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Exploration of Remaining Issues in Capital Asset Pricing Model: Hypothesis Bias and Effectiveness of β Variables

Jiawei Yang

School of Economics, Shenzhen University, Shenzhen, China
immmwendyyy@gmail.com

Abstract: The Capital Asset Pricing Model (CAPM) is a cornerstone of modern finance, yet its strict assumptions—particularly market equilibrium and homogeneous investor expectations—limit real-world accuracy. This study examines two key limitations: (1) how market equilibrium assumptions overlook dynamic price adjustment processes, and (2) why the beta coefficient fails to fully explain asset pricing risks. Using numerical examples and empirical tests, people demonstrate that CAPM's predictions systematically diverge when comparing pre- and post-equilibrium states, while asset scale variations and anomalies like firm size effects further weaken beta's reliability. These results reveal fundamental constraints: equilibrium logic ignores transitional market behaviors, and beta oversimplifies multidimensional risks. Although CAPM remains a vital theoretical tool, its practical application requires acknowledging these limitations. To enhance relevance, future research should develop multi-factor models integrating dynamic equilibrium frameworks, behavioral insights, and technological innovations like machine learning. This evolution could bridge the gap between CAPM's theoretical elegance and the complexity of actual financial markets.

Keywords: CAPM, Market equilibrium assumption, Beta, Limitations

1. Introduction

The capital asset pricing model is a set of predictions concerning equilibrium expected returns on risky assets[1].Harry Markowitz laid down the foundation of modern portfolio management in 1952. The CAPM was published 12 years later in articles by William Sharpe[2], John Lintner[3], and Jan Mossin[4]. This paper focuses on investigating two major unresolved issues of the model: assumption deviations and the validity of the beta variable. The CAPM directly assumes that the market is in equilibrium, thereby bypassing the question of how market adjustments should be made under conditions where the market is not in equilibrium[5]. In early empirical analyses, scholars identified two issues with Sharpe's predictions regarding the intercept and slope in the CAPM. The first issue concerns the inaccuracy of β predictions for individual assets. The second issue is that the regression residuals share common sources of variation[6]. The research progress on the limitations of CAPM includes Unrealistic Assumptions, Limitations of the Single-Factor Framework, Empirical Performance Issues, Time Variability and Nonlinear Effects and Applicability in International Markets. While there are still some research gaps in the field, involving Investor Heterogeneity and Behavioral Biases, Integration of ESG Factors, Applications of Machine Learning and Non-parametric Methods, Unique Characteristics of Emerging Markets and Impact of Digital Assets and Block-chain Technology.

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As Roll highlighted, the Capital Asset Pricing Model (CAPM) is fundamentally untestable in empirical settings due to the inherent challenge of demonstrating that a market index portfolio represents an efficient market portfolio, thereby rendering the accurate estimation of the true beta coefficient unattainable [7]. In fact, the CAPM is not only untestable in practice but also remains theoretically incomplete. This paper will illustrate the theoretical issues of the CAPM by presenting a simplified example, highlighting the problems inherent in its assumptions.

The CAPM relies on two primary assumptions: first, the market reaches full equilibrium, with investors agreeing on the joint distribution of asset returns over time, validated by empirical evidence; second, all investors can access risk-free borrowing and lending at a uniform rate. This paper derives the CAPM formula to assess the feasibility of unrestricted risk-free borrowing in real-world scenarios, emphasizing the limitations of beta. It critically examines biases in the assumptions and the validity of the β variable, arguing that the foundational assumptions of the CAPM do not fully hold in practical economic situations. The goal is to propose a framework for enhancing CAPM adaptations, including: (1) relaxing unrealistic assumptions; (2) transitioning from a single-period to a multi-period model; (3) incorporating new factors to test the model, addressing previously overlooked elements.

