

Turnover Liquidity in Over-the-Counter Markets and Its Role in Monetary Transmission

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Abstract. We develop a dynamic over-the-counter (OTC) asset-market model in which monetary policy acts through a turnover-liquidity channel. Investors and dealers meet with probability τ , bargain with investor weight θ , and trade a money claim and an equity claim. The market equilibrium features a valuation cutoff ε^* : conditional on meeting a dealer, investors with $\varepsilon > \varepsilon^*$ rebalance into equity while others hold money. Because the cutoff is tied to the money-equity wedge, an increase in money growth μ lowers ε^* , raises the trader mass $\tau[1 - G(\varepsilon^*)]$, and increases turnover and the price-dividend ratio. Two modeling ingredients sharpen identification and comparative statics: (i) payout timing is decoupled from survival via temporary dividend suspensions with probability ι and a trend-adoption parameter Ω in dividend dynamics; (ii) a two-speed intertemporal structure separates a real rate r from a delisting hazard p . We derive closed-form money-growth thresholds that bound the region where money is valued, delivering state-dependent policy pass-through. The framework yields testable cross-sectional predictions—stronger transmission when meeting intensity and investor surplus share are high, and when payout suspensions or adoption intensity amplify the cash-flow block—and organizes measurement around sufficient statistics such as standardized OTC turnover and reallocation flows.

Keywords: OTC trading, monetary transmission, bargaining, dividend suspension, technological adoption

1. Introduction

How does monetary policy move asset prices when trading is decentralized, balance sheets are cash-constrained, and investors can only intermittently reshuffle portfolios through dealers? A growing literature emphasizes a turnover-liquidity channel [1]: by altering the relative convenience yield of money, policy shifts the mass of investors willing to rebalance into (or out of) risky claims at over-the-counter (OTC) meetings, which in turn affects prices and quantities in asset markets. We build a tractable dynamic OTC model of equity and money that embeds this mechanism and delivers sharp, testable comparative statics for the equity price-dividend ratio and trading intensity [2].

The environment proceeds in discrete time with two subperiods per period. A unit mass of investors derives utility from consumption financed by dividends; a unit mass of dealers provides intermediation. Equity represents claims on a continuum of productive units that pay dividends before the Walrasian subperiod; intrinsically useless money is the sole medium of exchange.

Investors and dealers meet bilaterally in an OTC round with probability τ and bargain with investor weight θ [3]. Dealers rebalance in a competitive inter-dealer market and charge an intermediation fee that splits the OTC surplus. A monetary authority sets the gross money growth rate μ . The structure implies a simple cutoff ε^* in investor valuations: conditional on meeting a dealer, investors with $\varepsilon > \varepsilon^*$ buy equity and those with $\varepsilon < \varepsilon^*$ hold only money. Since the cutoff is proportional to the money–equity price wedge, monetary policy that changes the relative value of money shifts the trader mass $\tau [1 - G(\varepsilon^*)]$ into equity, moving both turnover and the price–dividend ratio through the turnover–liquidity channel.

This paper builds on the core insights of [1] and extends their framework along three substantive dimensions that broaden the economic scope and sharpen empirical content. First, we decouple payout risk from survival and embed trend–adoption into cash–flow dynamics. Specifically, beyond permanent shutdowns, a surviving productive unit may temporarily suspend payouts with probability ι , and its dividend growth mixes inertia with frontier adoption via a parameter $\Omega \in [0, 1]$. This separation allows the same productive unit to experience lumpy payout timing without conflating it with survival risk, and it lets the dividend law interpolate between pure inertia and rapid catch–up. Economically, the temporary payout suspension injects payout illiquidity into the cash–flow block, while trend–adoption governs how technological diffusion interacts with turnover liquidity and the price–dividend ratio. These features generate richer state dependence in the cutoff ε^* and in the region of money–growth rates that sustain a monetary equilibrium.

Second, we introduce a two–speed intertemporal discounting structure that cleanly maps to a real rate and a delisting hazard. Rather than a single reduced–form discount factor, agents’ intertemporal trade–offs are parameterized by $\beta_0 = 1/(1 + r)$ and $\beta_1 = 1/(1 + p)$, which combine to deliver the effective period weight in Euler equations. The decomposition isolates how the real interest rate r (a money–bond trade–off) and the survival component p (a mortality hazard for equity claims) enter the pricing conditions for money and equity. The mapping clarifies existence conditions for monetary equilibria and highlights the role of observed delistings or index exits when taking the theory to data.

Third, leveraging linear pricing of real objects to dividends, we derive closed–form money–growth thresholds that bound the set of monetary equilibria with interior money and equity holdings. These thresholds are transparent functions of turnover τ , bargaining θ , survival parameters, and the cross–section of valuations G . They make concrete how expansionary policy relaxes the money Euler inequality by lowering the cutoff ε^* , raising the mass of equity buyers, and thereby tightening the link between policy, turnover, and the price–dividend ratio. The formulas organize the transmission mechanism into sufficient statistics—most notably the trader mass $\tau [1 - G(\varepsilon^*)]$ —that can be proxied in data by reallocation flows between money–like assets and equities or by measures of OTC trading intensity.

The model delivers three sets of predictions. First, in any monetary equilibrium with interior holdings, an increase in μ reduces the valuation cutoff and increases the fraction of investors that rebalance into equity upon meeting, raising turnover and putting upward pressure on the price–dividend ratio. Second, the strength of this transmission is increasing in effective OTC surplus, which is governed by meeting intensity τ and bargaining weight θ , and in the thickness of the upper tail of the valuation distribution G . Third, payout timing and trend–adoption reshape the sensitivity of prices to policy by affecting the cash–flow block that underlies linear pricing, delivering cross–sectional differences in policy pass–through across industries and periods.

