

Research on Portfolio Optimization under Markowitz Model (MM) and the Index Model (IM): A Comparative Analysis Across Phases from 2004 to 2024

Guochang Wan

*Faculty of Life Sciences and Medicine, King's College London, London, The United Kingdom
K23164625@kcl.ac.uk*

Abstract. This paper investigates the impact of excluding the S&P 500 Index (SPX) on investment outcomes within portfolio optimization, focusing on the Group 4 stock pool over the period from September 2004 to September 2024. This paper compared the performance of the Markowitz model (MM) and the index model (IM) under the constraint $W_{\text{spx}}=0$. The analysis is divided into four phases: 2004–2007 (pre-crisis expansion), 2008–2011 (financial crisis and recovery), 2012–2019 (technology-led expansion), and 2020–2024 (pandemic and artificial intelligence wave). Results indicate portfolio performance is highly contingent upon market conditions. During the 2008–2011 financial crisis, excluding SPX yielded the most pronounced negative impact: IM, overly reliant on β , substantially allocated to financial stocks, leading to excessive risk concentration; conversely, MM reduced financial weightings via a complete covariance matrix, redirecting capital towards industrials and technology to achieve higher risk-adjusted returns. Conversely, during technology-driven expansion phases (2012–2019, 2020–2024), the exclusion of SPX had relatively minor effects as technology stocks dominated portfolio performance. In these periods, IM portfolios achieved higher absolute returns but exhibited excessive concentration and heightened risk; MM maintained balanced diversification, consistently delivering superior Sharpe ratios. For investors, MM should be prioritized during high-volatility or crisis phases. In technology-dominated expansionary phases, IM may be considered for those with higher risk tolerance. For policymakers, reliance on simplified index models during crises may amplify systemic risks, necessitating guidance for investors and institutions towards more robust risk management frameworks.

Keywords: Markowitz Model, Index Model, Portfolio Optimization, Efficient Frontier, Sharpe Ratio.

1. Introduction

The investment portfolio in the capital market is a part that investors are particularly concerned about. When market fluctuations are significant, investors will use the investment portfolio to better diversify risks and thereby achieve greater returns.

Many studies have conducted a series of analyses centered on portfolios. For example, Guo explored the selection of portfolio models by investors with differing preferences [1]. Some scholars employed the mean-variance model as a foundation to elucidate diversified portfolios, summarizing portfolio optimization models and solution methodologies under regularization [2]. Moreover, some literature elucidated the significance of portfolios, measuring portfolio risk and the optimal value of expected returns, utilizing portfolios to secure greater returns [3]. Employing the Markowitz Model (MM) and Index Model (IM) to construct optimal portfolios enables the observation and measurement of their potential as financial investment instruments within capital markets. Utilizing these models provides investors with superior options for decision-making when selecting components for optimal portfolios [4].

The aim of this paper is to analyze the effect of excluding the S&P 500 index (SPX) from the portfolio optimization of group 4 across four different periods: 2004-2007, 2008-2011, 2012-2019, 2020-2024. To gain a better understanding of portfolio performance across different time periods, this paper applies both the Markowitz Model (MM) and the Index Model (IM) to investigate portfolio optimization across four distinct time frames [5,6]. Through this analysis, it can gain insight into the differences between two distinct portfolio models, thereby providing investors with a reference point for selecting the more optimal portfolio model under varying market conditions. The focus is on excluding SPX ($W_{spx}=0$) and comparing the portfolio efficiency, diversification performance shifts under this constraint.

2. Data and methods

2.1. Data

The compositions of group 4 include Financials (JPM, MS, USB, WFC), Industrials (EMR, HON, CAT, DE, MMM, FDX, UNP, UPS), and Technology (AAPL, ACN, IBM, AMD, INTC, NVDA, QCOM, TXN), plus LIN (Basic Materials). Data daily returns from 2004-09-17 to 2024-09-20 were aggregated into monthly returns to reduce non-Gaussian noise. The risk-free rate is 1M Fed Funds rate. The division by period reflects different market system (pre-crisis, crisis, expansion, pandemic recovery). All the data from Bloomberg.

2.2. Models

2.2.1. MM (mean-variance model)

This model represents a seminal research achievement. Its core principle posits that investors pursue returns while simultaneously considering risk, measured by the variance or standard deviation of returns. As correlations between assets typically fall below 1, diversification can reduce overall risk. The Markowitz model employs a covariance matrix to determine optimal asset weights, thereby maximizing returns for a given level of risk or minimizing risk for a given level of return. Variance is used to represent the risk of a portfolio, and the mean is used to represent the expected return of a portfolio [7].

2.2.2. IM (single-factor index model)

This model represents a simplification of the Markowitz model. This model posits that asset returns are primarily driven by market returns (such as the S&P 500 Index), while accounting for the idiosyncratic risk of individual assets ($R_i = \alpha_i + \beta_i R_m + \epsilon_i$). This logic significantly reduces the

number of parameters requiring calculation: while the Markowitz model necessitates a full covariance matrix, the Index Model only requires estimating each asset's beta value and residual variance. Consequently, the Markowitz model is more precise, capable of reflecting complex correlations between assets, but relies heavily on substantial data; the Index Model is computationally straightforward and suitable for large-scale portfolios, though it assumes market factor dominance [6].

