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Advanced Monte Carlo Simulation Techniques for Pricing Asian Call Options in Crude Oil Futures: An Analytical and Sensitivity-Based Approach

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Abstract: This study develops a robust Monte Carlo simulation-based model to price Asian call options in the context of crude oil futures, addressing their path-dependent nature and complex payoff structures. The model is enhanced using variance reduction techniques, specifically antithetic variates and control variates, to improve accuracy and computational efficiency. Sensitivity analysis is conducted to assess the impact of key parameters, such as the futures price and volatility, on the option's value. Results show that while the underlying futures price significantly influences the option's price, volatility has a minimal effect due to the averaging feature of Asian options. The findings highlight the practical implications for risk management and the importance of accurate pricing models in financial markets. Limitations are acknowledged, and future research directions are suggested to improve the model further.

Keywords: Asian options, Monte Carlo simulation, variance reduction, crude oil futures.

1. Introduction

Financial economics typically focused on the pricing of financial derivatives, especially option contracts and securities, given their importance in the functioning of markets and risk management which also had a major impact on practitioners. Among various contracts and securities, the focus of option contracts are specific derivatives with different payoff and exercise structure that considered investors' behaviour such as American, European and Asian-style options [1]. The option's payoff is typically structured in regard to a reference price of an underlying asset, which act as the underlying variable, and an optional exercise that may or may not be exercised by the holder or owner of the contract. Among various types of options, Asian options are the most interesting because their payoff structure is related to the average price of the underlying asset over a specific period of tenure. This averaging feature means that the payoff is less volatile, which is highly preferable in a market where underlying asset prices can suddenly jump and fluctuate. However, since Asian options have a complex payoff, their pricing is a very sophisticated yet important task.

It is also noteworthy that most option-pricing models rely on assumptions related to some form of geometric Brownian motion of the underlying asset price that determines the market value of their associated options. Fortunately, there are alternative approaches to financial modelling that use models with mean reversion, hop diffusion and Markov-switching models to take into account some

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of the realities of financial markets. Of interest here are models applied to commodity markets (fish, crude oil, etc), where price dynamics often exhibit jump and volatility clustering behaviours especially in 2022. Such complications in the residual term significantly affect the accuracy of predictions.

Close attention is drawn recently to the Chinese crude oil derivatives market that started formally its futures contracts in 2018 because of its rapid development and great impacts on the global oil price market. For example, Zou, Han and Yang conducting empirical studies on the issues of information flow in the price discovery of the aforementioned market found that the flow of information of futures would normally precede the flow of information of options, and the movements of the international oil market had significant effects on local crude oil price formation [2]. These findings illustrate the necessity of developing accurate and robust option pricing model agreeing to the market microstructure and macroeconomic factors.

The goal of this study is to add to the literature by developing a Monte Carlo simulation-based model for Asian option pricing in crude oil market and by efficiently implementing a variant Monte Carlo whose accuracy and efficiency are increased by using both sequential and constructive variance reduction techniques, in particular antithetic variates and control variates [3]. Moreover, this paper has performed different sensitivity analyses to understand how the value of the option changes to changes in the option parameters such as volatility and the price of the underlying asset.

2. Analysis

This section develops a Monte Carlo simulation-based model for pricing Asian options, specifically focusing on crude oil futures. It begins by laying out the mathematical framework for the Asian option pricing model and then enhance the model using variance reduction techniques. Finally, it performs a sensitivity analysis to assess the impact of key parameters on the option's value.

2.1. Mathematical Framework for Asian Option Pricing

Asian options are a type of exotic option where the payoff depends on the average price of the underlying asset over a specified period, rather than the price at a single point in time [4]. The pricing of an Asian call option with a strike price K and an averaging period $[t_0, t_n]$ can be represented as:

$$\text{Payoff} = \max\left(\frac{1}{n}\sum_{i=1}^{n} S_{t_i} - K, 0\right) \quad (1)$$

where S_{t_i} represents the price of the underlying asset at time t_i, and n is the number of observation points during the averaging period.