Relative to the existing theory, decoupling payout risk from survival allows the model to distinguish liquidity–driven trading from cash–flow news in environments where dividends are

lumpy or subject to discretionary suspension. Embedding trend–adoption provides a disciplined way to map technological diffusion into asset–pricing sensitivity to monetary shocks. The two–speed discounting clarifies identification of the survival component in the pricing kernel and lends itself to empirical proxies based on observed delistings. Finally, the explicit money–growth bounds and sufficient–statistic representation of turnover facilitate both calibration and event–study designs that use heterogeneity in meeting intensities or bargaining environments as sources of variation in exposure to monetary policy.

2. Literature Review

The literature on the interaction between liquidity frictions and monetary policy transmission has expanded rapidly in recent decades. A central strand of this research emphasises how decentralised markets and over-the-counter (OTC) frictions affect asset pricing and the propagation of monetary shocks.

Early foundations were laid by [4], who developed the seminal search-theoretic model of money as a medium of exchange, highlighting how frictions in bilateral trade determine the acceptability of money. Building on this, [2] formalised OTC market structures, showing how search frictions and bargaining shape asset prices and allocations. Subsequent work by [3] extended these insights toward liquidity within asset markets alongside search frictions, while [5] analysed how asset illiquidity interacts with macroeconomic fluctuations.

A parallel literature has stressed the role of liquidity and funding constraints in amplifying shocks. [6] proposed the influential “liquidity spiral,” linking market liquidity with funding liquidity and showing how shocks propagate through margin and collateral channels. Similarly, [7] investigated adaptive trading with foreseeable returns under transaction costs, capturing the interplay between liquidity and asset allocation.

More directly related to monetary policy, [8] embedded liquidity frictions into an exchange economy, exploring implications for asset prices, while [9] provided a broad synthesis of illiquidity mechanisms in financial markets. The most relevant advance comes from [1] who developed a dynamic OTC market model with turnover liquidity. Their work introduced the concept of a valuation cutoff that governs whether investors rebalance into equity or hold money, demonstrating how money growth influences turnover and the price–dividend ratio through a liquidity channel.

The present paper builds on [1] and extends the framework in three important dimensions. First, it separates payout timing from survival risk, allowing for temporary dividend suspensions and embedding technological adoption into cash-flow dynamics. This captures richer state dependence in policy pass-through, distinguishing liquidity-driven trading from fundamental cash-flow shocks. Second, it introduces a two-speed intertemporal structure, distinguishing between the real rate and a delisting hazard, which clarifies identification in empirical applications. Third, it derives closed-form thresholds for money growth that bound monetary equilibria, providing transparent sufficient statistics for policy analysis.

Together, these extensions refine our understanding of how monetary shocks transmit through turnover liquidity in decentralized markets. They broaden the economic scope of the model, connect it to empirical observables such as OTC turnover and reallocation flows, and offer a tractable framework for calibration and event-study designs.

3. Methodology

Time is modeled as an infinite series of distinct intervals, numbered by $t = 0, 1, 2, \dots$. In this model, each period t is split into two sub-stages, during which agents perform distinct activities to be described subsequently. The economy consists of two agent groups—investors and dealers—each represented by a continuum, captured by sets $I[0,1]$ and $D[0,1]$. Each agent has a discount factor $\beta_0 + \beta_1$. $\beta_0 \equiv \frac{1}{r+1}$, which r is denoted as the real interest rate. $\beta_1 \equiv \frac{1}{1+p}$, which p accounts for the possibility of the death or the delisting of the agents and could also discount the utility value in the future. At each date t , a continuum of production units of measure $A^s \in \mathbb{R}_+$ operates, with each active unit generating a dividend $y_t \in \mathbb{R}_+$ in the subperiod one in each period t . However, two potential exogenous shocks could influence the dividends. Firstly, an idiosyncratic shock could make $(1-\delta)$'s proportion of firms permanently unproductive, where $\delta \in [0,1]$. Secondly, for the remaining production units, there is a probability of $1 \in [0,1]$ to choose not to pay dividends. When a firm continues to operate, its payout at time t is given by $y_t = \gamma_t y_{t-1}$, with γ_t denoting that nonnegative random variable defined via a cumulative distribution function F , i.e., $\Pr(\gamma_t \leq \gamma) = F(\gamma)$, and mean $\bar{\gamma} \in [0, 1/((\beta_0 + \beta_1)\delta)]$. In addition, a new parameter $\Omega \in [0,1]$ is introduced to interpret the firm's ability to chase up the market trend. If the firm is more likely to take advantage of new technology, e.g., then it would follow $y_t = \gamma_t y_{t-1}$, or there is also a probability of $1 - \Omega$ for the firm to stay in the dividend level as before. Thus, $y_t = \Omega \gamma_t y_{t-1} + (1 - \Omega)y_{t-1}$. Agents will capture these shocks at the beginning of each period t , so the dividend is also known at the very beginning. Meanwhile, the shutdown firms will be replaced by identical new entry firms immediately and follow the same procedure explained above. The only difference is that they do not have dividends in the first period since they have not started to produce.

Every production unit is associated with a fully divisible equity claim, which signifies its ownership and grants the holder the right to receive dividends. There is another financial instrument—intrinsically useless money, contributing nothing to the holders' utility, but can act as the only medium of exchange. All financial instruments are assumed to be perfectly recognisable and can be traded throughout the period.

