The Investigation into the Extent of Investors' Irrationality Based on GARCH-M Model

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Abstract: Research in risk preference suggests that, in violation of economic man hypothesis, most investors are economically irrational, which is reflected through the phenomenon of preference reversal. However, there is a scant of study to measure the extent how irrational individuals are. Inspired by the way how efficient market is defined, this paper proposes a measurement of preference reversal and divides irrationality into two types. Furthermore, based on the data of market index and GARCH-M model, this paper finds that risk preference inconsistency always exists and investors in smooth time are weakly irrational.

Keywords: irrational man, garch-m, market index

1. Introduction

Economic man theory performs an essential role in classical economics. What is economically rational man? By definition, rational man assumes that people are always rational and self-interested and pursue their subjective goals in an optimal manner. In 1971, an experiment that confirms the presence of preference reversals was published[1], challenging the traditional hypothesis in the expected utility theory that humans are economically rational, i.e., homo economicus[2, 3]. As an increasing number of empirical research emerge, now it's not refreshing to see this hypothesis are refuted by real data[4].

Although now the risk preference reversal has been regarded as the evidence that people are not economically rational, investigation into the irrational man is still needed because there is a scant of studies which could uncover how irrational people are. Specially in the context of epidemic, people's preference tends to be more violate and thus easier to reverse and be irrational. Therefore, measuring the extent how individuals are irrational plays a significant role in figuring out people's behaviors.

The rest of this article is structured as follows. Previous work regarding this field will be reviewed in Part Ⅱ. Subsequently, theoretical discussion is held in Part Ⅲ. Then the empirical design, models, variables as well as data we need will be detailly described in Part Ⅳ. Later, Part Ⅴ shows the empirical process and discuss the regression results we have obtained. Finally, Part Ⅵ draw a conclusion.

2. Literature Review

Rational man hypothesis is closely correlated to expected utility. The term expected utility dates back to Bernoulli[5]. He suggested that under the condition of risk and uncertainty, the goal of individual decision is to obtain the maximum expected utility value rather than the maximum expected amount value.

Later, the earliest formal survey on expected utility was conducted by Von Neumann and Morgenstern[6] in 1944. Based on a set of axiomatic hypotheses, they concluded that in the presence of risks, the utility level of the final decision result is obtained by the decision-making individual after weighted evaluation of various possible results. The decision-making individual pursues the maximization of expected utility after weighted evaluation, and the weight of the utility of each result is the probability of the occurrence of them. This is a groundbreaking progress in the utility theory.

However, following studies showed that the real situations are against the strict axiomatic hypotheses which lay the foundation for the expected utility theory.

In 1954, Savage proposed the subjective expected utility theory , in which the subjective probability was applied to replace the objective probability, thus implying the individual differences among diverse decision makers to some extent [7]. Additionally, Aumann established an incomplete expected utility theory through a discussion about lottery choices, which excluded the axiom of completeness [8]. Moreover, by releasing certain hypothesis, Hausner eliminated one of the axiom mentioned above, the axiom of continuity, and then extended Von Neumann, Morgenstern expected utility theory to the multidimensional expected utility theory [9]. All the research shed light on the further studies regarding the inconsistency in preference.

With the development of empirical research in economics, the previous axiomatic hypotheses are faced with more challenging doubts. Two of the most famous experiments are proposed by Allais [10]and Ellsberg [11]. Both of their experiments show the identical result. That is, the inconsistency in preference did exist because the experiments' results are contrary to typical utility theory.

Subsequently, due to the proposal of Allais Paradox and Ellsberg Paradox, a series of studies with supplement and amendment come along successively, such as rank dependent expected utility theory[12, 13], weighted utility function, prospect expected utility model, etc. The two most outstanding models are Quiggin's expected utility theory[13] and Machina's extended utility theory[14]. In Machina's theory, he revised the expected utility model and proposed the Machina triangle, which expanded the indifference curve into a sector reflecting local linear approximation and explained the paradox in the subsequent research[14].

In 1979, Kahneman and Tversky put forward the prospect theory [15, 16]and this theory questions the whole expected utility theory. In their framework, the utility function and objective probability are replaced by the value function and subjective probability function, respectively. What they indicate contradicts all the axiomatic hypotheses used in the traditional expected utility theory.

