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Modeling and Forecasting the Stock Volatility and Daily Returns of Apple Inc. Using ARIMA-GARCH Models

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Abstract: As a component of the capital of a stock corporation, stocks impact the yield of companies, whose volatility is greatly influencing risk management, investment and macroeconomic analysis. Therefore, if the future volatility is predictable, it can help the investors to grasp the trend of stocks to some extent. The paper mainly studies the yield rate of Apple Inc. based on the calculation of the data from the first half of 2020, using the ARIMA-GARCH model to analyze and forecast the logarithmic returns of individual stocks. The findings show that the prediction for the next ten days indicates the volatility of Apple Inc. will be gradually increasing and the fluctuation range of its yield will also expand. This reflects the enhancement of the uncertainty and the increased risk for short-term investment of Apple market. For calculation results, MAE is 1.15%, RMSE is 1.32% and MSE is 0.02%, showing that the error between predicted values and actual values is relatively small. The research further demonstrates that the GARCH model can grasp the dynamic changes of the market.

Keywords: ARIMA model, GARCH model, volatility, rate of return, stock prices.

1. Introduction

Stocks are a form of securities and the stock market is a market that enables all sorts of enterprises to share equity with each other and optimize the national industrial structure. Studying stock market fluctuations is meaningful for risk control, investment, and macroeconomic analysis. It contains market sentiment and information changes, which can both help investors and enterprises assess risks. In recent months, global stock markets have experienced epic turmoil ever since the COVID-19 pandemic in 2020. When the "Freedom Day" tariff policy of the United States was announced, tariffs of at least 10% were imposed on all imported goods from almost all countries, causing several stock indices in the United States to drop sharply. As of April 5, 2025, the Dow Jones Industrial Average had decreased by more than 2,000 points, the S&P 500 Index had fallen by 6%, and the Nasdaq Index has dropped by 5.9%, with a total market value loss of over 6.6 trillion US dollars. This means that market turbulence has aggravated and trade has been more tense, which may indicate future economic inflation and recession [1,2].

In past studies, using data cycles to reflect market conditions in different time periods, the behavior of Apple stocks price has been more comprehensively reflected and more accurate price patterns have been obtained by analyzing fluctuations to obtain the current status of the stock market [3]. Some studies have used ARIMA models to derive future prediction values for several periods. However, the prediction of overall stock market fluctuations is also an important part of the analysis. Apple Inc.

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is a leading technology enterprise in global market capitalization in the S&P 500 and Nasdaq indices which means its stock price fluctuations have a considerable influence on the overall market. Thus, studying Apple stocks is beneficial to understand the mechanism of large enterprises influencing the financial market.

This paper mainly explores the clustering of Apple's stock market fluctuations and analyzes the changing patterns of its daily returns. It also focuses on verifying the forecasting effect of the GARCH model on Apple's stock market volatility and daily returns. The research will process data from January 2, 2020 to June 30, 2020, a total of 125 initial samples. Starting with constructing an ARIMA model, the ARCH effect test was conducted on the data after that. Then, once the ARIMA model passed the test, the GARCH model would be further fitted on the basis of the tested model. Finally, the model was used to predict the logarithmic returns for the next 10 days and compared with the actual values of the logarithmic returns for the 10 days from July 2, 2020 to July 21, 2020.

2. Data processing

2.1. Data collection and cleaning

The data used in this study are collected from Yahoo Finance. A total of 125 sample points from January 2nd, 2020 to June 30th, 2020 were selected from the existing data, including opening price (Open), highest price (High), lowest price (Low), closing price (Close), trading volume (Volume), adjusted closing price (Adjusted), and trading date. The rate of return is the entire summary of investment and the degree of return on investment has more intuitive statistical attributes than the initial data. Logarithmic rate of return can ensure that the model is non-stationary, so this paper selects logarithmic rate of return as the indicator [4].

The cleaning process ensures that the dates are of Date type and sorted by time, adding the date interval column and calculating the rate of return. The US stock market is closed on weekends and holidays, such as Christmas, Thanksgiving, Independence Day, etc., and the calculation of the rate of return demands data from two consecutive days. If rates of return are calculated continuously from all dates, it will lead to data errors. Therefore, only when the number of days between the next day and the previous day is 1 day, the data will be calculated, otherwise, NA values will be seen as missing values and the first value of the rate of return will be removed. After calculating the rate of return, logarithmic processing will be performed on the data, because the logarithmic rate of return can indicate the cumulative earnings of Apple Inc. over a period of time through the addition form. Compared with the simple arithmetic rate of return, the logarithmic rate of return weakens the influence of extreme values and is likely to be closer to the normal distribution, which is conducive to analysis and prediction of the stock market situation.