2. The Capital Asset Pricing Model

2.1. The Content of CAPM Theory

Based on Markowitz's mean-variance analysis, Sharpe, Lintner, and Mossin investigated the pricing mechanism of financial assets in a competitive equilibrium market[2-4]. Investors are assumed to be risk-averse, have uniform return expectations, and optimize portfolios using mean-variance utility functions. Under these conditions, it is shown that the market portfolio is efficient in equilibrium. Additionally, the ratio of an asset's expected excess return (risk premium) to the market portfolio's expected excess return equals the covariance of the asset's return with the market portfolio's return, divided by the market portfolio's return variance. This relationship, termed the asset's beta, underpins the Capital Asset Pricing Model (CAPM) [8].

2.2. Assumptions of CAPM

(1) Investors evaluate portfolios based on their expected returns and standard deviations over a specific time period.

(2) Given a certain level of investment risk, investors always aim to maximize returns; conversely, given a certain level of expected return, investors strive to minimize risk.

(3) All investors share the same investment horizon, allowing them to borrow and lend at the same risk-free rate.

(4) The capital market is perfectly competitive, with no barriers to the flow of capital or information. It is assumed that no individual investor is large enough to influence the entire securities market, there are no transaction costs or taxes, and all investors have free access to all valuable information.

(5) Assets are infinitely divisible, meaning investors can purchase fractional shares, enabling them to allocate their investments in any proportion.

(6) Investors have homogeneous expectations, sharing the same understanding of expected returns, standard deviations, and covariances among securities.

Under the premise of complete information and negligible transaction costs, all investors operate under the same informational framework and possess the liberty to engage in asset transactions without limitations. Should the price of an asset diverge from its equilibrium state (for instance, becoming overvalued or undervalued), investors will promptly correct this discrepancy through their trading actions. This adjustment mechanism is swift and unobstructed, ultimately guaranteeing that

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the prices of all market assets embody a harmonious relationship between their associated risks and anticipated returns, thereby facilitating market equilibrium.

2.3. Related Formulas for CAPM

The basic formula of CAPM can be expressed as:

$$E(R_i) - R_f = \beta_i[E(R_m) - R_f] \tag{1}$$

The beta value reflects the systematic risk relationship between asset i and the market portfolio, and the calculation formula is:

$$\beta_i = \frac{Cov(R_i,R_m)}{Var(R_m)} \tag{2}$$

By substituting βi into the CAPM formula, it can be seen that:

$$E(R_i - R_f) = \frac{Cov(R_i,R_m)}{Var(R_m)} \cdot [E(R_m) - R_f] \tag{3}$$

This represents the ratio of the expected excess return of an asset [E (Ri) - Rf] to the expected excess return of a market portfolio [E (Rm) − Rf], which is equal to the covariance between the asset's return and the market portfolio return divided by the variance of the market portfolio return. This is the so-called beta value of assets.

To summarize this entire paragraph, people can use a concise formula:

$$\text{Expected excess returns} = \beta \cdot \text{market risk premium}$$

Among them, β is determined by the covariance between asset and market portfolio returns, as well as the variance of market portfolio returns. This is a core viewpoint of CAPM, used to assess the systemic risk of assets in the market and help investors evaluate reasonable expected returns.

3. Limitations of the Market Clearing Assumption

In order to clear the market, the CAPM assumes that at equilibrium, when the market is cleared, everyone chooses the same risk fund to match with risk-free assets when constructing their investment portfolio. And this only venture fund must be the same as the market portfolio.

To this end, the CAPM model first constructs an asset i whose holding weight exceeds the market portfolio, such as:

Invest in asset i at a ratio of α and in market asset portfolio M at a ratio of 1- α. The expected rate of return for this combination is

$$\bar{r}_\alpha = \alpha\bar{r}_i + (1-\alpha)\bar{r}_M \tag{4}$$

Its standard deviation is

$$\sigma_\alpha = [\alpha^2\sigma_i^2 + 2\alpha(1-\alpha)\sigma_{iM} + (1-\alpha)^2\sigma_M^2]^{\frac{1}{2}} \tag{5}$$

When α changes, the trajectory of $(\bar{r}_\alpha,\sigma_\alpha)$ on a plane with standard deviation as the x-axis and expected rate of return as the y-axis is a curve.

