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Hedging Longevity Risk with Longevity Bonds: Modeling, Design, and Valuation

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Abstract. The increasing life expectancy of humans poses significant longevity risk to life insurance companies and pension providers. This paper provides a systematic review of longevity bonds as a financial instrument for hedging longevity risk, examines stochastic mortality models, with a focus on the Lee-Carter model, and discusses the classification, operational mechanisms and pricing methods of longevity bonds. The study compares continuous and triggered longevity bonds, highlights their respective applicability to different risk profiles, and evaluates pricing methods such as the risk-neutral pricing method and the Wang transformation method. The study concludes that longevity bonds effectively transfer longevity risk faced by life insurers and pension funds, with continuous bonds suited for gradual risk hedging over extended periods and triggered bonds designed to protect against extreme longevity events. The Wang transformation method proves superior for long-term pricing due to its robustness, outperforming risk-neutral and instantaneous Sharpe ratio approaches in longevity bond valuation.

Keywords: Longevity Bonds, Lee-Carter model, risk-neutral pricing method, Wang transformation method

1. Introduction

As medical technology, public health, health management and income levels of the population continue to improve, people's actual life expectancy is generally higher than projected. The continued decline in mortality rates creates uncertainty, leading to longer benefit periods and higher total payments for pension and annuity products, thus creating an increasingly prominent longevity risk for life insurance companies and governments. As a systemic risk, longevity risk cannot be effectively diversified through the traditional law of large numbers, making effective management a critical issue for both academia and practice.

Effective management of longevity risk needs to start from two core aspects: firstly, constructing scientific stochastic mortality models to improve the accuracy of predicting future mortality changes; and secondly, designing effective financial instruments to realise risk transfer and diversification. Against this background, longevity risk securitisation has gradually emerged as a mainstream solution. The mechanism refers to the issuance of securities by annuity insurance companies through special purpose vehicles (SPVs) that are conditional on repayment triggered by specific longevity risk events, thus transferring the risk to the capital market. Since the 2008

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international financial crisis, the practice of longevity risk securitisation has achieved significant breakthroughs, with innovative derivative products such as longevity bonds, longevity swaps and q-forward contracts emerging in the market. Among them, longevity bonds have become a hot research topic due to their clear structure and wide application.

This paper focuses on longevity bonds, systematically analyzes mainstream mortality prediction models, summarizes their operational mechanisms and pricing methodologies, with a view to providing theoretical references and practical guidance for the innovation and risk management of related financial products.

2. Mortality projections

Prior to 1992, mortality projections were predominantly based on deterministic approaches. Commonly used static mortality models included the exponential model (Gompertz model) introduced by Gompertz, the age-specific mortality model (HP model) developed by Helligman and Pollard, and the CR model proposed by Calliere, among others [1-3].

Considering the uncertainty of mortality changes and the correlation between mortality changes and age and time, Lee et al. proposed a simple dynamic mortality model [4]. Commonly used discrete stochastic models also include Age-period-cohert model (APC model), Cairns-Blake-Dowd model (CBD model), etc.

Continuous stochastic mortality models are also an important class of mortality forecasting models, drawing on the continuous interest rate models widely used in financial and economic fields, and are mainly divided into short-term mortality models and forward mortality models. Milevsky and Promislow first proposed the use of continuous stochastic models to forecast mortality [5], and Dahl analyzed the similarity between mortality force and continuous interest rate and proposed evolving the stochastic interest rate model into a mortality model based on the maturity structure of the mortality force [6].

Among the above three types of models, static models ignore the uncertainty of future mortality rates and fail to capture their dynamic changes, making them usually only suitable for fitting historical data rather than for extrapolated projections. Continuous stochastic mortality models are innovative in their fitting methods, but due to their relatively short development time, the biological soundness of their long-term dynamics and the robustness of their predictions have yet to be further verified. As for the most commonly used discrete stochastic models, Jarner and Moller compared the Lee-Carter model, the APC model, and the CBD model, and pointed out that the Lee-Carter model has a relative advantage [7]. Zuo, Damle and Tuljapurkar mathematically discussed the possibility of accurately measuring the variability of LC model parameters by calculating the sensitivity of these parameters using regression theory. The results show that the LC model is robust to random perturbations, but the distribution of parameter variability is heterogeneous across mortality changes at different ages and years [8].

