Proceedings of ICMRED 2025 symposium: Effective Communication as a Powerful Management Tool
DOI: 10.54254/2754-1169/184/2025.BL23216

The Feasibility of Identifying the Portfolio Frontier in the Stock Market Using the Markowitz Model: A Demonstration with One Year of Daily Data

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Abstract: With the rapid development of the global economy, an increasing number of people are investing in the stock market to pursue higher returns. As a result, identifying the portfolio frontier has become an increasingly popular topic of discussion. This paper explores the feasibility of determining the portfolio frontier using the Markowitz model. After reviewing recent literature on the subject, I derive the relevant formulas. To validate the practical application of the theory, I collect daily closing prices of 15 different stocks from April 30, 2020, to April 30, 2021, and apply the data to the model. The results demonstrate that utilizing the Markowitz model to determine an optimal investment strategy can yield significantly higher returns with only a marginal increase in risk.

Keywords: Markowitz Model, Portfolio Frontier, Reality constraints

1. Introduction

With the rapid development of China's capital market, the stock market has become a key investment channel for Chinese residents. Consequently, portfolio management has emerged as a central issue in financial research, aiming to address the challenge of balancing risk and return. In 1952, Harry Markowitz published the seminal paper Portfolio Selection, which introduced risk quantification into investment decision-making through mathematical modeling. This work marked the inception of modern portfolio theory.

In China, as the capital market continues to open and evolve, the size of the stock market has expanded substantially, making it the second largest in the world. As of February 7, 2025, the total transaction volume of the Shanghai and Shenzhen stock exchanges reached 1,960.2 billion yuan. Due to significant market volatility and relatively immature investor behavior, Chinese investors face both substantial opportunities and risks when formulating investment strategies. Applying the mean-variance model to investment decisions can provide a solid theoretical foundation for asset allocation strategies.

This paper investigates how to determine the portfolio frontier using the Markowitz model. First, it reviews the current research and practical applications of the mean-variance model, with a focus on studies from the past decade. Next, it derives the relevant formulas. Then, daily data from 15 selected stocks are used as inputs to the model. Finally, the resulting data are plotted to visualize the portfolio frontier and demonstrate the model's feasibility.

2. Literature review

In recent years, with the rapid development of the global economy, personal wealth has been steadily increasing. Consequently, determining the most profitable investment portfolio has become a topic of growing interest. In this context, economists have conducted extensive research, among which the most representative is the mean-variance model proposed by Markowitz. In recent years, many scholars have sought to enhance the model's practicality by introducing various improvements. Zuo [1], in Study on Portfolio Strategy Based on Mean-Variance Model published in Research on Financial Management Theory, examines investment strategies in the decision-making process using the mean-variance model. By analyzing stock data from four Australian companies, Zuo found that the average return rate did not conform to the conventional risk-return relationship. Subsequently, 12 months of time-series data were selected to evaluate the risk and expected return of a portfolio composed of two stocks as risky assets. Using the Sharpe ratio for analysis, a 1:1 risky asset portfolio was selected and combined with risk-free assets to form a new portfolio. The study concludes that investors with different levels of risk aversion can identify optimal investment strategies that align with their individual risk tolerance, regardless of the presence of risk-free assets. Moreover, it was found that in markets with risk-free assets, the expected return of the optimal investment strategy is often higher than in markets without risk-free assets.

Zheng and Liu [2], in their article Portfolio Optimization Model Based on Entropy Variable Risk: Data Analysis from Shenzhen 100 published in the Journal of Shandong University of Finance and Economics, propose a portfolio optimization model incorporating entropy as a risk measure. Building on the Markowitz mean-variance framework, they introduce the mean-entropy model and conduct empirical research using data from 10 selected stocks in the Shenzhen Component Index. By comparing stock variances and corresponding entropy values, they verify the feasibility and necessity of the new model. The investment schemes generated by both models were calculated using MATLAB. Results indicate that the mean-entropy model offers greater practicality, as it enables investors to construct portfolios with fewer securities while maintaining comparable returns. This approach reduces transaction and management costs, conserves informational resources, and enhances investors' information-processing capacity.

Huang [3], in Construction Analysis of Stock Portfolio Based on Mean-Variance Model published in Shangye Jingji, conducts both theoretical and empirical analyses on stock portfolio construction using Markowitz portfolio theory and factor analysis. A comprehensive evaluation system is established, and top-performing listed companies are selected via factor analysis to form a stock pool. The mean-variance model is then applied to determine the optimal investment ratios, and the resulting portfolio is evaluated using Jensen's alpha and the Treynor ratio. The study concludes that the constructed portfolio is sound, and the methodology is both practical and generalizable.

