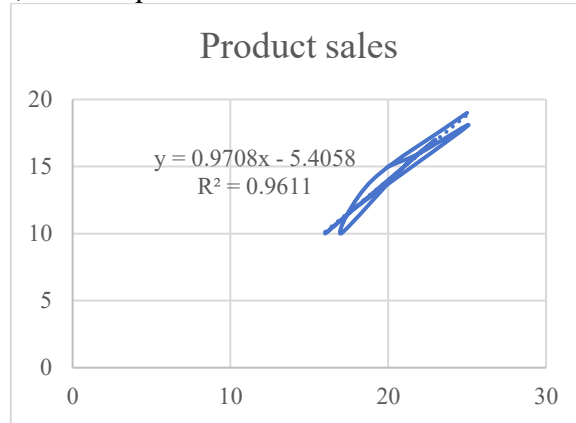


Figure1: Product sales

Figure 1 illustrates the linear equation $y = 0.9708x - 5.4058$, which describes the relationship between advertising investment and product sales. The slope of 0.9708 indicates that a 1-unit increase in advertising leads to an average increase of 0.9708 units in sales. The intercept of -5.4058 suggests that with no advertising investment, sales would be -5.4058. Additionally, the coefficient of determination is 0.9611, demonstrating a strong correlation, as approximately 96.11% of the variation in product sales can be attributed to advertising expenditure.

6. Formulation of the Best Advertising Strategy

6.1. Advertising Investment Decision Based on Model Results

The predicted value of product sales under different levels of advertising investment can be determined by the results of linear regression model. If a corporation aims to optimize product sales, it can determine the appropriate level of advertising expenditure that aligns with its specific objectives and financial constraints.[7]. An enterprise can either boost advertising to enhance product sales or, with a limited budget, reduce advertising while ensuring minimum sales levels.

In the case of an electronics firm, advertising investment decisions should rely on model outcomes. To achieve specific sales targets, the company can calculate required advertising spend using linear regression.

6.2. Optimization of Advertising Strategy

Businesses aim to enhance product sales by not only elevating advertising expenditure but also by refining their advertising strategies. This includes selecting suitable advertising platforms, crafting effective advertising messages, and timing advertisements appropriately. The effect of different advertising strategies on product sales can be selected according to the results of linear regression model. Electronic product companies should select appropriate advertising media for their target audience, create compelling content, and boost investment during peak seasons.

6.3. Other Factors

In the formulation of advertising strategy, enterprises should also consider other factors, such as product price, product quality and the strategy of competitors. These elements must be incorporated into the advertising framework; otherwise, they could considerably affect product sales.[7].

When analyzing the advertising strategy of electronic product companies, other factors should be considered comprehensively. I think the following aspects should be taken into account: first, keep an eye on market trends and adjust product prices in time to improve competitiveness; second, improve product quality to increase customer satisfaction; third, pay attention to competitors' advertising strategies and respond to them in time.

7. Conclusion

This study analyzes the relationship between product sales and advertising expenditure through linear regression to formulate effective advertising strategies. Findings reveal a significant linear correlation, suggesting that higher advertising investment can enhance sales, within a certain limit. When developing strategies to optimize sales and profitability, it is crucial to consider factors such as budget, media channels, content, and timing. Despite limitations, linear regression is vital for analyzing sales-ad spending relationships. Continuous data collection and model refinement are recommended to enhance advertising effectiveness and competitiveness. Additionally, leveraging big data analytics can yield deeper insights into market trends and consumer behavior, refining strategies further. Utilizing social media analytics for consumer interest and real-time metrics for performance evaluation enables timely strategic adjustments. The relevance of linear regression models differs across sectors, requiring customized adjustments. In conclusion, while linear regression provides a robust foundation for advertising strategy, effective execution demands attention to various influencing factors for best outcomes. Future studies should incorporate additional variables such as consumer behavior and market dynamics, possibly integrating other statistical methods and AI technologies to enhance efficiency.

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