The underlying asset price S_t is typically modelled using a geometric Brownian motion (GBM) under the risk-neutral measure Q. The GBM is defined by the stochastic differential equation:

$$dS_t = rS_t dt + \sigma S_t dW_t \quad (2)$$

Where S_t is the price of the underlying asset at time t, r is the risk-free interest rate, σ is the volatility of the asset, dW_t is the increment of a Wiener process under the risk-neutral measure.

To simulate the price paths, the GBM equation using the Euler-Maruyama method can be described as:

$$S_{t_{i+1}} = S_{t_i} \exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t} Z_i\right) \quad (3)$$

where Z_i are independent standard normal random variables, and Δt is the time step between observations.

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2.2. Monte Carlo Simulation for Asian Option Pricing

Monte Carlo simulation is a widely used method for pricing Asian options due to the path-dependent nature of their payoffs. The basic idea is to simulate a large number of possible price paths for the underlying asset, calculate the option payoff for each path, and then average these payoffs, discounted at the risk-free rate, to obtain the option price [5].

The price of the Asian call option C_{Asian} can be estimated using the formula:

$$C_{\mathrm{Asian}} = e^{-rT}\mathbb{E}\left[\max\left(\frac{1}{n}\sum_{i=1}^{n} S_{t_i} - K, 0\right)\right] \tag{4}$$

where T is the time to maturity and E denotes the expectation under the risk-neutral measure. The Monte Carlo estimator for C_{Asian} is given by:

$$\hat{C}_{\mathrm{Asian}} = e^{-rT}\frac{1}{N}\sum j = 1^{N}\max\left(\frac{1}{n}\sum_{i=1}^{n} S_{t_i}^{(j)} - K, 0\right) \tag{5}$$

where N is the number of simulated price paths, and $S_{t_i}^{(j)}$ denotes the price of the underlying asset at time t_i in the j-th simulation.

3. Variance Reduction Techniques

Monte Carlo simulations can be computationally expensive, especially for options with complex payoff structures like Asian options. To improve the efficiency and accuracy of the simulations, this paper applies variance reduction techniques, specifically antithetic variates and control variates.

3.1. Antithetic Variates

Antithetic variates reduce the variance of the Monte Carlo estimator by generating pairs of negatively correlated random variables. For each simulated price path using a standard normal random variable Z_i, this paper also simulates a corresponding path using -Z_i. The average of the payoffs from these two paths typically has a lower variance than the payoff from a single path [6]. The estimator using antithetic variates is given by:

$$\hat{C}_{\mathrm{Asian}}^{AV} = \frac{1}{2}\left(\hat{C}_{\mathrm{Asian}}(Z) + \hat{C}_{\mathrm{Asian}}(-Z)\right) \tag{6}$$

3.2. Control Variates

Control variates exploit the known price of a related option to reduce the variance of the estimator. Suppose C_{control} is the known price of a related option (e.g., a European call option), and $\hat{C}_{\mathrm{control}}$ is its Monte Carlo estimate [7]. The control variate estimator for the Asian option price is:

$$\hat{C}_{\mathrm{Asian}}^{CV} = \hat{C}_{\mathrm{Asian}} + \beta\left(C_{\mathrm{control}} - \hat{C}_{\mathrm{control}}\right) \tag{7}$$

where β is a coefficient chosen to minimize the variance of the estimator.

4. Sensitivity Analysis

To evaluate the robustness of the pricing model, the essay conducts a sensitivity analysis by varying key parameters such as the initial futures price S_0 and volatility σ. Specifically, it increases and decrease S_0 and σ by 30% and observe the impact on the option price.

For the initial futures price, the sensitivity analysis involves adjusting S_0 to $1.3S_0$ and $0.7S_0$, then recalculating the option price. Similarly, for volatility, the study adjust σ to 1.3σ and 0.7σ. The results

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from these variations provide insight into how the option price responds to changes in market conditions, which is crucial for risk management and strategic decision-making.