Furthermore, in 1982, Looms and Sudgen proposed the regret theory [17]. This theory suggests that people will choose certain gains over easy risks because they are more likely to regret them, meaning they are regret aversion. Such behaviors prove that the preference with independence consistency may not exist.

Up to now, there have been increasing number of papers emerging to show that the inconsistency in people's preference did exist. Therefore, recent studies begin to focus on this topic.

Some research attribute this inconsistency symptom to psychological anomaly [18], while others concentrate on breaking new ground through the empirical methods. One of the mature theories is the preference reversals, which is firstly proposed by Sarah Lichtenstein and Paul Slovic[1]. Through a gambling decision test, the authors concluded that people's actual choices are inconsistent with their utility judgments.

The conclusion has been proven by other researchers many times. For example, through a test, Lindman obtained a similar result [19]. In 1980, Mowen and Gentry found that the groups that can make joint decisions will show a stronger tendency to reverse preferences than individuals [20]. In

1982, after a series of revision in the factors that may influence people's preference, economists Grether and Plott also got the identical result [21].

All these preference reversals mentioned above are embodiment of preference inconsistency, the opposite of the classic hypothesis of economics, named economic man, or homo economicus [2, 3]. However, a corresponding question arises as well. That is, although previous studies on preference reversals have been well documented, there is still a scant of research that dig into it to reveal the underlying logic against the economic man, i.e., the man is uneconomic or economically irrational.

The existing studies with respect to the irrational man are limited. Some scholars may argue that never in the history of the economic discipline has selfishness constituted the core of the Homo economicus model, thus unnecessary to discuss this hypothesis [22], while others consider it as a strict hypothesis in typical economic studies and it did contributes to the basis of expected utility theory and subsequent derivative theories, as the paper Beyond Economic Man summarizes [23]. Up to now, mainstream literature have been long focusing on the irrationality of managers in the corporate, which helps develop the behavioral asset pricing.

To sum up, both reality and empirical research show the preference reversals, imposing a challenge on the expected utility theory. In other words, these studies show that decision makers or investors are economically irrational. Most existing papers, however, stay on the discovery of this phenomenon while insufficient literatures are conducted, and not enough models are established to illustrate how irrational we are. Specially, in the context of epidemic, people tend to exhibit systematically irrational behaviors [25] and thereby it is more crucial to study the extent of irrationality.

3. Theoretical Background

3.1. Theoretical Discussion

In traditional expected theory[6], decision makers' expected utility can be captured by the equation as follows:

$$U = u(\sum_{i=1}^{n} P_i w_i) \tag{1}$$

where the U denotes the utility; u is the utility function; P_i is the subjective probability of various results and w_i represents the wealth under diverse conditions.

Because of risk aversion[6], the utility relations can be written as:

$$U = u(\sum_{i=1}^{n} P_i w_i) > \sum_{i=1}^{n} P_i u(w_i) \tag{2}$$

To describe risk aversion, a new concept, the coefficient of risk aversion, should be introduced into the utility function.

To capture the coefficient of risk aversion, an effective empirical model is GARCH-M model[26]. According to this model, investors' risk preference is measured by the risk premium coefficient:

$$\begin{gathered} r_t = c + \gamma\sigma + \varepsilon_t \\ \varepsilon_t = \sigma \cdot \eta_t \\ \sigma^2 = \alpha_0 + \alpha_1 \cdot \varepsilon_{t-1}^2 + \alpha_2 \cdot \sigma_{t-1} \end{gathered} \tag{3}$$

DOI: 10.54254/2754-1169/7/20230247

where r_t is the daily stock return; σ^2 is measured by the third equation and γ denotes the risk premium the investor requires and thus represents the risk aversion.

Here, we use the γ in the regression to denote the coefficient of risk aversion. Different sign of γ denotes individual's different risk preference. If $\gamma > 0$ then we say the investor is risk averse and ask for γ compensation for taking a unit more risk.

After introducing the coefficient and inspired by the method adopted by DeBont and Thaler[27], we can now define the preference reversal.

First of all, we define:

$$E(\gamma_t|F_{t-1}) \quad (4)$$

The F_{t-1} represents the collection of all information at the time $t-1$, and the conditional expectation denotes the mean of coefficient of risk aversion at time t with reflecting all the information at time $t-1$. This thought is inspired by the research methods proposed by Fama[28].