2.2. Augmented Dickey-Fuller test

Since conducting time series analysis directly on the cleaned data may lead to the phenomenon of "spurious regression", where there is no actual correlation between variables, but the regression results still have the possibility to show significant correlation and ultimately result in unreliable conclusions and false judgments. The ADF test aims to determine whether the data has a unit root, thereby judging whether the data is stationary. If the data is not stationary, differencing is required in the next step to achieve the purpose of making the data stationary. The null hypothesis of this test is that there is a unit root and the sequence is non-stationary; the alternative hypothesis is that there is no unit root and the sequence is stationary [5].

This study conducted an ADF test on the data, and the result showed that the test statistic of the hypothesis was -9.33 while the critical values were: 1% critical value -3.51, 5% critical value -2.89 and 10% critical value -2.58. Since the test statistic was significantly smaller than all the critical

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values, the null hypothesis was rejected, indicating that the data does not have a unit root, that is, the earnings of Apple Inc. are stationary.

2.3. White noise test

The main function of the white noise test is to determine whether a time series is a purely random sequence. Because a purely random sequence cannot predict its patterns or trends, if the white noise test is passed, then the next step of modeling can be carried out. According to the result of Box-Ljung test, the p-value is 0.01, which is less than 0.05, thus rejecting the null hypothesis. Therefore, the original sequence has autocorrelation and can be used for the modeling steps.

2.4. Check for fluctuations and aggregations

Volatility clustering is a phenomenon in the financial market where the fluctuations of asset prices exhibit persistent concentration. That means during periods of high volatility, there are often subsequent periods of high volatility, and during periods of low volatility, there is a tendency for the volatility to persistently remain low. Here, the research observed the clustering of volatility through logarithmic returns, squared returns, and absolute returns.

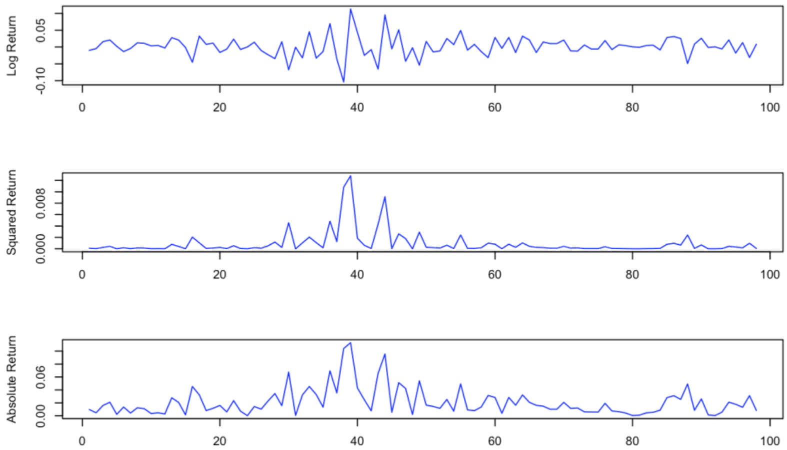

Figure 1: Image of aggregate volatility of yield rates

As shown in Figure 1, the three graphs all fluctuate significantly around the horizontal coordinate 40, while the fluctuations in the preceding and subsequent regions tend to be smoother. This indicates that the market of Apple Inc. oscillated intensively from mid-February to mid-March, and was relatively stable in other periods.

2.5. Check for normality

From Figure 2, most of the data points in the middle section of the figure lie on a straight line, but there are obvious deviations at both ends. The whole yield is approximately normally distributed only in the middle part, while there are differences at both ends from the normal distribution, indicating a fat-tailed feature. Moreover, the fat tail on the left is obviously longer, suggesting a higher probability of negative returns. Therefore, the data do not fully conform to the normal distribution. The paper further provided the commonly used descriptive statistics of the data and conducted a Jarque-Bera test to examine the normality of the data. The p-value is $0.00 < 0.01$. Thus, the null hypothesis is rejected, indicating that the data do not follow a normal distribution [6].