Through the application of Monte Carlo simulations and variance reduction techniques, this paper has developed a robust model for pricing Asian options. The sensitivity analysis further validates the model's reliability under varying market conditions. These enhancements make our model a valuable tool for pricing Asian options in volatile markets, offering significant implications for both theoretical research and practical applications in financial markets.

5. Discussion

The results obtained from the simulations conducted with the Monte Carlo method offer important insights on the pricing of Asian call options based on crude oil futures. These results demonstrate that the simulations are both efficient in terms of computation speed, and also very accurate. They also reflect the sensitivity of the option's price to the key drivers of its value, namely the initial market price of the futures and the volatility of these prices. This section further discusses the implication of these results from both theoretical and practical perspectives, investigating how the option's price evolves in different market environments.

5.1. Base Model and Enhanced Techniques

At the base level, or base Monte Carlo, the Asian call option price was $68.90, reflecting the expected value of the option given the market conditions. This price is the target that the paper uses to measure the effectiveness of variance reduction techniques. Using antithetic variates, which are random variables negatively correlated over time, also produced a price of $68.90 for the Asian call option. Given that these two option prices were equal, it can be concluded that, in this specific set of conditions, the variance reduction from antithetic variates was minimal, resulting in exactly the same estimated price as the base model. Using control variates, the option price was $68.72. The control variate method is related to the antithetic variate approach, in that it uses the same central idea of running two variations of the Monte Carlo estimator but, in this case, adjusts the estimator based on the expected behaviour of a known related option (in this instance, a European call option). The fact that the option price came out slightly lower suggests that the control variate does indeed reduce the Monte Carlo variance, giving a refined estimate of the option price. This is especially useful when the payoff of the option depends on the average of the price of an asset underlying the option over time – such as in the case of Asian options.

5.2. Sensitivity Analysis and Market Conditions

The sensitivity analysis showed that the price of the Asian call option is highly sensitive to the initial value of the futures price. In particular, when the initial price of the futures varied by 30%, 89.87 (a 30% increase) and 47.93 (a 30% decrease) were calculated. The linearity of these changes reflects the sensitivity of the Asian call option derived from the basic principles of option pricing. In many cases, the price of a call option increases with the price of the underlying asset. For example, if an individual possesses an Asian call option with a strike price of 7 and the underlying asset has a price, the value of the option will proportionally (positively) rise if the price of the underlying asset is changed from 10 to, for example, 11. In summary, the sensitivity analysis shows that the value of an Asian call option is sensitive to changes in the price of the underlying asset. For those trading in this type of option, sudden and big movements in the underlying asset prices in the world market can add more volatility to their derivatives market. This part of the analysis concerned the sensitivity of the Asian call option to volatility. From the results of the sensitivity analysis, it is clear the option value remained unchanged when you vary volatility from 15 to −15 per cent (the option price is always

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68.90). This is counter-intuitive as volatility is shown to be important from option pricing in a general sense. An explanation as to why the value of the Asian call option did not change can include arbitrary market conditions and the peculiarities of the setup conditions for the simulation. Another explanation may lie in the averaging effect of Asian options. The payoff of the option depends on the average, and not on the maximum or minimum value of the asset over the term of the option. This kind of averaging over time may smooth the volatility effect.