Then, assume we only study one period situation, and we can define:

$$\theta = E(\gamma_{t+1}|F_t) - E(\gamma_t|F_{t-1}) \quad (5)$$

It's easy to see, if the investor's risk preference is consistent, i.e., the preference will not reverse, the risk aversion will not change as time goes and thus $\theta = 0$; However, if $\theta \neq 0$, it means the inconsistency of preference exists. But does the preference reversal happen? The answer may be not because here we only know that θ is different from 0 but do not know it just increases (or decreases) a bit or it totally changes plus or minus sign.

Here, we assume all the investors are risk averse and thereby at the initial time t, $E(\gamma_t|F_{t-1}) > 0$. If the preference reversal happens, the γ will totally turns its positive value to the negative one. So, at last, $\theta < 0$.

In conclusion, under the assumption that all the decision makers are risk averse initially: If $\theta = 0$, the preference is consistent. If $\theta \neq 0$, the preference is inconsistent, but it may not be due to preference reversal. If $\theta < 0$, the preference reversal happens. If $\theta > 0$, the preference is inconsistent but not reversal.

Then, we assume that the inconsistency in investors' risk preference is not affected by macro factors and thereby we could let the preference inconsistency to prove and measure people's irrationality.

When the reversal happens, it can undoubtedly prove that the irrational man exists. But when $\theta \neq 0$ without reversal happens, i.e., $\theta > 0$ can it also indicate that people are irrational? The answer is yes since investors are biased in their preferences at different time.

How to measure the extent? Using the idea similar to "net profit margin", we can define the ratio of θ over initial risk aversion:

$$\varphi = \frac{E(\gamma_{t+1}|F_t) - E(\gamma_t|F_{t-1})}{E(\gamma_t|F_{t-1})} \quad (6)$$

Thus, the ratio can be used to roughly estimate the extend how irrational we are. Given the fixed time period (here we only consider one period), the larger the ratio is, the more irrational we will be.

Furthermore, we can also roughly divide irrationality into two forms based on the absolute value of φ compared with 1. If $|\varphi| < 1$, we call it weak irrationality because of a subtle change in risk preference. If $|\varphi| > 1$, we call it strong irrationality because of a significant change in risk preference.

For an intuitive understanding, the risk aversion coefficient should follow the path as the figure below shows:

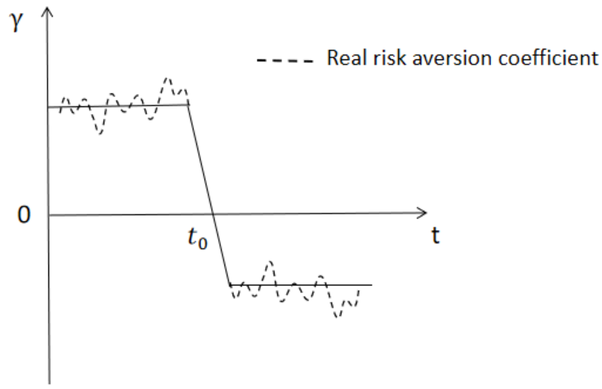

Figure. 1: Risk Aversion Coefficient Over Time.

If we assume (1) the initial preferences of investors are risk averse; (2) the inconsistency in investors' risk preference is not affected by macro factors; (3) we only consider one time period time, then the real change path of risk aversion coefficient over time will initially go up and down around a positive value and after a reversal, the value will experience a steep fall to go around a negative value. This process will reflect investors are economically irrational, and the degree of change measures the extent how they are irrational.

3.2. Hypothesis

After all the analysis mentioned above, we now can conclude our hypothesis as follows:

(1) The inconsistency of preference exists.

(2) In a smooth economic climate, investors are always weak irrational.

(3) In unsteady economic situations, such as during epidemic, investors are strongly irrational.

4. Model Specification

To quantitatively investigate into the investors' irrationality, the first thing is to obtain the risk preference coefficient. Based on the theoretical analysis above, the risk preference is usually correlated with returns and the most common scenario is the returns in capital market. So, the GARCH-M model can be applied to obtain the risk preference coefficient:

$$r_t = c + \gamma\sigma^2 + \varepsilon_t$$

$$\varepsilon_t = \sigma \cdot \eta_t$$

$$\sigma^2 = \alpha_0 + \alpha_1 \cdot \varepsilon_{t-1}^2 + \alpha_2 \cdot \sigma_{t-1}$$

All coefficients in the equation have been illustrated in theoretical part. In this paper, r_t and σ will be replaced by the returns and risks of market index, respectively. Besides, considering the hypothesis we need to test and the epidemic variable, this paper divides the GARCH-M into three phases, taking 2012 and 2020 as cut-off points, respectively.