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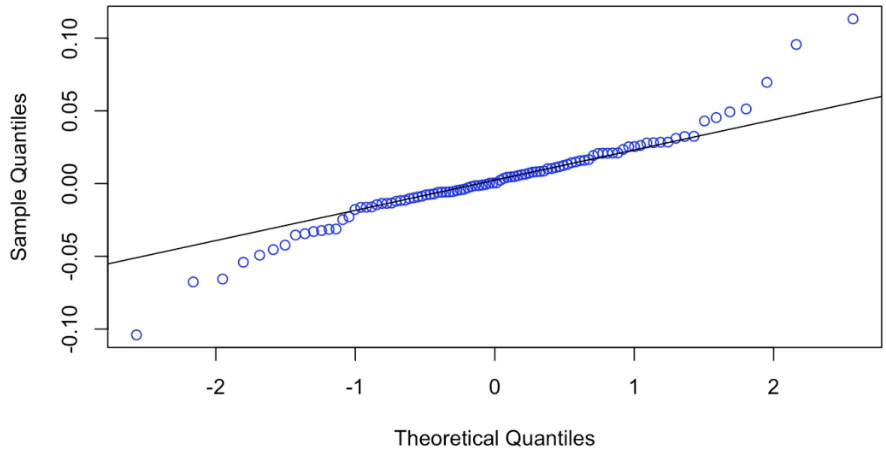

Figure 2: Daily yield rate of Apple Inc. QQ plot

3. Construction of ARIMA model

3.1. Fitting ARIMA model

Once the difference order has been determined, the parameters (p, d, q) of the ARIMA model can be selected by using information criteria [7]. The research results show that the minimum value of AIC is -414.26 and the minimum value of BIC is -406.51. Both of them jointly support the selection of ARIMA(0, 0, 1), indicating that the model has no autoregression and difference, but only first-order moving average with a mean of zero. Under the same data and variables, the values of information criteria are all small, and the residual variance and log-likelihood value both show that the fitting error of the model is small, and the model is superior.

Calculation formulas are:

$$ME = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i| \tag{1}$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{m}\sum_{i=1}^{m} (y_i - \hat{y}_i)^2} \tag{2}$$

According to the calculation formula, this paper obtains ME = 0.01 and RMSE = 0.03, both of which are close to 0, indicating that the model has relatively high prediction accuracy in the future.

3.2. Autocorrelation test

The classical linear regression model assumes that the residuals are mutually independent. If there exists autocorrelation among the residuals, the least squares estimators are no longer the best linear unbiased estimators (BLUE). Therefore, in this paper, the study conducts a test on the residuals of the model.

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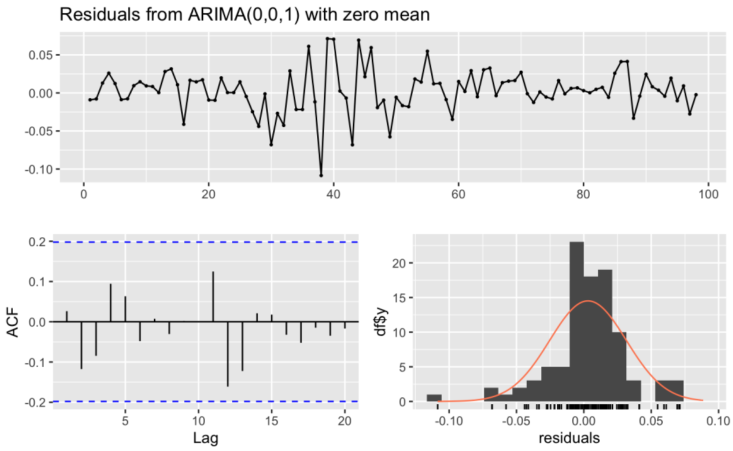

Figure 3: Visualization of ARIMA model residuals

The Figure 3 tells that p-value = 0.92 > 0.05, accepting the null hypothesis and concluding that the residuals of the ARIMA (0,0,1) model do not exhibit autocorrelation. The results indicate that the model has fully extracted the information from the data and has a good fitting effect on the data.