At the beginning of subperiod one, we experience the shutdown of $(1-\delta)$'s proportion of firms, which new entries will immediately make up. Hence, investors are allocated an initial bundle of $(1-\delta)A^s$ equity claims linked to the recently established production units. Then, the trade will be organised in a random bilateral OTC market between dealers and investors. Meanwhile, a Walrasian interdealer market will enable a dealer to trade with other dealers. The parameter $\tau \in [0,1]$ is used to denote the market friction, which is a likelihood which the investor could successfully contact a dealer within the OTC market. When an investor encounters a dealer, they negotiate both the price and the number of shares that the dealer will later trade in the interdealer market on the investor's interest. Upon completion of the trade, the dealer charges an intermediation fee φ . The trading arrangement is governed by an equality bargaining framework, where the investor's bargaining strength is represented by $\theta \in [0,1]$. Importantly, the OTC transaction takes place prior to dividend realization, with the exception of newly established units that have not yet begun distributing dividends.

In the second subperiod, production units become active, providing each agent with a proportional technology that transforms labor into a standard, temporary consumption good. All the consumption goods generated, equity shares, and currency can be exchanged in an immediate in the Walrasian market. Money can be used to purchase consumption goods to increase investors' utility.

Meanwhile, monetary authority will control the money supply, denoted as $A_{t+1}^m = \mu A_t^m$ through lump-sum disbursements or taxation imposed to investors.

The preferences of a representative dealer are given by:

$$E_0^d \sum_{t=0}^{\infty} (\beta_0 + \beta_1)^t (c_{dt} - h_{dt})$$

,

Where c_{dt} denotes the utilization of the homogenous commodities in the subsequent subperiod of period t , and quantifies the endeavors to manufacture the commodity. The expectation operator is with respect to the stochastic transactions process assessed by γ in the OTC market. Dealers derive no utility from dividends. A trader's tastes are expressed by:

$$E_0^i \sum_{t=0}^{\infty} (\beta_0 + \beta_1)^t (\varepsilon_{it} y_{it} + c_{dt} - h_{dt})$$

,

Unlike dealers, investors derive utility from dividends. Each investor i is subject to a valuation shock ε_{it} , drawn independently across time and agents from a distribution G with support $[\varepsilon_L, \varepsilon_H] \subseteq [0, \infty)$ and mean $\bar{\varepsilon} = \int \varepsilon dG(\varepsilon)$. At the start of period t , prior to OTC trading, investor i observes his realization of ε_{it} . Consequently, the expectation operator E_0^i for investors, unlike that of dealers, incorporates this idiosyncratic valuation shock.

When a social planner seeks to maximize the aggregate expected discounted utility of all agents under the outlined conditions, two key features emerge. First, equity holdings across periods are retained exclusively by dealers. Second, at the close of the first period, equity shares remain only with those traders who assign them the highest valuations. The preceding propositions help to shape allocation incentives that determine the equilibrium results discussed in the following section. Specifically, because investors derive linear utility from dividends while dealers place no value on them, efficiency dictates that assets should be concentrated among those investors assigning the highest valuations to the stocks, namely ε_H . Moreover, assigning all assets to dealers at the close of a period ensures that, in the subsequent OTC trading round, the shares will be reallocated to those investors who place the greatest value on them.

4. Results

Consider in the period t , α_{dt} denotes the portfolio of a dealer, and α_{it} denotes the portfolio of an investor. The valuation of the investor is ε [10]. Make $\alpha_{it}^+ = (\alpha_{imt}^-, \alpha_{ist}^-)$ represent the investor's portfolio after trading, while φ_t indicates the intermediation fee imposed by the dealer, which are paid by investors within the following sub-interval. We postulate that $(\alpha_{it}^-, \varphi_t)$ represents the outcome of the Equality Bargaining in which $\theta \in [0, 1]$ constitutes the negotiation strength of shareholders.

Consider $W_{Dt}^\Delta(\alpha_{dt}, \varphi_t)$ represent the greatest return the dealer anticipated to obtain using portfolio α_{dt} and realized φ_t when he reorganizes his portfolio within the interdealer market during the interval t [7]. Consider $W_{It}(\alpha_{it}^-, \varphi_t)$ signify the highest projected gain that the investor can get at the beginning of the second subperiod whose portfolio is α_{it}^- and has paid a fee φ_t to the dealer.

As a result, standing on the point of a social planner, the optimized consequence would be:

$$\text{Max} [\varepsilon y_t \alpha_{ist}^- + W_{It}(\alpha_{it}^-, \varphi_t) - \varepsilon y_t \alpha_{ist} - W_{It}(\alpha_{it}, 0)](\theta) [W_{Dt}^\Delta(\alpha_{dt}, \varphi_t) - W_{Dt}^\Delta(\alpha_{dt}, 0)](1-\theta) \quad (1)$$

Subject to:

1. $\alpha_{imt}^- + p_t \alpha_{ist}^- \leq \alpha_{imt} + p_t \alpha_{ist}$
2. $W_{Dt}^\Delta(\alpha_{dt}, 0) \leq W_{Dt}^\Delta(\alpha_{dt}, \varphi_t)$
3. $\varepsilon y_t \alpha_{ist} + W_{It}(\alpha_{it}^-, 0) \leq \varepsilon y_t \alpha_{ist}^- + W_{It}(\alpha_{it}^-, \varphi_t)$

In period t , p_t represents the equity price in the interdealer market.

Let $W_{Dt}(\alpha_{dt}, \varphi_t)$ capture the dealer's maximum attainable payoff, conditional on holding portfolio α_{dt} and receiving the intermediation fee φ_t from the OTC trade.