DOI: 10.54254/2754-1169/7/20230247

5. Empirical Research

5.1. Summary Statistics

In this part, summary statistics will go around the returns of market index and here, HS300(000300) is selected as example here to be detailly described.

The summary statistics of HS300 are concluded in Table 1 as follows.

Table 1: Summary Statistics of HS300.

	Year<2020			Year>2020			All Years
	Mean	Std	Median	Mean	Std	Median	Mean
Returns	.00056	.01725	.0009	.000112	.013256	.0008	.00235
Opening Price	2954.7	987.84	3094.7	4713.6	485.28	4841.2	3193.3
Closing Price	2951.7	988.20	3087.3	4715.7	484.11	4843.9	3193.2
Rise	.05481	1.7241	.08709	.00865	1.3254	.073903	.23270

Besides, the variation in the daily rise of HS300 is also plotted to show its trend. It's easy to see from the figures that the daily rise and fall of the HS300 has strong fluctuation aggregation, proving that the returns of HS300 apply to GARCH-M.

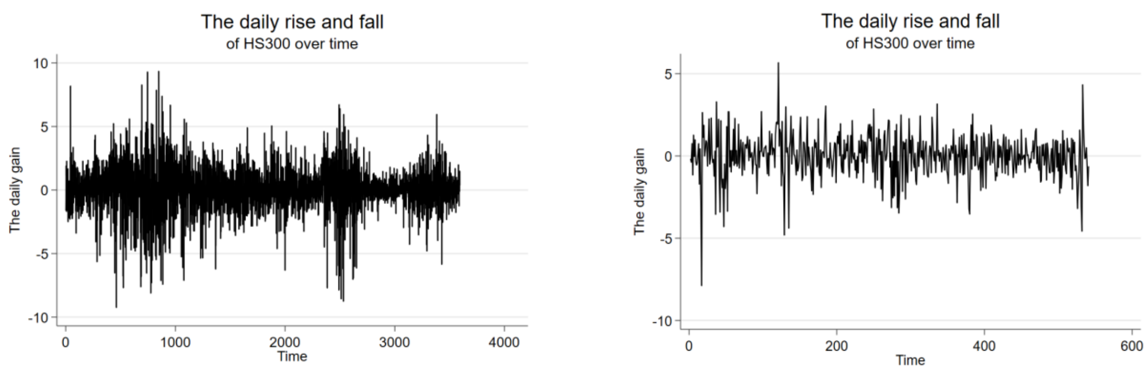

Figure. 2. The Figure shows the daily gain of HS300 over time. The first one shows the daily gain before 2020 while the second one shows corresponding data after 2020. The high volatility proves that the data can be applied to GARCH-M model.

5.2. Test of Stationarity and Arch Effect

Before regression, corresponding tests should be performed regarding GARCH-M. They are stationarity test and Arch effect test, which are premises of such time series model. For stationarity test, this paper take Dickey-Fuller test because the data HS300 is simple and limited.

Because of three phases we plan to divide this model, corresponding three tests are required here. The results of stationarity test and Arch effect tests are listed in Table 2 and Table 3, respectively.

Table 2:Results of Stationarity Test.

	Year>2020	2012<Year<2020	Year<2012
Test-Statistic	-23.314	-42.783	-41.739
P-value	0.0000	0.0000	0.0000

DOI: 10.54254/2754-1169/7/20230247

As we can see from the results of the Dickey-Fuller test for respective three phases, all of their P-values are euqal to zero, meaning that all of them passes the stationarity tests and apply to into time series regression model.

Then, Arch effect test should be performed to test how much lag should be adopted in time series regression for each phase. Here we test four lag for each phase to see the effect.

Table 3: Arch Effect Test.

Lags	Year>2020	2012<Year<2020	Year<2012
1	0.0001	0.0000	0.0001
2	0.0152	0.0000	0.0000
3	0.2375	0.0000	0.0000
4	0.2277	0.0000	0.0057

As is shown, the value in the table denotes the probability larger than chi-square. The smaller the value is, the more significant it will be when adopt corresponding lag in the regression. When it's after 2020, the probability of the one lag is the smallest, meaning that one lag is the best lag this phase should adopt. Similarly, the values of year after 2012 and before 2020 are all zero, means all lag is good. To simplify the process, one lag is chosen as well. For year before 2012, however, two lag is best.