4. Construction of GARCH model

4.1. Heteroscedasticity test

The heteroscedasticity test which is proposed by Engle is conducted to determine whether there is an ARCH effect. This test is carried out by predicting the residuals and then conducting a regression on the squared residuals to determine if there is an effect [8]. Especially in the time series analysis of economic data, there is often a phenomenon of volatility clustering, which indicates that there may be an ARCH effect. If it shows an ARCH effect, the model under the assumption of homoscedasticity will no longer be applicable, and GARCH models need to be adopted to capture the dynamic changes of variance more accurately. Thus, the Ljung-Box test was conducted on the squared returns, with a p-value of 0.01 < 0.05. The study believes that there is significant autocorrelation in the squared returns. Next, test the ARCH effect for the logarithmic returns. The null hypothesis of the ARCH-LM test is that there is no ARCH effect. The result shows that the p-value is 0.00, which is less than 0.01. Therefore, the null hypothesis is rejected, and it is concluded that the logarithmic returns can be modeled using ARCH.

4.2. Estimation under normal disturbance model

Firstly, the parameters of the ARCH model are estimated by MLE. The model is set as ARCH(1) model (single-factor GARCH model, without lag variance term), and the mean model is ARIMA(0,0,0), with the distribution assumption being normal distribution. Then, the coefficients are subjected to robustness tests. According to the Nybloom stability test results, the joint statistic 0.60 < 0.85, the 10% asymptotic critical value, indicates that the model parameters are stable. Although the ARCH(1) model captures some volatility characteristics, the standardized residuals have serial correlation, and the ARCH-LM test shows that the residuals still have ARCH effects. The model does not provide a comprehensive description of the fluctuations. Next, fit the GARCH model.

Similarly, for the GARCH model, through Nybloom test results, the joint statistic 0.60 < 1.07, the 10% asymptotic critical value, indicates that from the perspective of joint testing, the model parameters have stability in statistical significance.

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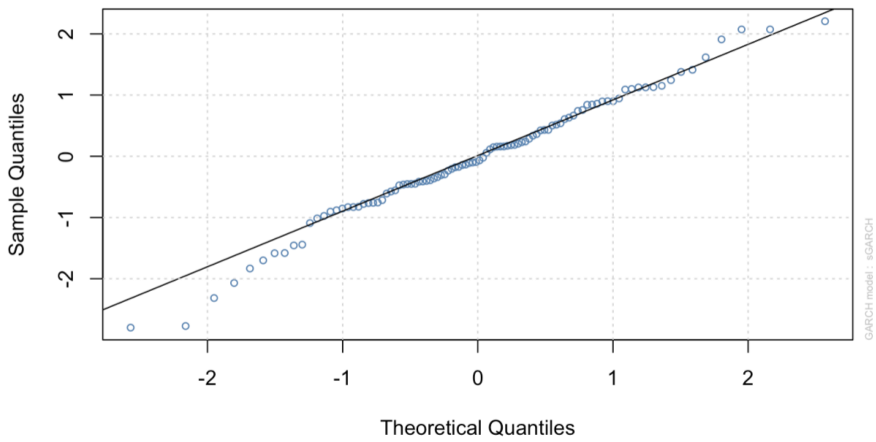

Figure 4: The QQ plot estimated under normal disturbance

However, from Figure 4, it can be seen that both sides show thick tails. Therefore, using ARCH (1)-GARCH (1,1) under normal information to estimate this model may not be very appropriate. In the following step, the paper will consider the ARCH and GARCH estimations under non-normal conditions.

4.3. Estimation of the model

The asymmetric ARCH model needs to be considered in the case of non-normal distribution, named the highly nonlinear volatility model, which is used for situations such as volatility clustering and fat-tailed distribution [9]. Repeating previous operation, it can obtain the estimated parameters after MLE estimation. The joint statistic of 0.65 based on the Nybloom test is greater than the statistics of all critical values. Therefore, the parameters are stable. Before estimating the asymmetric GARCH, a test needs to be conducted. GARCH model generally characterizes the model volatility, while the model assumes that the residual sequence does not have a specific direction of systematic deviation. Through the test, it can be judged whether there is an error in the model residuals. This study conducted the Sign Bais Test, and the results demonstrates that the p-values are all greater than 0.05. Hence, under the common significance level, the null hypothesis of no sign bias is accepted, that is, the model performs well in terms of sign bias, and the estimation results are not significantly affected by significant sign-related bias.