Li [4], in Study on Markowitz Portfolio Model with Mean and Variance Variation published in Bohai Economic Outlook, investigates a version of the Markowitz model in which both the mean and variance vary simultaneously. The paper establishes a modified model and derives the efficient frontier under this condition. Four stocks from the CSMAR database are selected for application analysis. The study highlights the importance for investors of considering changes in both the mean return and variance when constructing portfolios, in order to mitigate investment uncertainty.

Chen and Chen [5], in their article The Problem of Optimal Reinsurance Strategy under the Principle of Relative Wealth Preference and Mean-Variance Premium Rate published in the Journal of Systems Science and Mathematical Sciences, study the optimal reinsurance strategies for insurance companies under model ambiguity. They theoretically and numerically examine the impact of model parameters on optimal reinsurance decisions and derive several meaningful economic insights. First, the greater the competition among insurance companies, the higher the reinsurance contract prices, which in turn motivates insurance firms to increase their retention ratios. Second, the more uncertain insurance companies are about the model, the more inclined they are to reduce their retention levels. Third, increased uncertainty also leads to higher reinsurance prices. Dang et al. [6], in Advancing Mean-Variance Portfolio Optimization with Base Constraint published in Fuzzy Systems and Mathematics, explore how to construct a portfolio frontier to maximize profits. Incorporating real-world conditions from the financial market, they introduce a permissible mean-variance portfolio model that accounts for transaction costs, borrowing constraints, base constraints, and upper and lower bound constraints. The study concludes that, under the same level of risk, investors with higher emotional coefficients achieve higher profits compared to those with lower emotional coefficients. Additionally, the relationship between risk and expected return varies with different levels of expected return. Gu et al. [7], in Study on the Effectiveness of Mean-Variance Portfolio Model with Background Risk in Cross-Border Investment also published in Fuzzy Systems and Mathematics, assess the applicability of the Markowitz model in international investment scenarios. Recognizing the influence of background risks from non-financial sources—often overlooked in traditional models—the authors extend the mean-variance framework to include these external risks. Based on an analytical formulation and empirical analysis involving five representative stock indices, their findings suggest that incorporating background risk into the model improves investor performance and enhances the effectiveness of cross-border portfolio strategies.

Zhang and Lin [8], in Research on Allowable Mean-Variance Portfolio Optimization Based on Bayesian Theory published in the Journal of Applied Statistics and Management, construct a Bayesian-based admissible mean-variance portfolio model. Bayesian theory is used to adjust model parameters, accounting for the subjective expectations of investors. The study reveals that investor sentiment significantly influences both returns and risks, and thereby shifts the position of the efficient frontier. Optimistic investors see the efficient frontier shift upward and to the left, indicating higher returns for lower risk, while pessimistic sentiment causes the frontier to move downward and to the right. Finally, Zhang and Shen [9], in Return Rate Prediction and Portfolio Model Based on Deep Learning published in the Journal of Fuyang Normal University (Natural Science), apply the Transformer deep learning model to predict the return rates of alternative assets with the aim of improving expected return accuracy. Their results show that the Transformer model outperforms LSTM and SVR models in return prediction and significantly enhances the performance of the mean-variance (MV) portfolio model. When integrated with the MV model, the Transformer-based approach delivers superior portfolio construction outcomes compared to combinations involving LSTM, SVR, or the MV model alone.

3. Methodology

This paper focuses on calculating the portfolio frontier using the Markowitz model, which is based on idealized assumptions and a well-established mathematical framework. The model quantifies the trade-off between risk and return by constructing portfolios that either maximize the expected return for a given level of risk or minimize risk for a specified target return. Below is a detailed explanation of the symbol system, modeling process, and solution procedure. We suppose that there are $N \geq 2$ risky assets traded in a frictionless market where unlimited short selling is allowed. Furthermore, it is assumed that the return on any asset cannot be expressed as a linear combination of the returns on other assets. Under this assumption, asset returns are linearly independent, and the variance-covariance matrix V is non-singular. w represents a N-vector, which is a constant, and $w \neq 0$. A portfolio is a frontier portfolio if it has the minimum variance among portfolios that have the same expected rate of return. A portfolio p is a frontier portfolio is and only if w_p, the N-vector portfolio weights of p, is the solution to

