

Multiple Equilibria in the Vaccination Game: Nash Equilibrium and Socially Optimal Policy Choices

Xichen Xie

*Business School, Macau University of Science and Technology, Macau, China
XXC1220007291@outlook.com*

Abstract: In recent years, global infectious disease pandemics have emerged as one of the most pressing environmental issues, posing a major threat to public health and social stability worldwide. According to previous research, the rational decisions of individuals often lead to lower vaccination rates than the optimal level of society. In this paper, we investigate the difference between Nash equilibrium and socially optimal solution of individual vaccination decisions in the context of a global pandemic. The aim is to establish a theoretical model, analyze the gap between the two, and then propose suggestions for optimizing public health policies to improve vaccination rates, achieve herd immunity, and maximize social welfare. This paper uses the game theory method to set up the basic hypothesis, construct the game model, and the social welfare function, analyze the Nash equilibrium, the social optimal solution and the dynamic model, and make a comparative analysis. Studies have found that individual rational decision-making leads to a lower vaccination rate than the socially optimal level, and it is difficult to achieve herd immunity. On this basis, it is suggested that the government should adopt policy intervention measures such as subsidies, publicity and education, long-term incentives, and improving information transparency. The conclusion shows that policy intervention can effectively improve the current situation, but the model has limitations. Further studies can be conducted from the aspects of parameter sensitivity analysis, incomplete information and individual heterogeneity.

Keywords: Vaccination, Nash equilibrium, Socially optimal, Dynamic game, Public health policy

1. Introduction

With the acceleration of globalization, the risk of spreading infectious diseases has risen sharply, and its impact on public health security and social stability cannot be underestimated. According to the data of the World Health Organization (WHO), in recent decades, new infectious diseases have emerged continuously, such as Ebola virus and Zika virus, seriously threatening the lives and health of hundreds of millions of people around the world. Take the COVID-19 epidemic as an example, as of February 27, 2020, more than 882,000 cases of COVID-19 (the illness caused by SARS-CoV-2) and 2,800 deaths have been reported, with about 95% of the cases and 97% of the deaths occurring in China[1]. Its rapid spread around the world has put countries' healthcare systems under enormous pressure and hit the global economy hard. Vaccination, as a key means to prevent the spread of infectious diseases and build a herd immune barrier, is crucial to controlling outbreaks. However, in reality, there are differences in individual decision-making on vaccination, which makes it difficult

for the vaccination rate to reach the optimal level of society, which has aroused widespread concern. From the policy perspective, although governments actively promote vaccination, they still face many challenges in the process of policy formulation and implementation. How to optimize public health policies to improve the vaccination rate has become an urgent problem to be solved.

In the field of vaccine-related research, many scholars have carried out exploration. Bauch and Galvani [2] introduced game theory into vaccination decision-making for the first time and revealed the relationship between individual behavior and herd immunity. Fu et al.[3] further explored the impact of social networks and group behaviors on vaccination. Lv. H et al. [4] analyzed the costs and benefits of vaccination from an economic perspective. These studies provide an important theoretical basis for understanding vaccination behavior, but there are still some limitations. Most of the existing studies focus on the construction of theoretical models, and the consideration of complex factors in reality is not comprehensive enough, for example, the impact of individual heterogeneity and incomplete information on vaccination decision-making is not fully explored. At the same time, in terms of policy suggestions, there is a lack of specific measures for the characteristics of different regions and different populations, resulting in the actual operability and effectiveness of the policy being improved.

Based on the above background, this study aims to deeply analyze the difference between Nash equilibrium and social optimal solution in individual vaccination decisions, and explore how to narrow this gap through policy intervention, so as to maximize herd immunity and social welfare. By considering a variety of realistic factors, this paper builds a more realistic game theory model and fills the gap of comprehensive factors in the existing research. The research results can not only provide theoretical support for the formulation of public health policies, help decision-makers to formulate more targeted and effective vaccination policies, but also provide new ideas for optimizing infectious disease prevention and control strategies, which have important theoretical value and practical significance.

2. Literature Review

2.1. Economic Benefits of Vaccination

Vaccination programs offer significant long-term economic benefits, including cost savings and substantial social value. Studies show that vaccines have averted millions of deaths and saved billions in treatment costs, particularly in low- and middle-income countries. Ozawa, S. et al[5] stated that the economic benefits of vaccination in low - and middle-income countries go beyond health gains. Quilici S. et al[6] shows that in addition to the impact on healthcare resources and productivity, reduced mortality and morbidity contribute to increased consumption and gross domestic product.