5.3. Regression Results

The regression results of three phases are shown in the following table.

Table 4: Results of GARCH-M.

	Year>2020	**2012<Year<2020**	**Year<2012**
R/ARCHM			
_cons	-.0079347 (-1.48)	-.0023285 (-1.64)	-.0074518** (-2.10)
σ^2	.652636 (1.51)	.2230018** (2.01)	.428467** (2.26)
ARCH			
Arch L1.	.1084256** (2.43)	.2297346*** (8.61)	.1134044*** (5.81)
Garch L1.	.6669369*** (2.79)	.6884693*** (12.69)	1.80325*** (5.56)
_cons	.0000312 (0.87)	.0000103 (1.39)	-.0003721*** (-3.05)

What we want here is the coefficient of σ^2, which indicates the risk aversion coefficient, i.e., risk preference should be focused on. As the regression results show, the coefficients in the three phases are all positive, meaning the systematic risk preference is risk aversion. In addition, both this coefficient between 2012 and 2020 and the one before 2012 are significant at 5% significance level, showing the results are reliable and it's safe to say that we can adopt them as the coefficient we need.

The coefficient after 2020, however, is not significant at all. To some extent, it reflects the instability of systematic risk preference after the eruption of epidemic and thus it seems to keep in line with the logic of our hypothesis, that is, the risk preference in unstable situations is more fluctuate.

But due to its insignificance, it's hard for us to jump to conclusions. From another point of view, this implies that the following study regarding 2020 based on PSM-DID is necessary.

Before that, we might as well take the coefficient in 2020 as the reliable risk preference and then we can discuss them in more detail to test our hypothesis.

First, we discuss the smooth economic climate. According to:

$$\theta = E(\gamma_{t+1}|F_t) - E(\gamma_t|F_{t-1})$$

We know the fact that from the time before 2012 to the time between 2012 and 2020, the change of risk preference did happen, and $\theta = -0.2054652$. Based on the analysis above, here $\theta < 0$ and the initial risk preference is risk aversion, so the preference reversal happens, implying that less risk averse people are between this time. Furthermore, according to:

$$\varphi = \frac{E(\gamma_{t+1}|F_t) - E(\gamma_t|F_{t-1})}{E(\gamma_t|F_{t-1})}$$

Here, $\varphi = -0.47953565$, that means, $|\varphi| < 1$. By our definition before, this is weak irrationality because just a subtle change happens. These results prove our hypothesis (1) and (2), i.e., the inconsistency of risk preference did exist due to θ derives from zero and people are always weak irrational in smooth economic climate due to $|\varphi| < 1$.

Then, we come to the epidemic time, the unsteady economic situations. Consider the two formulas again. Here, $\theta = 0.4296342$, a positive value, indicating that people become more risk averse than before. Besides, $\varphi = 1.92659521$, whose absolute value is distinguishingly larger than one, indicating a strong irrationality here due to our definition before. This result proves our third hypothesis, that is, in unsteady economic situations, such as during epidemic, people are economically strong irrational.

The two discussions above perfectly satisfy our expectations with testing all our hypothesis proposed. These results can also be described in an economic intuition. In smooth time, people can guarantee their quality of life and the overall living standard of society is relatively high, so people's risk perception is weaker thus causing the risk preference to change without rationality and this trend is always weak. However, in the time that is not so smooth, affected by some negative news, it'll be easier for people to be anxious and thus their risk aversion increases as well. This behavior is to protect themselves from external risks.

6. Conclusion

In conclusion, our results significantly support the first two hypothesis that this paper proposed before. That is, inconsistency of risk preference did exist and in smooth time, investors are always weakly irrational. The third hypothesis seems to be also proved but the result is not significant, which may be due to insufficient data because of the short duration of the epidemic up to now. Therefore, this hypothesis needs to be verified by follow-up studies.

Although it's an ideal result meeting our expectations, there are still a few of shortcomings. The main one is that the data we use are all market index data, behind which has an underlying assumption that the market index can perfectly reflect the investors' risk preference. This assumption, however, is not quite consistent with the facts and this may cause representative bias. To solve this problem, various market index can be used to test the hypothesis.

All in all, our first two hypothesis was basically confirmed, and more work needs to be done to polish the research.

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