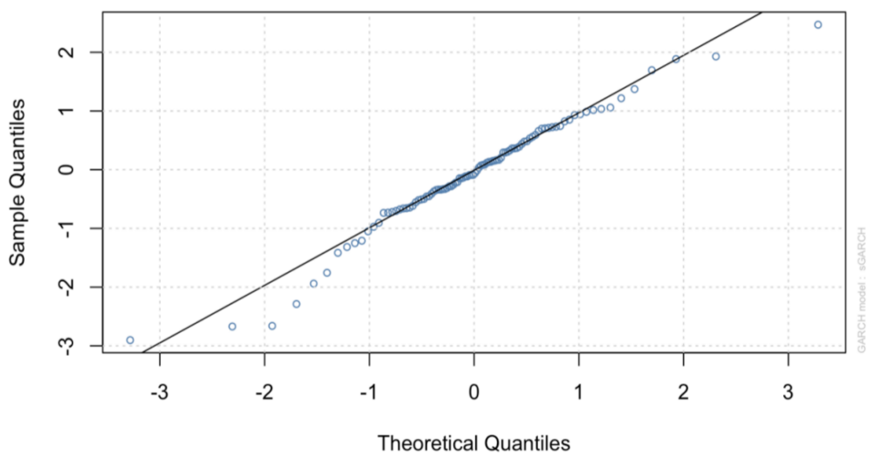

Figure 5: QQ plot under the estimation of t-distribution

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According to Figure 5, for the GARCH (1,1) model under the t-distribution, more points in the middle section are concentrated on the line, but both ends still exhibit fat-tailed feature.

4.4. Model selection

These criteria in Table 1 take both the goodness of fit of the model and the complexity of the model into account. Generally, the smaller the value, the better the model is relative. According to the comparison results, regardless of which information criterion is used, the value of GARCH(1,1) is the smallest. Therefore, GARCH (1,1) is chosen as the optimal model.

Table 1: Selection result of information criteria

	ARCH(1,1)	ARCH(1,1)-t	GARCH(1,1)	GARCH(1,1)-t
Akaike	-4.28	-4.34	-4.51	-4.34
Bayes	-4.20	-4.24	-4.40	-4.24
Shibata	-4.28	-4.34	-4.51	-4.34
Hannan-Quinn	-4.25	-4.30	-4.46	-4.30

Based on the previous parameter estimation results, it can be concluded that $\omega = 0.000047$, $\alpha_1 = 0.411465$, $\beta_1 = 0.587535$. Therefore, the conditional variance equation of GARCH (1,1) is

$$\sigma_t^2 = 0.000047 + 0.4114465 \times \varepsilon_{t-1}^2 + 0.587535\sigma_{t-1}^2 \tag{3}$$

5. Forecasting

5.1. Results of forecasting

The forecasting results from Table 2 indicate that the Forecast Volatility shows a continuous upward trend, rising from 0.02 on T+1 day to 0.03 on T+10 days. It reflects that the model predicts an increasing future volatility. Forecast Return fluctuates significantly, with both positive and negative values, demonstrating the model's dynamic prediction of the direction and magnitude of returns. Future_10_Returns also has both positive and negative values, differing from the Forecast Return, indicating that the model's prediction has certain deviations, but it can still be used as a reference to analyze the fluctuation trend and the characteristics of return changes.

Table 2: Comparison between predicted yield rate and actual yield rate

Day	Date	Forecast Volatility	Forecast Return	Future_10_Returns
T+1	2020-07-02	0.02	0.01	0.00
T+2	2020-07-07	0.02	0.01	0.00
T+3	2020-07-08	0.02	0.00	0.02
T+4	2020-07-09	0.02	0.00	0.00
T+5	2020-07-10	0.02	0.01	0.00
T+6	2020-07-14	0.02	0.00	0.02
T+7	2020-07-15	0.02	0.00	0.01
T+8	2020-07-16	0.02	0.00	-0.01
T+9	2020-07-17	0.02	0.01	0.00
T+10	2020-07-21	0.03	0.01	-0.01

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Figure 6 vividly presents the prediction characteristics of the ARMA - GARCH (1,1) model for market fluctuations and returns, providing a visual basis for analyzing market trends and risks. The blue line (Forecast Volatility) represents the predicted volatility, showing a continuous upward trend. The red line (Forecast Return) represents the predicted return rate, with a relatively large fluctuation range.