2.2. Game Theory in Vaccination Decisions and Optimal Vaccination Rates

Game theory has been applied to understand individual vaccination behavior and its impact on herd immunity, highlighting the interplay between personal choices and public health outcomes. Research focuses on defining optimal vaccination rates. Stephenson, B. et al [7] considered different hospital statuses and compared time-varying rates to constant rates in order to determine in which scenarios a time-varying rate would be preferred. They saw that with this increase in the number of susceptible patients, the optimal vaccination rate decreased. In this case, a time-varying rate of vaccination was shown to be more beneficial.

2.3. Policy Interventions

Many researches evaluate the effectiveness of policy interventions to enhance vaccine uptake and public health outcomes. A comparison of 12 counties in China found that six counties with government-funded vaccination programs had significantly higher rates of influenza virus infection among older adults than six counties without government-funded vaccination programs.[8] And K Gravagna et al.[9]. analyzed the impact of national mandatory vaccination policies and a global assessment of the consequences of non-compliance.

3. Model setting based on Game theory

3.1. Basic Assumption

Assumption 1: Rational behavior of individual decision makers.

Individuals make vaccination decisions based on their perceived costs and benefits.

Assumption 2: Costs and benefits of vaccination

These include direct financial costs, time costs, health benefits and externalities of vaccination(e.g., herd immunity).

3.2. Game Theory Models

3.2.1. Define the policy space for participants

Participants: N individuals in the community who need to decide whether to be vaccinated each period.

Strategy: Individuals have two choices per phase: vaccinate (V) or not vaccinate (NV).

3.2.2. Payoff Function

The payoff function of vaccination: $U_v = -C + S + B(q)$

c: Cost of vaccination (including financial cost, time cost, side effects, etc.)

s: Subsidies or other incentives provided by the government (if any).

The payoff function of not vaccinating: $U_{NV} = -p \times r$

p: The proportion of the community not vaccinated

r: Risk of contracting diseases when not vaccinated

3.2.3. Social Welfare

Suppose there are N individuals in the community, where q is the proportion who are vaccinated and $p=1-q$, which is the proportion who are not vaccinated. The total social welfare function W can be expressed as: $W = q \times U_v + p \times U_{NV}$

This function takes into account the benefits of vaccinated and unvaccinated individuals and reflects the welfare level of the entire community. Maximizing social welfare is one of the goals pursued by public health policies. By analyzing the social welfare function, we can evaluate the impact of different vaccination strategies on the overall social welfare and provide reference for policy formulation.

4. Equilibrium Analysis

4.1. Nash Equilibrium

In Nash equilibrium, given the strategies of others, an individual cannot improve his own returns by unilaterally changing his strategies. The equilibrium conditions are:

$$U_{NV} = U_V$$

$$-C + S + B(q) = -p \times r$$

$$q^* = \frac{C - S - r}{700 - r}$$

This implies that individuals will vaccinate if the perceived benefits outweigh the costs.

However, this Nash equilibrium vaccination rate may not be socially optimal because individuals only consider their own interests when making decisions and do not fully consider the impact of the externalities of vaccination on the overall welfare of society.

4.2. Socially Optimal

The socially optimal solution is to determine the optimal vaccination rate from the perspective of maximizing the overall social welfare. It considers all costs and benefits of vaccination, including individual benefits and externalities. Different from Nash equilibrium, the social optimal solution seeks to maximize the welfare of the whole society rather than the individual benefit.

$$q = \frac{-(S - C - 2r) \pm \sqrt{(S - C - 2r)^2 - 4(700 + r)(-r)}}{2(700 + r)}$$

4.3. Dynamic Model

Taking into account individual decisions affects not only current health status, but future risk of disease transmission and health gains. Static models cannot capture the effects of this time dimension. But in a dynamic model, individuals can adjust their strategies according to the behavior of others and the dynamic changes in the spread of disease.

In the short term, individuals make vaccination decisions based on current costs and benefits.

$$U_v(t) = -C + S + B(q) + \delta \cdot V_{t+1}$$

$$U_{NV}(t) = -p_r t + \delta \cdot V_{t+1}$$

$$q^* = \frac{C - S - r}{k - r}$$

The discount factor δ reflects the individual's preference for future benefits.

The larger the discount factor, the more the individual pays attention to the future income; Instead, they focus more on current earnings.

V_{t+1} captures the long-term health and economic impacts of vaccination decisions.

In the long run, q_t converges to a stable state q_t^* .

4.4. Comparative Analysis

When Nash equilibrium is larger than socially optimal, it indicates that individual rational decision-making leads to a lower vaccination rate than the socially optimal level, and herd immunity cannot be realized. This difference indicates that the market mechanism has certain limitations in the field of vaccination, and the government and society need to take measures to intervene in order to guide individuals to make decisions that are more in line with the interests of society.