According to the design of the CAPM, this curve is tangent to the capital market line of a single fund composed of market portfolios as risk assets at α= 0. That is to say, the slope of this curve at α= 0 is

$$\frac{\bar{r}_M - r_f}{\sigma_M} \tag{6}$$

Therefore, the CAPM draws the following conclusion:

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If the market portfolio M is effective, then the expected return on any asset i satisfies:

$$\bar{r}_i - r_f = \beta_i(\bar{r}_M - \bar{r}_f) \tag{7}$$

Where,

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} \tag{8}$$

Here, β_i is seen as a standardized expression of the covariance between the asset and the market portfolio, representing the so-called systematic risk of this asset. Moreover, adventurous companies are considered to have higher β values, while conservative companies have lower β values.

This paper has already calculated $\bar{r}_M$ and σ_M^2. Next, it will calculate σ_{iM} and β_i.

Due to the zero covariance of the returns of these two assets. Therefore, this essay has

$$\sigma_{iM} = w_i\sigma_i^2, i = 1,2 \tag{9}$$

Thus, it can be concluded that

$$\sigma 1M = 0.036, \sigma 2M = 0.016 \tag{10}$$

Therefore

$$\beta 1 = 1.06, \beta 2 = 0.47 \tag{11}$$

This paper has noticed that assets with lower risk (represented by the standard deviation of returns) actually have higher values. This is contrary to the market sentiment of economics professors [5]. Due to the fact that almost all textbooks attempt to examine values from regression analysis in the stock market, without directly examining values from theoretical construction, they only see a constantly changing value without understanding the true meaning of the value and the true reasons for its changes.

Using the CAPM model and the above calculation results, this essay calculates the expected rate of return $\bar{r}_i^C$ (i=1,2) under the CAPM model using the previous data example:

$$\bar{r}_1^C = \frac{18}{17}(0.22 - 0.1) + 0.1 = 0.227 > \bar{r}_1 \tag{12}$$

$$\bar{r}_2^C = \frac{8}{17}(0.22 - 0.1) + 0.1 = 0.156 > \bar{r}_2 \tag{13}$$

The findings indicate that CAPM is invalid due to market inefficiencies; investors cannot achieve market clearance at current stock prices without violating CAPM's optimization criteria. Thus, the asset market is not in equilibrium, making CAPM inapplicable. To theoretically validate CAPM, the market must be realigned for equilibrium. While the model's unrealistic assumptions are acknowledged, empirical studies often overlook that investors typically hold only a subset of stocks, complicating market equilibrium assessments. In the absence of equilibrium, CAPM must identify mechanisms to restore it and support its theoretical framework. However, CAPM's fundamental assumption of market equilibrium neglects adjustment mechanisms, rendering the model inherently flawed.

When the market cannot clear, prices will face pressure to adjust. Here, if every investor constructs their own investment portfolio based on $w_1 = \frac{4}{7}$ and $w_2 = \frac{3}{7}$, there will be some unsold stocks in the first stock, and there will also be some supply shortages in the second stock. In this way, the first stock will lower its price to increase the return of stock buyers and attract purchases, while the second stock will raise its price to lower its return and reduce purchases.

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4. Limitations of β

4.1. The Effectiveness of β Existence

CAPM adds two assumptions on the basis of the mean variance model assumption: one is complete agreement, that is, when the market is completely cleared in the T-1 period, investors agree to the joint distribution of asset returns from T-1 to T, which is true and can be obtained through empirical analysis; Secondly, for all investors, there is the possibility of borrowing at risk-free interest rates. Under the first assumption, all investors have the same asset portfolio, which means that all investors hold the same asset portfolio as the market portfolio. The formula is:

$$E(R_i) = E(R_{zm}) + [E(R_m) - E(R_{zm})]\beta_i \ , i=1,.....,N. \quad (14)$$

Among them, β represents the relevant risk of a certain asset in the market portfolio, and from an economic perspective, β contributes to the risk of the market asset portfolio with every dollar invested in a certain asset i.