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$$\min_{\{w\}} \frac{1}{2} w^T V w$$
$$w^T e = E[\widetilde{r_p}] \tag{1}$$
$$w^T 1 = 1,$$

where e denotes the N-vector of expected rates of return on the N risky assets, $E[\widetilde{r_p}]$ denotes the expected rate of return on portfolio p, and 1is an N-vector of ones. The programming problem problem $\min_{\{w\}} \frac{1}{2} w^T V w$ minimizes the portfolio variance subject to the constraints that the portfolio expected rate of return is equal to $E[\widetilde{r_p}]$ and that the portfolio weights sum to unity. Forming the Lagrangian, w_p is the solution to the following:

$$\min_{\{w,\lambda,\gamma\}} L = \frac{1}{2} w^T V w + \lambda\left(E[\widetilde{r_p}] - w^T e\right) + \gamma(1 - w^T 1) \tag{2}$$

and the result would be:

$$w_p = g + hE[\widetilde{r_p}] \tag{3}$$

where

$$g = \frac{1}{D}[B(V^{-1}1) - A(V^{-1}e)],$$
$$h = \frac{1}{D}[C(V^{-1}e) - A(V^{-1}1)],$$
$$A = 1^T V^{-1} e = e^T V^{-1} 1, \tag{4}$$
$$B = e^T V^{-1} e,$$
$$C = 1^T V^{-1} 1$$
$$D = B * C - A^2$$

With these equations, A/C can be calculated. After that, the value of $E[\widetilde{r_p}]$ can be set (roughly evenly distributed around A/C). Using the equation $\sigma(\tilde{r}) = \sqrt{w^T V w}$, $\sigma(\tilde{r})$ is calculated. Then, take $\sigma(\tilde{r})$ as the horizontal axis and $E[\widetilde{r_p}]$ as the vertical axis, and make a graph of the function by plotting points, which is a parabola in variance-expected rate of return space with vertex (1/C,A/C). That means the portfolio having the minimum variance of all possible portfolios,or the minimum variance portfolio, is at (1/C,A/C).

4. Discussion

To apply the aforementioned theoretical methodology in practice, 15 different stocks were selected: ASIANPAINT, ADANIPORTS, AXISBANK, BAJAJ-AUTO, BAJAJFINSV, BAJFINANCE, BHARTIARTL, BPCL, BRITANNIA, CIPLA, COALINDIA, DRREDDY, EICHERMOT, GAIL, and GRASIM. These stocks were analyzed using their daily closing prices from April 30, 2020, to April 30, 2021. The first step in the data processing involved calculating the average daily return (or growth rate) of the closing price for each stock. These values were compiled into a vector denoted as the eee matrix, representing the expected returns for the assets. The matrix is presented as follows:

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Table 1: Average daily growth rate of closing price of 15 stocks ranging from 30/4/2020 to 30/4/2021

ASIANPAINT	1.60‰
ADANIPORTS	4.00‰
AXISBANK	2.30‰
BAJAJ-AUTO	1.70‰
BAJAJFINSV	3.40‰
BAJFINANCE	3.90‰
BHARTIARTL	0.40‰
BPCL	0.80‰
BRITANNIA	0.40‰
CIPLA	1.90‰
COALINDIA	-0.20‰
DRREDDY	1.30‰
EICHERMOT	1.40‰
GAIL	1.70‰
GRASIM	4.30‰

Besides, the V matrix and its inverse matrix,which represents the covariance between pairs of 15 stocks (too long to show here), and a 1 by 15 unit matrix are required. Then, using these data and the formula above, the value of A, B, C, and D can be calculated, with which g and h matrix can be gained. Next, take a few value evenly around A/C as the $E[\tilde{r_p}]$, and put them into $w_p = g + hE[\tilde{r_p}]$ to find w_p. Here are five.

Table 2: The investment proportion of each stock under different $E[\tilde{r_p}]$

$E[\tilde{r_p}]$	0.0006	0.0007	0.0008	0.0009	0.001
ASIANPAINT	13.45%	13.37%	13.29%	13.21%	13.13%
ADANIPORTS	-1.79%	-1.10%	-0.40%	0.29%	0.99%
AXISBANK	6.52%	6.29%	6.05%	5.82%	5.59%
BAJAJ-AUTO	12.46%	12.42%	12.39%	12.36%	12.33%
BAJAJFINSV	4.16%	4.20%	4.25%	4.29%	4.33%
BAJFINANCE	-12.38%	-11.80%	-11.23%	-10.65%	-10.07%
BHARTIARTL	11.96%	11.12%	10.28%	9.44%	8.59%
BPCL	2.24%	2.23%	2.22%	2.21%	2.19%
BRITANNIA	39.89%	39.14%	38.38%	37.63%	36.88%
CIPLA	7.30%	7.62%	7.94%	8.26%	8.58%
COALINDIA	6.06%	5.01%	3.96%	2.92%	1.87%
DRREDDY	8.90%	8.96%	9.03%	9.10%	9.16%
EICHERMOT	0.28%	0.22%	0.17%	0.11%	0.05%
GAIL	2.95%	3.12%	3.30%	3.48%	3.65%
GRASIM	-1.98%	-0.81%	0.37%	1.55%	2.73%

Finally, $\sigma(\tilde{r})=\sqrt{w^T V w}$ is used to calculate $\sigma(\tilde{r})$, and the Frontier curve is drawn as followed.