Since at the beginning of the second subperiod, the dealer holds the portfolio α_{dt}^Δ . Then, the behaviour of the dealer in the interdealer market can be expressed as:

$$W_{Dt}^\Delta(\alpha_{dt}) = \frac{\max}{\alpha_{dt}^\Delta \in R_+^2} W_{Dt}(\alpha_{dt}^\Delta, \varphi_t) \quad (2)$$

Subject to

1. $\alpha_{dmt}^\Delta + p_t \alpha_{dst}^\Delta \leq \alpha_{dmt} + p_t \alpha_{dst}$

We make $\alpha_{dt}^\Delta(\alpha_{dt}) = (\alpha_{dmt}^\Delta(\alpha_{dmt}), \alpha_{dst}^\Delta(\alpha_{dst}))$ denote the solution of (2).

Consider $V_{Dt}(\alpha_{dt})$ represent the highest anticipated adjusted return of the dealer that joins the OTC session of time t with portfolio $\alpha_{dt} \equiv (\alpha_{dmt}, \alpha_{dst})$. Meanwhile, make $\phi_t \equiv (\phi_{mt}, \phi_{st})$, which is the genuine cost of funds and equity respectively in the second subperiod. Then,

$$W_{Dt}^\Delta(\alpha_{dt}, \varphi_t) = \text{Max}[c_t - h_t + (\beta_0 + \beta_1)E_t V_{D(t+1)}(\alpha_{d(t+1)})] \quad (3)$$

Subject to

1. $c_t + \phi_t \alpha_{d(t+1)}^\sim \leq h_t + \varphi_t + \phi_t \alpha_{d(t+1)}$
- $$\alpha_{t+1}^\sim = (\alpha_{m(t+1)}^\sim, \alpha_{s(t+1)}^\sim), \alpha_{t+1} = (\alpha_{m(t+1)}^\sim, ((\delta + 1)\alpha_{s(t+1)}^\sim))$$

E_t denotes a conditional expectation regarding a subsequent-period realization of a dividend.

Similarly, $V_{It}(\alpha_{it}, \varepsilon)$ demotes the the greatest anticipated discounted return for an investor given their valuation ε when he enters into the OTC round.

$$W_{It}(\alpha_{it}^-, \varphi_t) = \text{Max}[c_t - h_t + \beta_0 + \beta_1)E_t \int V_{I(t+1)}(\alpha_{I(t+1)}^\sim, \varepsilon') dG(\varepsilon'). \quad (4)$$

Subject to

$$1. c_t + \phi_t \alpha_{i(t+1)}^\sim \leq h_t - \varphi_t + \phi_t \alpha_{i(t+1)} + T_t$$

T_t is the limp-sum monetary transfer. $\alpha_{I(t+1)} = (\alpha_{Im(t+1)}^{\sim}, ((\delta+1)\alpha_{Is(t+1)}^{\sim} + (1-\delta) A^s)$

The value function of an investor:

$$V_{It}(\alpha_{it}, \varepsilon) = \tau \{ \varepsilon y_t \alpha_{ist}^-(\alpha_{it}, \varepsilon + W_{It}[\alpha_{it}^-(\alpha_{it}, \varepsilon), \varphi_t(\alpha_{it}, \varepsilon)] + (1 - \tau) [\varepsilon y_t \alpha_{ist} - W_{It}(\alpha_{it}, 0)] \}$$

The value function of a dealer:

$$V_{Dt}(\alpha_{dt}) = \tau \int W_{Dt}^{\wedge} [(\alpha_{dt}, \varphi_t(\alpha_{dt}, \varepsilon))] d H_{it}(\alpha_{dt}, \varepsilon) + (1 - \tau) W_{Dt}^{\wedge}(\alpha_{dt}, 0)$$

in which H_{it} represents the combined accumulated spread of shareholder holdings and assessments that a dealer might face within the OTC market throughout phase t .

5. Discussion

5.1 Lemma 1

Define $\varepsilon_t^* \equiv \frac{p_t \phi_{mt} - \phi_{st}}{y_t}$

And

$$\chi(\varepsilon_t^*, \varepsilon) = 1 \text{ if } \varepsilon_t^* < \varepsilon$$

$$\in [0, 1] \text{ if } \varepsilon = \varepsilon_t^*$$

$$= 0 \text{ if } \varepsilon > \varepsilon_t^*$$

Consider the investor's post OTC trade portfolio, is given by

$$\alpha_{imt}^-(\alpha_t, \varepsilon) = [1 - \chi(\varepsilon_t^*, \varepsilon)] (\alpha_{imt} + p_t \alpha_{ist})$$

$$\alpha_{ist}^-(\alpha_t, \varepsilon) = \chi(\varepsilon_t^*, \varepsilon) (1/p_t) (\alpha_{imt} + p_t \alpha_{ist})$$

Also the intermediation fee levied by the broker represents

$$\varphi_t(\alpha_t, \varepsilon) = (1 - \theta) (\varepsilon - \varepsilon_t^*) \{ \chi(\varepsilon_t^*, \varepsilon) (1/p_t) \alpha_{imt} - [1 - \chi(\varepsilon_t^*, \varepsilon)] \alpha_{ist} \} y_t$$

Lemma 1 describes the structure of investors' and dealers' portfolios after trading. When $\varepsilon_t^* < \varepsilon$, investors allocate their entire monetary holdings to equity purchases. Conversely, when $\varepsilon_t^* > \varepsilon$, investors liquidate all equity and retain only cash. The dealer's intermediation revenue, φ_t , corresponds to a fraction $1 - \theta$ of the investor's trading surplus.

The aggregate equity holds at a constant level over time, $A_{Dst} = A_{Ds(t+n)}$ and $A_{Ist} = A_{Is(t+n)}$ for all n . The real asset prices are linearly connected to aggregate dividends by a time-invariant constant, i.e. $\phi_{st} = \phi_s y_t$, $p_t \phi_{mt} \equiv \phi_{st}^- = \phi_s^- y_t$, $\phi_{mt} A_{Imt} = Z y_t$, and $\phi_{mt} A_{Dmt} = Z_D y_t$.