Let's briefly review the Lee-Carter model here. The model uses a sequence of three parameters: $\{\alpha_x\}$, $\{\beta_x\}$ and $\{k_t\}$ to describe the central mortality rate mx,t , expressed as:

$$\ln m_{x,t}=\alpha_x+\beta_x k_t+\varepsilon_{x,t}, \quad \varepsilon_{x,t}\sim N(0,\sigma^2). \tag{1}$$

Here the age-related parameter α_x gives the average level of mortality at each age and is a relatively time-fixed value; k_t is a time-varying parameter that gives the rate of improvement in mortality; the other age-dependent parameter β_x gives the sensitivity of each age to the parameter

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k_t , and $\varepsilon_{x,t}$ is the error term. By looking at the form of the model, we can see that it is a logarithmic bilinear model, and to avoid parameter non-uniqueness, Lee and Carter propose two constraints:

$$\sum_x \beta_x=1,\ \sum_t k_t=0 \tag{2}$$

In the application, k_t declines over time, corresponding to a decelerated increase in life expectancy at birth, while β_x is usually positive, reflecting age-specific improvements in mortality.

The method of applying the model to predict future mortality (the LC method), is usually divided into two stages: the first stage is to estimate the parameters α_x , β_x and k_t using historical data, and the second stage is to model the fitted values of the parameter k_t using the ARIMA process and to extrapolate the trend of k_t to obtain the prediction results. Among them, there are two major categories of parameter estimation methods, one is the non-likelihood estimation method, mainly including matrix singular value decomposition (SVD), least squares (OLS) and weighted least squares (WLS); the second is the likelihood estimation method, the most widely used is the great likelihood estimation method (MLE) [9].

The Lee-Carter model has been improved upon by many scholars. Basellini, Camarda, and Booth celebrated the 30th anniversary of the LC model by systematically reviewing the most prominent extensions to the Lee-Carter approach in the field of mortality forecasting since its introduction in 1992, as well as the limitations they attempted to address. In response to the validity of the extended models, the study also combed through a review of existing comparative assessments [10].

3. Classification and operation mechanism of longevity bonds

Blake and Burrows were the first to suggest that longevity risk could be hedged by issuing longevity bonds [11]. A longevity bond is a marketable bond whose debt service is linked to the survival probability of a specified population group, with its core function being to diversify longevity risk by separating it from the overall insurance company risk and transferring it to numerous capital market investors. Based on the cash flow structure, longevity bonds can be categorised as continuous and triggered.

Continuous-type longevity bonds achieve continuous longevity risk transfer by directly linking interest payments to the actual survival rate index of a specific population group. Their interest is usually adjusted according to the following formula:

$$Coupon_t = N[r_0 + k(S(t) - \widehat{S}(t))]\ t = 1, 2, \ldots, T. \tag{3}$$

Where, N refers to the par, the base rate r_0 is often referenced to LIBOR or SOFR plus a certain spread, k is the risk-sharing factor, $\widehat{S}(t)$ is the reference survival index set at the time of issuance and $S(t)$ is the actual survival index published for the observation period. The structure allows the issuer to gradually and smoothly transfer the additional cost of liabilities due to rising survival rates to investors, achieving an effective hedge against ongoing, moderate longevity risk. Advantages of this bond type include risk spreading across multiple periods, cash flows more closely matching liability growth patterns, and relatively controllable basis risk; however, the structure is complex, with high reliance on data quality and modeling, and limited market liquidity. Typical examples are the EIB/BNP bonds designed by BNP Paribas and proposed to be issued by the European Investment Bank in November 2004.