2.2.3. Optimization & constraints

Standard mean-variance optimization with $W_{\text{spx}}=0$ constraint. It is needed to get Efficient Frontier, Global Minimum Variance Portfolio (GMVP), Tangency Portfolio (Max Sharpe), Maximum and minimum return.

3. Results

3.1. Pre-crisis (2004-2007)

The research constructed portfolio models for the period Pre-Crisis (2004-2007). The results presented in Table 1 show the return is 14.2%, the risk is 12.5%, and the Sharpe is 0.69 under MM Tangency Portfolio and the return is 16.0%, the risk is 14.9%, and the Sharpe is 0.64 under IM Tangency Portfolio.

Weight Insights: MM favored a balanced allocation between Industrials (CAT, HON) and Tech (AAPL), while IM concentrated more in Financials (JPM, MS). Excluding SPX raised volatility. IM is influenced by High-beta financials,

3.2. Crisis & recovery (2008-2011)

The research constructed portfolio models for the period Crisis & Recovery (2008-2011). The results presented in Table 1 show the return is 7.9%, the risk is 15.4%, and the Sharpe is 0.27 under MM Tangency Portfolio and the return is 9.1%, the risk is 21.2%, and the Sharpe is 0.21 under IM Tangency Portfolio.

Weight Insights: MM allocates more to the Tech (IBM, AAPL) and Industrial (UPS) and reduced financials less than 10%. IM still allocates more financials (JPM, WFC), underestimating risks. MM achieved lower volatility and higher risk-adjusted performance than IM. Excluding SPX proved most damaging here. MM was relatively more robust because MM can detect the increase in financial risk and reduced the weight however the IM is relied on Beta which failed in crisis.

3.3. Tech boom (2012-2019)

The research constructed portfolio models for the period Tech Boom (2012-2019). The results presented in Table 1 show the return is 18.7%, the risk is 14.1%, and the Sharpe is 0.94 under MM Tangency Portfolio and the return is 20.3%, the risk is 16.8%, and the Sharpe is 0.88 under IM Tangency Portfolio.

Weight Insights: MM diversified across AAPL, TXN, NVDA, ACN. IM concentrated on AAPL and NVDA. Excluding SPX had little negative effect. The results align with Fama & French [8], Tech stock paly the dominant factor.

3.4. Covid-19 & AI surge (2020-2024)

The research constructed portfolio models for the period Covid-19 & AI Surge (2020-2024). The results presented in Table 1 show the return is 22.1%, the risk is 18.6%, and the Sharpe is 0.87 under MM Tangency Portfolio and the return is 25.5%, the risk is 22.9%, and the Sharpe is 0.84 under IM Tangency Portfolio.

Weight Insights: MM balanced allocation. IM concentrated in NVDA, AMD. SPX exclusion had limited bad effect because of tech in the dominant position. The finding's results agree with the opinions from Pastor & Veronesi on technological shocks reshaping returns [9].

The table 1 shows returns, risks and Sharpe ratios by using MM and IM during four different periods.

Table 1. Results of MM and IM

Period	Return		Risk		Sharpe ratio	
	MM	IM	MM	IM	MM	IM
Pre-Crisis (2004-2007)	MM	IM	MM	IM	MM	IM
	14.2%	16.0%	12.5%	14.9%	0.69	0.64
Crisis & Recovery (2008-2011)	MM	IM	MM	IM	MM	IM
	7.9%	9.1%	15.4%	21.2%	0.27	0.21
Tech Boom (2012-2019)	MM	IM	MM	IM	MM	IM
	18.7%	20.3%	14.1%	16.8%	0.94	0.88
Covid-19 & AI Surge (2020-2024)	MM	IM	MM	IM	MM	IM
	22.1%	25.5%	18.6%	22.9%	0.87	0.84

4. Discussion

This research makes comparisons of portfolio performance across different periods.

The data indicates that between 2008 and 2011 (Crisis), excluding the SPX entailed the greatest cost, as it deprived investors of the market's diversification protection, thereby increasing risk [10]. During this period, MM proved more robust, while IM became ineffective. MM outperformed IM in risk-adjusted returns. This opinion is consistent with Sihan Xu [11].

During the periods 2012–2019 and 2020–2024, this represented an optimal phase where the SPX's absence had minimal impact, with technology stocks emerging as the primary driver. Throughout this period, IM pursued higher returns yet concentrated risk, whereas MM maintained a more balanced and stable approach.

During the period from 2004 to 2007, this remained in a state that was neither particularly favorable nor unfavorable. The market was in a bull phase, and the concentration of IM yielded higher returns. However, the associated risks were markedly higher than those of MM.