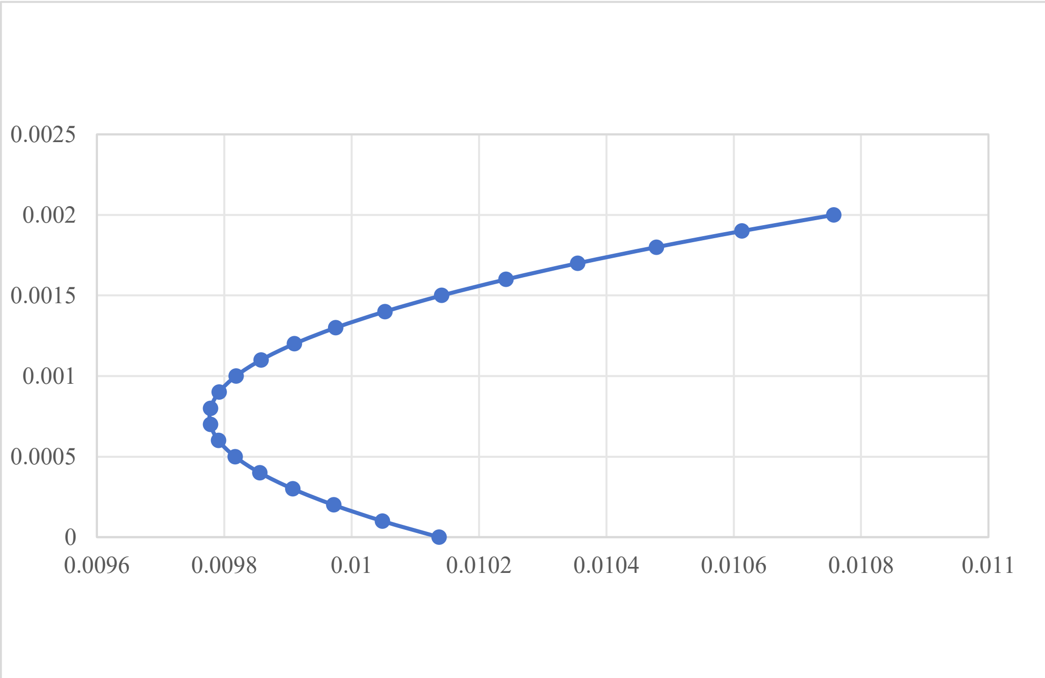

Figure 1: The frontier curve

It is evident from the figure that the plotted curve is a hyperbola, with its vertex corresponding to the vertical coordinate A/C, which represents the minimum-variance solution. This result confirms the validity of the theoretical framework outlined above. However, the presence of negative values in the portfolio weights indicates short-selling, which is not permissible under practical investment constraints. Therefore, it is necessary to construct an investment portfolio in which all asset weights are non-negative.

To address this, any stock with a weight less than 0 is excluded from the portfolio. The remaining assets are then re-evaluated using the same optimization procedure. This iterative process continues until all selected stocks have weights greater than or equal to 0, ensuring a feasible, long-only portfolio. The final optimized portfolio is presented below.

Table 3: Final investment proportion for each stock

ASIANPAINT	11.87%
ADANIPORTS	0.00%
AXISBANK	2.46%
BAJAJ-AUTO	11.71%
BAJAJFINSV	0.00%
BAJFINANCE	0.00%
BHARTIARTL	9.07%
BPCL	2.26%
BRITANNIA	36.28%
CIPLA	8.85%
COALINDIA	3.60%
DRREDDY	10.39%
EICHERMOT	0.23%
GAIL	3.28%
GRASIM	0.00%

In conclusion, the theoretical method can be used to find the portfolio frontier, but reality constraints should also be taken into account.

5. Conclusion

This paper investigates how to determine the portfolio frontier using the Markowitz model. First, the relevant mathematical formulas are derived. Then, daily closing prices from April 30, 2020, to April 30, 2021, for 15 selected stocks are applied to these formulas. The resulting data are plotted as a curve to illustrate the model's feasibility. However, a problem arises when negative weights appear in the portfolio, as all weights must be non-negative in a realistic investment scenario. To address this, stocks with negative weights below 0 are excluded, and the remaining data are re-entered into the model. This process is repeated iteratively until a final optimal portfolio is obtained, where all asset weights are non-negative. The results of this experiment demonstrate that the average return from a random investment portfolio (without optimization) is approximately 0.0007, with a risk level of 0.0098. After applying the Markowitz model, the return increases to 0.001, while the risk rises slightly to 0.01. This indicates a substantial improvement in return for a minimal increase in risk, highlighting the practical value and applicability of the Markowitz model in real-world investment decision-making.

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