Thus, within a recursive equilibrium framework,

$$\varepsilon_t^* = \phi_s^- - \phi_s \equiv \varepsilon^*$$

$$\frac{\phi_{s(t+1)}}{\phi_{st}} = \phi_{s(t+1)}^- / \phi_{st}^- = Y_{t+1}$$

$$\frac{\phi_{mt}}{\phi_{m(t+1)}} = \frac{\mu}{Y_{t+1}}$$

$$\mu = p_{t+1}/p_t \quad [4]$$

In the analysis, we set $\beta^- \equiv \beta\gamma^-$ and impose the assumption $\mu > \beta^-$.

For our analysis, it is easy to define

$$\mu^\wedge \equiv \beta^- [1 + \frac{(1-\tau\theta)(1-\beta^- \delta)(\epsilon^\wedge - \epsilon^-)}{\epsilon^\wedge}] \text{ and } \mu^- \equiv \beta^- [1 + \frac{\tau\theta(1-\beta^- \delta)(\epsilon^- - \epsilon_L)}{\beta^- \delta \epsilon^- + (1-\beta^- \delta)\epsilon_L}] \quad (5)$$

Where $\epsilon^\wedge \in [\epsilon^-, \epsilon_H]$ is a unique solution to

$$\epsilon^- - \epsilon^\wedge + \tau\theta \int_{\epsilon_L}^{\epsilon^\wedge} \epsilon^\wedge - \epsilon dG(\epsilon) = 0 \quad (6)$$

Lemma 4 shows that $\mu^\wedge$ is strictly less than μ^- . Based on this, the next proposition outlines the set of possible equilibria.

Proposition 1:

(i) For any parameter configuration, a nonmonetary equilibrium exists.

(ii) A recursive monetary equilibrium fails to exist whenever $\mu \geq \mu^-$.

(iii) In the absence of money, the condition $A_{St} = A_{St} - A_{SDt} = A_{St}$ holds, indicating that equity is entirely held by investors. Under such circumstances, no transactions occur in the OTC exchange, and the equity value in the subsequent interval becomes

$$\phi_{st} = \phi_{st} y_t, \text{ with } \phi_s = \frac{\beta^- \delta}{1 - \beta^- \delta} \epsilon^- \quad (7)$$

(iv) If $\mu \in (\beta^-, \mu^-)$, then there is one recursive monetary equilibrium, asset holdings of dealers and investors at the beginning of the OTC round of period t are

$$A_{Dmt} = A_{mt} - A_{Imt} = 0 \text{ and}$$

$$= \delta A_{st} \text{ if } \beta^- < \mu < \mu^\wedge$$

$$A_{DSt} = A_{st} - A_{Ist} \in [0, \delta A_{st}] \text{ if } \mu^\wedge = \mu$$

$$= 0 \text{ if } \mu^\wedge < \mu < \mu^-$$

And asset prices are

$$\phi_{st} = \phi_{st} y_t$$

5.2 Proof of Lemma 1

Notice that (3) implies

$$W_{Dt}(\alpha_{dt}, \varphi_t) = \phi_t \alpha_{dt} + \varphi_t + W_{Dt}^-$$

where

$$(A1) \quad W_{Dt}^- \equiv \max [-\phi_t \alpha_{d(t+1)}^\sim + (\beta_0 + \beta_1) E_t V_{D(t+1)} (\alpha_{m(t+1)}^\sim, (\delta+1) \alpha_{s(t+1)}^\sim)]$$

So (2) implies that

$$W_{Dt}^\wedge(\alpha_{dt}, \varphi_t) = \varphi_t + W_{Dt}^- + \max \phi_t \alpha_{dt}^\wedge$$

$$\text{s.t. } \alpha_{dmt}^\wedge + p_t \alpha_{dst}^\wedge \leq \alpha_{dmt} + p_t \alpha_{dst}$$

Hence,

$$\alpha_{dmt}^\wedge(\alpha_{dt}, \varphi_t) = \alpha_{dmt} + p_t \alpha_{dst}^* \text{ if } 0 < \epsilon_t$$

$$\in [0, \alpha_{dmt} + p_t \alpha_{dst}^*] \text{ if } 0 = \epsilon_t$$

$$= 0 \text{ if } 0 > \epsilon_t$$

$$\alpha_{dst}^{\wedge} \alpha_{dt, \varphi_t} = (1/p_t) [\alpha_{dmt} + p_t \alpha_{dst} - \alpha_{dmt}^{\wedge} \alpha_{dt, \varphi_t}]$$

And

$$(A2) \quad W_{Dt}^{\wedge}(\alpha_{dt}, \varphi_t) = \varphi_t + W_{Dt}^- + \max(\phi_{mt}, \phi_{st}/p_t)(\alpha_{dmt} + p_t \alpha_{dst})$$

Also, notice that (4) implies

$$(A3) \quad W_{It}(\alpha_{It}, \varphi_t) = \phi_t \alpha_{It} - \varphi_t + W_{It}^-$$

$$(A4) \quad W_{It}^- \equiv T_t + \max \{ -\phi_t \alpha_{I(t+1)}^{\sim} + (\beta_0 + \beta_1) E_t \int V_{I(t+1)} [\alpha_{m(t+1)}^{\sim}, (\delta+1) \alpha_{s(t+1)}^{\sim} + (1-\delta) A^s, \varepsilon] dG(\varepsilon) \}$$