5. Policy Implications

5.1. The Need for Policy Intervention

In vaccination decision-making, nash equilibrium reflects the individual's behavioral choice under rational decision-making, while socially optimal represents the ideal state from the perspective of the overall social welfare maximization. In order to narrow the gap between Nash equilibrium and the social optimum, achieve herd immunity and maximize social welfare, policy intervention is necessary. For example, the government can not only reduce the cost of individual vaccination through subsidies but also improve the enthusiasm of individuals to vaccinate. Through publicity and education, the public's awareness of herd immunity should be enhanced, the individual's sense of social responsibility should be enhanced, and the initiative of vaccination should be promoted.

5.2. Specific Policy Suggestions

Firstly, subsidies can be an effective tool to increase vaccination uptake by reducing direct costs for individuals. Governments can lower financial barriers through partial or full subsidies, making vaccines more accessible and appealing (e.g., free or discounted immunization programs). Secondly, publicity and education play a crucial role in enhancing public understanding of herd immunity (V_{t+1}) and fostering a sense of social responsibility. Improved scientific literacy [10] and targeted education campaigns for both youth and adults [11] can mitigate vaccine hesitancy and encourage compliance. Thirdly, long-term incentives, such as incorporating a discount factor (δ) into policy design, can encourage individuals to prioritize future benefits over short-term costs. By emphasizing the long-term economic and health advantages of vaccination, policymakers can improve participation rates[12,13]. Lastly, transparency in communicating vaccination rates and disease transmission risks helps individuals make informed decisions. Public access to real-time data enhances trust and enables better risk assessment, reducing uncertainty around vaccination choices.

5.3. Challenges in Policy Implementation

5.3.1. Public Acceptance and Behavioral Response

One of the most significant challenges in policy implementation, particularly in public health interventions such as vaccination programs, is securing public acceptance and encouraging compliant behavioral responses. Policies that mandate or strongly recommend vaccination often face resistance due to various factors, including misinformation, distrust in governmental or scientific institutions, and cultural or religious beliefs [14]. For instance, vaccine hesitancy has been identified as a major barrier to achieving herd immunity, as seen during the COVID-19 pandemic [15]. Effective policy design must incorporate strategies to address public concerns through transparent communication, community engagement, and targeted education campaigns. Failure to account for public attitudes and behaviors may lead to low adherence, ultimately undermining policy effectiveness.

5.3.2. Fairness and Efficiency of Policy Implementation

Another critical challenge in policy implementation is ensuring both fairness and efficiency in the distribution of resources and enforcement of regulations. For example, during the initial rollout of COVID-19 vaccines, disparities in access between high- and low-income regions raised ethical concerns and hindered global vaccination efforts [16]. To mitigate such issues, policymakers must establish clear, transparent criteria for resource allocation and ensure that implementation mechanisms are free from bias or favoritism. Additionally, efficiency in policy execution is essential to avoid bureaucratic delays and ensure timely delivery of services. Balancing fairness with

operational efficiency remains a persistent challenge, particularly in large-scale or emergency policy interventions.

6. Conclusion

In this paper, Nash equilibrium and socially optimal solutions in vaccination decision-making are analyzed by the game theory model, and the gap between individual rational decision-making and overall social optimal decision-making is revealed. Through model construction and equilibrium analysis, the internal mechanism of the vaccine coverage rate not reaching the optimal level of society was clarified. It then proposes targeted policy interventions, including subsidies, education, long-term incentives, and improved information transparency. It is of great significance to increase the vaccination rate, achieve herd immunity, and maximize social welfare.

While the current model provides valuable insights, it presents some limitations that warrant further study. In future research, special attention needs to be paid to four key areas to enhance the robustness and real-world applicability of the model. First, a comprehensive sensitivity analysis should be conducted to examine how changes in key parameters, such as vaccination costs and disease transmission rates, affect balanced outcomes. Second, the model's assumptions about complete information need to be addressed by incorporating uncertainties and their impact on the vaccination decision-making process. Third, the homogeneity assumption of the current framework fails to account for key individual differences in health status, socioeconomic factors, and behavioral attitudes that significantly influence real-world vaccination choices. Fourth, the rational agent hypothesis should be relaxed to accommodate empirically observed deviations from perfect rationality in human decision-making. Improving models to address these limitations will significantly improve our understanding of the dynamics of vaccination behavior and enhance the practical utility of models in policy-making.