Under the second assumption, investors can borrow at a risk-free rate, where $E(R_{zm})$ will be equal to the market risk-free rate R_f, and the model formula is:

$$E(R_i) = R_f + [E(R_m) - R_f)]\beta_i \ , i=1,.....,N. \quad (15)$$

Under these two assumptions, people would choose to borrow at risk-free rates to hold the same asset portfolio as the market portfolio.

Restricted risk-free lending is non-existent. Sharp's CAPM model faces two key issues: first, the prediction of β for a single asset is flawed, as regression analysis reveals the actual intercept is greater and β is lower than predicted, suggesting a smoother relationship between actual expected returns and β than the model indicates. Second, regression residuals exhibit a common source of variation, indicating positive correlation. These challenges underscore the limitations of β in empirical analysis [6].

4.2. Inspection of CAPM: Insufficient Explanatory Power of β Coefficient

Empirical research indicates that the capital asset pricing model (CAPM) is incomplete, as the beta coefficient fails to fully account for capital asset pricing. Emerging abnormal factors influencing pricing include economies of scale, profit-to-market ratio, cash flow per share to market ratio, book value to market value ratio, and calendar effects. Testing typically encompasses three areas: the risk-return relationship, CAPM time series analysis, and cross-sectional CAPM evaluation. Sharpe's seminal study on the risk-return relationship analyzed 34 US mutual funds from 1954 to 1963 [2], revealing that US stock market returns surpassed the risk-free rate, with a correlation coefficient between fund returns and standard deviations exceeding 0.8, suggesting a linear risk-return relationship[9]. The BJS method, established by Black, Jensen, and Scholes, serves as a key CAPM time series test, utilizing indicator variables to reduce β estimation bias. Their results highlighted inconsistencies between actual regression results and theoretical predictions: low-risk stocks met expected returns, while high-risk stocks underperformed. The most famous cross-sectional CAPM test was conducted by Fama and Macbeth (FM) using cross-sectional data for analysis [10]. The FM results indicate that there is a positive relationship between returns and beta values; Other non systematic risks do not play a major role in the pricing of stock returns.

5. Conclusion

The Capital Asset Pricing Model (CAPM) is essential in finance, offering a theoretical basis for the risk-return relationship. This paper examines CAPM's limitations regarding its assumptions and

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variables, aiming not to criticize its unrealistic aspects but to guide enhancements for better alignment with economic realities.

The concept of market equilibrium simplifies the model but overlooks the complex dynamics of market adjustments. This research demonstrates through numerical examples that the Capital Asset Pricing Model (CAPM) predictions falter without market equilibrium, emphasizing the need for a detailed analysis of the mechanisms achieving it. Additionally, the beta coefficient, crucial to the CAPM, inadequately captures asset pricing in empirical studies. Regression analyses reveal gaps between expected and actual relationships, indicating that beta alone does not account for all relevant risk factors. Anomalies such as firm size and value further weaken the model's explanatory power. These findings underscore the theoretical and practical limitations of the CAPM. While it remains a cornerstone of financial theory, the model's assumptions about investor behavior, market conditions, and risk-free borrowing fail to reflect the complexities of real-world financial markets.

To improve CAPM's relevance and accuracy, future research should relax restrictive assumptions, extend the model to multi-period frameworks, and incorporate factors like behavioral biases, ESG considerations, and technological advancements such as machine learning. Addressing these gaps will enable the next generation of asset pricing models to align more closely with the complexities of global financial markets.

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