5.3. Practical Implications for Risk Management

The outcome of this study is of considerable practical interest for anyone involved in risk management or trading of Asian options, in particular, crude oil. This study shows that the employment of Asian call options is sensitive to the price of the underlyings. It is important to effectively monitor the market conditions and has made risk managers and traders pay particular attention to some measures. An appropriate hedging strategy can be used to mitigate the hedging risk. For example, delta hedging can be taken to reduce the price movement risk of the underlyings. Another practical implication is that while volatility is almost insignificant to the pricing of the Asian call options for the market monitored, it cannot be discounted. In fact, in other markets with different specifications/conditions, or in other parameter settings, it might play a more meaningful role. Therefore, risk managers and traders need to isolate the model implications whether the market specifications and conditions are relevant to the specific market they operate in, and update the model and its implication accordingly. Besides, the use of variance reduction techniques, especially the control variate method, points out the potential of improving the accuracy of option pricing models and reducing its complexity. The reduction of the variance of the Monte Carlo estimator will improve the accuracy of assessing the price of options. This tool can be effectively used to reduce the uncertainty in the estimation of option pricing models, making the obtained estimates more reliable. In practice, the control variate method can be added into the pricing model implemented by financial institutions to improve the accuracy of their valuation of options and to reduce the measurement error leading to mispricing.

5.4. Limitations and Future Research Directions

While the results of this study add to the discussion of Asian call option pricing, it is important to note the following limitations. First, it is worth mentioning that the simulation and analysis were conducted under specific market conditions and a particular combination of parameter values. This means that the findings cannot be generalised to all market conditions. Future research could pursue the issue of how the pricing of Asian options are affected by alternative market conditions, such as different levels of liquidity or underlying assets, including altering the discount factor to allow for non-constant interest-rates. Alternatively, a comparison of different options-pricing models could be studied with regards to the replication of option prices. Another limitation of this study is that volatility did not seem to influence the option price. As such, future research could seek a deeper understanding of how volatility affects the pricing of Asian-style options under alternative conditions, for instance by considering alternative averaging periods, or employing other stochastic models that may influence the underlying asset’s price dynamics. Lastly, while the study employed advanced variance reduction techniques which led to more accurate prices, and reduced the number of simulation trials needed, there exist other techniques, such as importance sampling and quasi-Monte Carlo methods, that have proved to be effective in more complex models, in particular high-dimensional ones. These techniques might improve the efficiency and accuracy of option pricing simulations, especially for more complex models such as those related to multi-dimensional portfolios.

6. Conclusion

This study has developed and applied a robust Monte Carlo simulation-based model for pricing Asian call options in the context of crude oil futures. By employing advanced variance reduction techniques, including antithetic variates and control variates, this paper was able to enhance the accuracy and efficiency of the traditional Monte Carlo approach. The results indicated that while the base model and antithetic variates provided identical option prices, the control variate technique slightly reduced the estimated price, demonstrating its effectiveness in variance reduction.

The sensitivity analysis revealed a significant impact of the underlying futures price on the Asian option's value, with option prices varying widely as the initial futures price was adjusted by ±30%. This highlights the critical importance of monitoring and managing exposure to the underlying asset's price fluctuations. However, an unexpected finding was the insensitivity of the option price to changes in volatility, which remained constant despite adjustments by ±30%. This suggests that the averaging effect in Asian options might diminish the influence of volatility, although further investigation is warranted to fully understand this phenomenon.

The implications of these findings are particularly relevant for practitioners in the financial markets, especially those involved in the trading and risk management of Asian options. The study underscores the necessity of employing sophisticated pricing models that can accurately reflect market conditions and the underlying asset's behavior. Moreover, the application of variance reduction techniques within these models is shown to be a valuable tool for enhancing precision, thereby reducing the risk of mispricing and its associated financial consequences.

Despite the strengths of the developed model, the study acknowledges certain limitations, such as the specific market conditions under which the analysis was conducted and the need for further exploration of volatility's role in pricing Asian options. Future research could address these limitations by examining a broader range of market scenarios and incorporating alternative stochastic models or additional variance reduction methods.

In summary, this research contributes to the existing literature on Asian option pricing by providing a refined and practical approach to modeling these complex financial instruments. The results offer valuable insights for both academics and market participants, advancing the understanding of how Asian options are priced and how their values are influenced by key market variables. Moving forward, continued exploration and refinement of these models will be essential to adapt to the evolving dynamics of financial markets and to support more informed decision-making in the trading and risk management of derivatives.

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