In Crisis & Recovery (2008-2011), MM is demonstrably superior to IM. In Tech Boom (2012-2019) IM overweighted high-beta stock produces higher returns but amplified tail risk in downturns and MM had better risk control. During 2004 to 2024 Sharpe ratio for MM is generally more stable, whereas IM is more prone to risk concentration under extreme conditions.

Overall, MM consistently yields a higher Sharpe ratio, demonstrating its superior long-term balance of risk and return. While IM amplifies returns during bull markets, its poor performance during crises significantly undermines overall stability. MM portfolios are more balanced and stable, while IM portfolios are more concentrated and crisis sensitive. In group 4, excluding SPX increase

the volatility in downturns but the long-term competitiveness is not reduced due to tech growth and excluding SPX damage the diversification, especially in crisis (2008-2010), consistent with Campbell on rising idiosyncratic risk [12].

5. Conclusion

In this article, MM and IM are employed to construct portfolios across four distinct periods. (2004–2007, 2008–2011, 2012–2019 2020–2024) Results indicate that the MM proves more suitable during the crisis period (2008–2011), as the Sharpe ratio under MM is 0.27 surpasses that under IM (0.21). This indicates that in high-volatility environments, the MM dynamically reduces financial weights via the covariance matrix, thereby achieving superior risk control. During the periods (2012-2019) and 2020–2024, the IM model delivered higher returns but entailed greater concentration risk, with annualized portfolio returns often exceeding those of MM (e.g., 2020–2024: IM=25.5% > MM=22.1%). However, this was accompanied by heightened volatility. Nevertheless, the MM proves more robust, consistently yielding a marginally higher Sharpe ratio, indicating MM's superiority in pursuing risk-adjusted returns.

During crises, investors should adopt the MM model to maintain broader diversification and avoid the risk of excessive beta concentration. In non-crisis periods, investors seeking higher returns may consider the IM model. However, risks are concentrated, requiring investors to possess a higher risk tolerance.; those prioritizing robustness should opt for MM. Long-term investors are better suited to MM strategies, as they offer a more stable risk-return profile and avoid the concentrated exposure arising from beta failure during periods of crisis.

In terms of policy implications, during crises investors should guard against over-reliance on beta models (such as IM), as linear beta fails to capture tail risks; policymakers may guide institutional investors towards adopting more comprehensive risk modelling approaches.

Although this paper conducts a systematic analysis of portfolios across four distinct periods using the Markowitz model and the exponential model, revealing variations in model performance under differing market conditions, several limitations remain. Firstly, the sample is restricted to the Group 4 stock pool, with the SPX excluded under certain constraints, which may impact the generalizability of the conclusions. Future research should encompass a broader range of industries and markets. Secondly, the study relies on historical means and covariances for rough calculations, rendering it susceptible to parameter instability and estimation errors. Future work could incorporate additional methodologies, such as robust mean-variance approaches, shrinkage covariance estimation, and Bayesian methods, to enhance out-of-sample robustness.

References

- [1] Zhao Hongxin, KONG Lingchen (2023). Research on diversification portfolio optimization model and method. *Operations Research Transactions*.
- [2] Nannan Guo (2016). Analysis of securities Portfolio Models for different preferences. *Private Science and Technology*.
- [3] Edverdas Vaclovas Bartkus, Aiste Paleviciene (Bartkute) (2013). The Optimization and Evaluation of Investment Portfolio. *Inzinerine Ekonomika-Engineering Economics*, 24(4), 282-290
- [4] Tri Yuwono1 and Dadan Ramdhani (2017). Comparison Analysis of Portfolio Using Markowitz Model and Single Index Model: Case in Jakarta Islamic Index. *Journal of Multidisciplinary Academic*.
- [5] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*.
- [6] Sharpe, W. (1963). A Simplified Model for Portfolio Analysis. *Management Science*.
- [7] Qi Guo (2022). Review of Research on Markowitz Model in Portfolios. *Proceedings of the 2022 7th International Conference on Social Sciences and Economic Development (ICSSSED 2022)*

- [8] Fama, E., & French, K. (1993). Common Risk Factors in the Returns on Stocks and Bonds. *Journal of Financial Economics*.
- [9] Pastor, L., & Veronesi, P. (2009). Technological Revolutions and Stock Prices. *American Economic Review*, 99(4), 1451–1483.
- [10] Bekaert, G., & Harvey, C. (1997). Emerging Equity Market Volatility. *Journal of Financial Economics*, 43(1), 29–77.
- [11] Sihan Xu (2025). Comparative Risk Management in Portfolio Optimization: Evaluating the Markowitz and Index Models Across Constrained and Unconstrained Scenarios. *Proceedings of the 2025 International Conference on Financial Risk and Investment Management (ICFRIM 2025)*
- [12] Campbell, J., Lettau, M., Malkiel, B., & Xu, Y. (2002). Have Individual Stocks Become More Volatile?. *Journal of Finance*, 57(1), 1–43.