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In contrast, triggered longevity bonds rely on pre-set triggering conditions, with the occurrence of survival thresholds, life quartiles, etc., determining whether to activate the principal loss or deferral mechanism. This type of bond usually pays interest and repays capital like an ordinary bond when it is not triggered, while investors may face a partial or total loss of principal in the event of a triggering event, and the issuer is provided with funds to compensate for losses due to extreme risks. Trigger bonds feature a simple and transparent structure, are better suited for hedging sudden and catastrophic longevity risk events, and may be less expensive to issue. However, they are subject to higher basis risk and the "cliff effect", i.e., the risk transfer is not continuous and may not be able to effectively cover evolving longevity trends, as represented by the Kortis longevity bond.

Overall, continuous bonds are better suited for pension schemes needing to hedge liabilities over long, stable growth periods, while triggered bonds are better suited for life or reinsurance companies seeking protection against extreme longevity risk. The two bond types also differ in investor profile: continuous bonds attract organizations preferring stable cash flows and able to bear systemic risk, while triggered bonds suit investors seeking higher yields and willing to accept tail risk.

4. Pricing methodology for longevity bonds

The theoretical dynamics of longevity bond pricing models revolve around several core pricing approaches, including the risk-neutral approach, the Wang switching approach, the instantaneous Sharpe ratio approach, and other derivative pricing approaches, which are constantly being compared, modified, and extended through theoretical and empirical studies.

The risk-neutral pricing approach was first proposed by Cox and Ross and has been widely used in the valuation of financial derivatives under incomplete market conditions [12]. In pricing longevity risk securitisation products using the risk-neutral approach, Deng et al. pointed out that the most important thing in using risk-neutral pricing method is to choose an appropriate risk-neutral measure, on the basis of which other necessary parameters are calculated [13]. Cairns et al. assumed that the market prices longevity risk consistently and conducted risk-neutral pricing based on the issue price of EIB longevity bonds [14]. Song et al. priced EIB/BNP-type longevity bonds by introducing three relative entropies and determining the optimal risk-neutral measure based on the minimum relative entropy criterion [15].

The Wang transformation method is an important probabilistic distortion technique, which was systematically proposed by Wang on the basis of Venter's study, and has been gradually developed into both one-factor and two-factor forms [16,17]. The latter corrects for parameter uncertainty by introducing a t-distribution, and is particularly suitable for stochastic adjustment in mortality modelling.

The practicality and robustness of Wang's transformation method have been verified many times in subsequent studies: Lin and Cox apply this methodology to the pricing of longevity bonds and estimate the market price of longevity risk [18]. Denuit, Devolder, and Goderniaux successfully combine the Lee-Carter model with Wang conversion pricing to construct a complete pricing framework for Survivor Bonds based on a publicly available mortality index. They demonstrated the stability of the method by deriving the upper and lower bounds of bond prices under the Wang conversion risk measure and achieved effective calibration of the risk parameters based on the Belgian market data [19]. Chen et al. find by comparison that the Wang conversion method outperforms the risk-neutral pricing method and the instantaneous Sharpe ratio method in terms of long-term robustness and is more suitable for pricing long-term bonds [20]. The study of Fan et al. also continues this idea by designing longevity bonds with double touch-point characteristics and

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conducting pricing simulations using two-factor Wang transformation, which further enriches the application scenarios of the method [21].

5. Conclusion

This paper systematically reviews the operating mechanisms, classification, and pricing methods of longevity bonds, focusing on the application of stochastic mortality models—such as the Lee-Carter model—in longevity risk prediction, and compares mainstream pricing approaches, including risk-neutral pricing and the Wang transform. The study shows that longevity bonds are effective in transferring longevity risk faced by life insurers and pension funds, with continuous bonds suitable for gradual risk hedging and triggered bonds for extreme events, and that the Wang transformation method outperforms others in long-term pricing due to its robustness.

Future research should focus on integrating multi-factor mortality models, dynamic risk measurement, and AI techniques to improve pricing accuracy and market acceptance, thereby promoting the practical development of longevity risk securitisation.

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