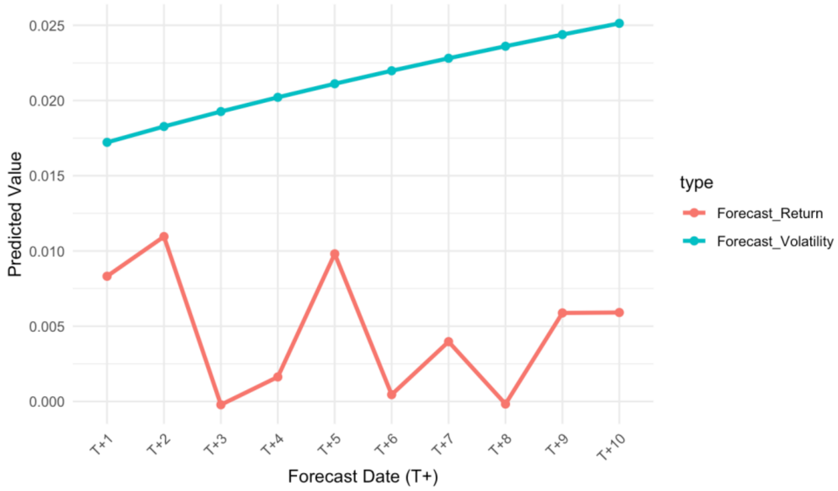

Figure 6: ARMA-GARCH(1,1) prediction: yield and volatility

5.2. Error calculation

In order to test the accuracy of the predicted values, it is necessary to calculate the error between them and the actual values. MAE represents the average absolute error of the predicted values and actual values, reflecting the average magnitude of the prediction error. The smaller this value is, the higher the prediction accuracy is. RMSE is the square root of MSE and has the same unit as MAE. It also measures the error size and is more sensitive to extreme values. MSE calculates the average after squaring the errors, giving more weight to larger errors and paying more attention to the influence of larger errors [10]. The calculation formulas are:

$$MAE = \frac{1}{m}\sum_{i=1}^{m} |y_i - \hat{y}_i| \tag{4}$$

$$MSE = \frac{1}{m}\sum_{i=1}^{m} (y_i - \hat{y}_i)^2 \tag{5}$$

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{m}\sum_{i=1}^{m} (y_i - \hat{y}_i)^2} \tag{6}$$

As shown from Table 3, these indicators assess the prediction effect of the model jointly. The smaller the value is, the closer the predicted value is to the actual value, and the higher the accuracy of the model is. From the calculation results, MAE is 1.15% and RMSE is 1.32%. The difference between the two is not significant. MSE is relatively small, indicating that the model has good accuracy in overall prediction.

Table 3: Calculation result of error value

MAE	MSE	RMSE
1.15%	0.02%	1.32%

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6. Conclusion

This study analyzes and predicts the stock market situation of Apple Inc. in the first half of 2020 using an ARIMA-GARCH model. The research results indicate that based on the AIC and BIC information criteria, the optimal model is ARIMA(0,0,1)-GARCH(1,1). Through the model, it is predicted that the volatility of Apple Inc. over the next ten days will show an upward trend, the fluctuation range of the return rate will gradually go up, and the error will be controlled at a low level, with a relatively high overall fitting accuracy. The prediction results of the research model reflect that the market uncertainty of Apple Inc. has increased, and the risk of investing in its stocks will continue to increase in the short term. Moreover, the positive and negative fluctuations of the predicted return rate indicate that the GARCH model can dynamically catch the development trend of the stock market.

This study is helpful for understanding the aggregation characteristics of the volatility of individual stocks, providing support for yield rate prediction, and filling the gap in the prediction of volatility at the individual stock level. The research results can provide short-term risk warnings for enterprises and investors, serving as a reference for strategy.

Although the model prediction effect demonstrated in this paper is good, the ARIMA model in the research may have overfitted, which will lead to errors in the model prediction results. Moreover, the sample is relatively limited, and after data cleaning, there are less than 100 available sample points. Therefore, in the future, a longer time period can be added, and macro variables can be included to enhance the robustness of the model. At the same time, models such as EGARCH or GJR-GARCH can also be introduced to explore further market mechanisms and volatility structures.

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