With (A2) and (A3), (1) can be written as:

$$\text{Max} [(\varepsilon_t - \varepsilon) (\alpha_{imt}^* - \alpha_{imt}) \frac{y_t}{p_t} - \varphi_t] (\theta) (\varphi_t) (1-\theta)$$

$$\text{s.t. } 0 \leq \varphi_t \leq (\varepsilon_t - \varepsilon) (\alpha_{imt}^* - \alpha_{imt}) \frac{y_t}{p_t}$$

with $\alpha_{ist}^* = \alpha_{ist} + (1/p_t) (\alpha_{imt} - \alpha_{imt}^*)$. Hence,

$$\begin{aligned} \alpha_{imt}^* \alpha_{it, \varepsilon, \varphi_t} &= \alpha_{imt} + p_t \alpha_{ist}^* \text{ if } \varepsilon < \varepsilon_t^* \\ &\in [0, \alpha_{imt} + p_t \alpha_{ist}^*] \text{ if } \varepsilon = \varepsilon_t^* \\ &= 0 \text{ if } \varepsilon > \varepsilon_t^* \end{aligned}$$

$$\alpha_{ist}^* \alpha_{it, \varepsilon, \varphi_t} = \alpha_{ist} + (1/p_t) [\alpha_{imt} - \alpha_{imt}^* (\alpha_{it, \varepsilon, \varphi_t})]$$

And

$$\begin{aligned} \varphi_t (\alpha_{it, \varepsilon}) &= (1-\theta) (\varepsilon_t - \varepsilon) \alpha_{ist}^* y_t \text{ if } \varepsilon < \varepsilon_t^* \\ &= (1-\theta) (\varepsilon_t - \varepsilon) (-\alpha_{ist}) y_t \text{ if } \varepsilon > \varepsilon_t^* \end{aligned}$$

This concludes the proof.

5.3 Lemma 2

Define $(\alpha_{dm(t+1)}^{\sim}, \alpha_{ds(t+1)}^{\sim})$ as the dealer's portfolio choices and $(\alpha_{im(t+1)}^{\sim}, \alpha_{is(t+1)}^{\sim})$ as the shareholder's portfolio decisions during the next step of term t . These allocations are required to fulfill this subsequent first-order essential as well as adequate requirements:

$$(A5) \quad \phi_{mt} \geq (\beta_0 + \beta_1) E_t \max(\phi_{m(t+1)}, \phi_{m(t+1)}/p_{t+1}), \text{ with "}" \text{ if } \alpha_{dm(t+1)}^{\sim} > 0$$

$$(A6) \quad \phi_{st} \geq (\beta_0 + \beta_1) \delta E_t \max(\phi_{s(t+1)}, \phi_{m(t+1)} p_{t+1}), \text{ with "}" \text{ if } \alpha_{ds(t+1)}^{\sim} > 0$$

$$(A7) \quad \phi_{mt} \geq (\beta_0 + \beta_1) E_t [\phi_{m(t+1)} + \tau \theta \int_{E_{t+1}^*}^{E_H} (E - E_{t+1}^*) y_{t+1} dG(E) / p_{t+1}], \text{ with "}" \text{ if } \alpha_{im(t+1)}^{\sim} > 0$$

$$(A8) \quad \phi_{st} \geq (\beta_0 + \beta_1) \delta E_t [\varepsilon y_{t+1} + \phi_{s(t+1)} + \tau \theta \int_{E_L^*}^{E_{t+1}^*} (E_{t+1}^* - E) y_{t+1} dG(E)], \text{ with "}" \text{ if } \alpha_{is(t+1)}^{\sim} > 0$$

5.4 Proof of Lemma 2

With Lemma 1, we can write $V_{It}(\alpha_t, \varepsilon)$ as

$$(A9) \quad V_{It}(\alpha_{it}, \varepsilon) = [\tau \theta (\varepsilon - \varepsilon_t^*) \Pi_{\{\varepsilon_t^* < \varepsilon\}} \frac{1}{p_t} y_t + \phi_{mt}] \alpha_{imt} + \{[\varepsilon + \tau \theta (\varepsilon_t - \varepsilon_t^*) \Pi_{\{\varepsilon_t^* > \varepsilon\}}] y_t \phi_{mt}\} \alpha_{ist} + W_{It}^-$$

And

$$V_{Dt}(\alpha_t) = \tau \int_t \varphi(\alpha_{it}, \varepsilon) dH_{It}(\alpha_{it}, \varepsilon) + \max(\phi_{mt}, \phi_{st})(\alpha_{imt} + \alpha_{ist}) + W_{Dt}^-$$

Since ε is time-independent, an investor's decision on the portfolio to hold into period $t + 1$ does not depend on ε . Hence, it can be expressed as $dH_{It}(\alpha_{it}, \varepsilon) = dF_{It}(\alpha_{it})dG(\varepsilon)$, where F_{It} represents the combined accumulated spread of investors' financial holdings and equity positions at the start of the OTC trading phase in period t . Hence,

$$(A10) \quad V_{Dt}(\alpha_t) = \max(\phi_{mt}, \phi_{st})(\alpha_{imt} + \alpha_{ist}) + V_{Dt}(0)$$

Where

$$V_{Dt}(0) = \tau(1-\theta) \int (\varepsilon - \varepsilon_t^*) [\Pi_{\{\varepsilon_t^* < \varepsilon\}} \frac{1}{p_t} A_{Imt} + \Pi_{\{\varepsilon_t^* > \varepsilon\}} A_{Ist}] dG(\varepsilon) y_t + W_{Dt}^-$$

From (A10) we have

$$V_{D(t+1)}(\alpha_{m(t+1)}, \delta \alpha_{s(t+1)}) = \max(\phi_{m(t+1)}, \phi_{s(t+1)})(\alpha_{m(t+1)} + \delta \alpha_{s(t+1)}) + V_{D(t+1)}(0)$$