From a practical point of view, the results of this study can provide a theoretical basis for the formulation of public health policies. Future studies could focus on these limitations to further expand and refine the vaccination game model. By introducing more realistic assumptions and considering more complex factors, the model is made closer to reality. Policymakers can draw on the findings to formulate more targeted policies based on regional characteristics and the prevalence of infectious diseases. For example, increasing subsidies and strengthening education in areas with low vaccination rates. In the future, research and practice will complement each other and make greater contributions to the prevention and control of infectious diseases worldwide and to the protection of public health and social stability.

References

- [1] C. del Rio, C., & Malani, P. N. (2020). COVID-19—New Insights on a Rapidly Changing Epidemic. *JAMA*, 323(14), 1339-1340. <https://doi.org/10.1001/jama.2020.3072>
- [2] Bauch, C. T., & Galvani, A. P. (2013). Social factors in epidemiology. *Science*, 342(6154), 47-49.
- [3] Fu, F., Rosenbloom, D. I., Wang, L., & Nowak, M. A. (2011). Imitation dynamics of vaccination behaviour on social networks. *Proceedings of the Royal Society B: Biological Sciences*, 278(1702), 42-49.
- [4] Lv, H., Chen, F., Wang, Y., Chen, Y., & Hu, Y. (2023). Cost-effectiveness analysis of introducing rotavirus vaccine into immunization program in Zhejiang province, China: A decision tree-Markov model study. *Human Vaccines & Immunotherapeutics*, 19(2), 2217075.
- [5] Ozawa, S., Mirelman, A., Stack, M. L., Walker, D. G., & Levine, O. S. (2012). Cost-effectiveness and economic benefits of vaccines in low-and middle-income countries: a systematic review. *Vaccine*, 31(1), 96-108.
- [6] Quilici, S., Smith, R., & Signorelli, C. (2015). Role of vaccination in economic growth. *Journal of market access & health policy*, 3(1), 27044.
- [7] Stephenson, B., Lanzas, C., Lenhart, S., & Day, J. (2017). Optimal control of vaccination rate in an epidemiological model of *Clostridium difficile* transmission. *Journal of mathematical biology*, 75, 1693-1713.

- [8] Jiang, X., Shang, X., Lin, J., Zhao, Y., Wang, W., & Qiu, Y. (2020). Impacts of free vaccination policy and associated factors on influenza vaccination behavior of the elderly in China: A quasi-experimental study. *Vaccine*, 39(5), 846-852.
- [9] Gravagna, K., Becker, A., Valeris-Chacin, R., Mohammed, I., Tambe, S., Awan, F. A., ... & Basta, N. E. (2020). Global assessment of national mandatory vaccination policies and consequences of non-compliance. *Vaccine*, 38(49), 7865-7873.
- [10] Austin, E. W., Austin, B. W., Willoughby, J. F., Amram, O., & Domgaard, S. (2021). How media literacy and science media literacy predicted the adoption of protective behaviors amidst the COVID-19 pandemic. *Journal of health communication*, 26(4), 239-252.
- [11] Khubchandani, J., Sharma, S., Price, J. H., Wiblishauser, M. J., Sharma, M., & Webb, F. J. (2021). COVID-19 vaccination hesitancy in the United States: a rapid national assessment. *Journal of community health*, 46, 270-277.
- [12] Bloom, D. E., Canning, D., & Weston, M. (2017). The value of vaccination. In *Fighting the diseases of poverty* (pp. 214-238). Routledge.
- [13] Matrajt, L., Eaton, J., Leung, T., Dimitrov, D., Schiffer, J. T., Swan, D. A., & Janes, H. (2021). Optimizing vaccine allocation for COVID-19 vaccines shows the potential role of single-dose vaccination. *Nature communications*, 12(1), 3449.
- [14] Hornsey, M. J., Harris, E. A., & Fielding, K. S. (2018). The psychological roots of anti-vaccination attitudes: A 24-nation investigation. *Health psychology*, 37(4), 307.
- [15] Lazarus, R., Baos, S., Cappel-Porter, H., Carson-Stevens, A., Clout, M., Culliford, L., ... & Ryan, K. A. (2021). Safety and immunogenicity of concomitant administration of COVID-19 vaccines (ChAdOx1 or BNT162b2) with seasonal influenza vaccines in adults in the UK (ComFluCOV): a multicentre, randomised, controlled, phase 4 trial. *The Lancet*, 398(10318), 2277-2287.
- [16] Bollyky, T. J., Hulland, E. N., Barber, R. M., Collins, J. K., Kiernan, S., Moses, M., ... & Dieleman, J. L. (2022). Pandemic preparedness and COVID-19: an exploratory analysis of infection and fatality rates, and contextual factors associated with preparedness in 177 countries, from Jan 1, 2020, to Sept 30, 2021. *The Lancet*, 399(10334), 1489-1512.