And from (A9) we have

$$V_{I(t+1)}(\alpha_{m(t+1)}, \delta \alpha_{s(t+1)}, (1-\delta)A^s) = [\tau \theta \int_{E_{t+1}^{E_H}} (E - E_{t+1}^*) y_{t+1} dG(E) / p_{t+1} + \phi_{m(t+1)}] \alpha_{m(t+1)} + \delta \{ [\varepsilon^- + \tau \theta \int_{E_L}^{E_{t+1}^*} (E_{t+1}^* - E) dG(E)] y_{t+1} + \phi_{s(t+1)} \} \alpha_{s(t+1)} + K_{t+1},$$

$$\text{Where } K_{t+1} \equiv \{ [\varepsilon^- + \tau \theta \int (\varepsilon_{t+1}^* - \varepsilon) \Pi_{\{\varepsilon_{t+1}^* > \varepsilon\}} dG(\varepsilon)] y_{t+1} + \phi_{s(t+1)} \} (1-\delta) A^s + W_{I(t+1)}^-$$

With the first order conditions, then it is sufficient to get the corresponding statement in the lemma.

5.5 Lemma 3

Throughout interval, the market-clearing condition regarding equity in the interdealer market can be expressed as

$$\tau[1-G(\varepsilon^*)] (A_{Ist} + A_{Imt}/p_t) + \chi(0, \varepsilon^*) (A_{Dst} + A_{Dmt}/p_t) = A_{Dst} + \tau A_{Ist}$$

Which is equivalent to

$$\{ \tau[1-G(\varepsilon^*)] A_{Imt} + \chi(0, \varepsilon^*) A_{Dmt} \} \frac{1}{p_t} = \tau G(\varepsilon^*) A_{Ist} + [1 - \chi(0, \varepsilon^*)] A_{Dst}$$

5.6 Proof of Lemma 3

Recall that

$$-A_{Dst}^- = \int (1/p_t) [\alpha_{dmt} + p_t \alpha_{dst} - \alpha_{dmt}^\wedge \alpha_{dt}, \varphi_t] dF_{Dt}(\alpha_{dt})$$

If $\varepsilon^* < 0$

$$A_{Dst}^- = \chi(0, \varepsilon^*) (A_{Dst} + A_{Dmt}/p_t)$$

While, $A_{Ist}^- = \tau \int \alpha_{st}(\alpha_{it}, \varepsilon) dH_{It}(\alpha_{it}, \varepsilon)$

$$A_{Ist}^- = \tau[1-G(\varepsilon^*)] (A_{Ist} + A_{Imt}/p_t)$$

Given such formulas, since one understand, a market-clearing state is $A_{Ist}^- + A_{Dst}^- = A_{Dst} + \tau A_{Ist}$. Then, we can prove this LEMMA

5.7 Corollary 1: Equilibrium characterisation

A sequence of prices, $\left\{ \frac{1}{p_t}, \phi_{mt}, \phi_{st} \right\}$, t is from 0 to infinity, together with the bilateral terms of trade in the OTC market, $\left\{ \alpha_{dt}^-, \phi_t \right\}$, t is from 0 to infinity, dealer portfolios,

$$\left\{ \langle \alpha_{dt}^{\Delta}, \alpha_{d(t+1)}^{\sim}, \alpha_{d(t+1)} \rangle_{d \in D} \right\}_{t=0}^{\text{infinity}},$$

And investor portfolios, $\left\{ \langle \alpha_{i(t+1)}^{\sim}, \alpha_{d(t+1)} \rangle_{i \in I} \right\}_{t=0}^{\text{infinity}}$, form an equilibrium precisely when the subsequent criteria apply for each t :

(i) the intermediation fee together alongside a best post-transaction holdings within a OTC market $\phi_t(\alpha_t, \varepsilon) = (1 - \theta)(\varepsilon - \varepsilon_t^*) \left\{ \chi(\varepsilon_t, \varepsilon) \left(\frac{1}{p_t} \right) \alpha_{imt} - [1 - \chi(\varepsilon_t, \varepsilon)] \alpha_{ist} \right\} y_t$

$$\alpha_{imt}^{\sim}(\alpha_t, \varepsilon) = [1 - \chi(\varepsilon_t, \varepsilon)](\alpha_{imt} + p_t \alpha_{ist})$$

$$\alpha_{ist}^{\sim}(\alpha_t, \varepsilon) = \chi(\varepsilon_t, \varepsilon) \left(\frac{1}{p_t} \right) (\alpha_{imt} + p_t \alpha_{ist})$$

(ii) the interdealer market clearing

$$\left\{ \tau [1 - G(\varepsilon^*)] A + \chi(0, \varepsilon^*) A \right\} \frac{1}{p_t} = \tau G(\varepsilon^*) A_{Ist} + [1 - \chi(0, \varepsilon^*)] A_{Dst}$$

(iii) the optimal end-of-period portfolios

$$\phi_{mt} \geq (\beta_0 + \beta_1) E_t \max(\phi_{m(t+1)}, \phi_{m(t+1)} / p_{t+1})$$

$$\text{with "}" \text{ if } \alpha_{dm(t+1)}^{\sim} > 0$$

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t \max(\phi_{s(t+1)}, \phi_{m(t+1)} p_{t+1})$$

$$\text{with "}" \text{ if } \alpha_{ds(t+1)}^{\sim} > 0$$

$$\phi_{mt} \geq (\beta_0 + \beta_1) E_t \left[\phi_{m(t+1)}, \tau \theta \int_{E_{t+1}}^{E_H^*} (E - E_{t+1}^*) y_{t+1} dG(E) / p_{t+1} \right]$$

$$\text{with "}" \text{ if } \alpha_{im(t+1)}^{\sim} > 0$$

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t \left[\varepsilon^- y_{t+1} + \phi_{s(t+1)}, + \tau \theta \int_{E_L^*}^{E_{t+1}^*} (E_{t+1}^* - E) y_{t+1} dG(E) \right]$$

$$\text{with "}" \text{ if } \alpha_{is(t+1)}^{\sim} > 0$$

For all $d \in D$ and all $i \in I$, and

$$\alpha_{jm(t+1)} = \alpha_{jm(t+1)}^{\sim}$$

$$\alpha_{js(t+1)} = \delta \alpha_{js(t+1)}^{\sim} + \text{if } \{j \in I\} (1 - \delta) A_s$$

For all $j \in D, I$

(iv) End-of-period market clearing

$$\tilde{A}_{Ds(t+1)} + \tilde{A}_{Is(t+1)} = A_s$$

$$\tilde{A}_{Is(t+1)} + \tilde{A}_{Im(t+1)} = A_{m(t+1)}$$

5.8 Lemma 4

Consider $\mu^\wedge$ and μ^- as defined in (5). Then $\mu^\wedge < \mu^-$

5.9 Proof of Lemma 4

Define $\Upsilon(\chi) \equiv \beta^- [1 + \tau\theta (1 - \beta^- \delta) \chi]$. Let $\chi^\wedge \equiv \frac{(1-\tau\theta)(\epsilon^\wedge - \epsilon^-)}{\epsilon^\wedge \tau\theta}$ and $\chi^- \equiv \frac{\epsilon^- - \epsilon_L}{\beta^- \delta \epsilon^- (1 - \beta^- \delta) \epsilon_L}$, so that $\mu^\wedge = \Upsilon(\chi^\wedge)$ and $\mu^- = \Upsilon(\chi^-)$. Since Υ is strictly increasing, $\mu^\wedge < \mu^-$ if and only if $\chi^\wedge < \chi^-$. With (6) and the fact that $\epsilon^- \equiv \int_{\epsilon_L}^{\epsilon_H} \epsilon dG(\epsilon) = \epsilon_H - \int_{\epsilon_L}^{\epsilon_H} G(\epsilon) d\epsilon$

$$\chi^\wedge = \frac{\int_{\epsilon^\wedge}^{\epsilon_H} [1 - G(\epsilon)] d\epsilon}{\epsilon^- + \tau\theta \int_{\epsilon_L}^{\epsilon^\wedge} G(\epsilon) d\epsilon},$$

So clearly,

$$\chi^\wedge < \frac{\int_{\epsilon^\wedge}^{\epsilon_H} [1 - G(\epsilon)] d\epsilon}{\epsilon^-} = \frac{\epsilon^- - \epsilon_L}{\epsilon^-} < \chi^-$$

Hence, $\mu^\wedge < \mu^-$.

5.10 Proof of Proposition 1

Under an equilibrium without money (or where money holds no value), no transactions arise within the OTC arena. Per Lemma 2, the first-order criteria are:

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t \phi_{s(t+1)}, \text{ with } \prime = \prime \text{ if } \alpha_{ds(t+1)}^\sim > 0$$

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t (\epsilon^- y_{t+1} + \phi_{s(t+1)}), \text{ with } \prime = \prime \text{ if } \alpha_{is(t+1)}^\sim > 0$$

In the recursive equilibrium, $E_t \left(\frac{\phi_{s(t+1)}}{\phi_{st}} \right) = \gamma^-$, and $\beta \gamma^- \delta < 1$ is taken as a standing assumption, implying that dealers do not hold equity. Under this condition, the Walrasian equity market achieves clearance only when

$$\phi_{st} = (\beta_0 + \beta_1) \delta E_t (\epsilon^- y_{t+1} + \phi_{s(t+1)})$$

$$\phi_{st} = \beta \delta E_t (\epsilon^- y_{t+1} + \beta \delta \gamma^- \phi_{st})$$

$$\phi_{s} y_t = \beta \delta E_t (\epsilon^- \gamma y_t) + \beta \delta \gamma^- \phi_{s} y_t$$

$$\phi_{s} y_t = \beta \delta \epsilon^- \gamma^- y_t + \beta \delta \gamma^- \phi_{s} y_t$$

$$\phi_s = \beta \delta \epsilon^- \gamma^- + \beta \delta \gamma^- \phi_s$$

$$\phi_s = \frac{\delta \epsilon^- \beta \gamma^-}{1 - \beta \gamma^- \delta}$$

$$\phi_s = \frac{\beta^- \delta}{1 - \beta^- \delta} \varepsilon^-$$

It confirms section (i) and (iii) in the assertion of the proposition

Then, we turn to monetary equilibrium,

From (A5) to (A12)

$$\phi_{mt} \geq (\beta_0 + \beta_1) E_t (\phi_{m(t+1)})$$

$$1 \geq \beta E_t \left(\frac{\phi_{m(t+1)}}{\phi_{mt}} \right)$$

$$1 \geq \beta \left(\frac{\gamma^-}{\mu} \right)$$

$$\mu \geq \beta^-, \text{ with } = \text{ if } \alpha_{dm(t+1)}^{\sim} > 0 \quad (A12)$$

From (A6) to (A13)

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t (\phi_{m(t+1)} p_{t+1})$$

$$\phi_{st} \geq \beta \delta E_t (\phi_{s(t+1)}^-)$$

$$\frac{\phi_{st}}{y_t y_{t+1}} \geq \beta \delta E_t \left(\frac{\phi_{s(t+1)}^-}{y_t y_{t+1}} \right)$$

$$\frac{\phi_s}{y_{t+1}} \geq \beta \delta E_t \left(\frac{\phi_s^-}{y_t} \right)$$

$$\phi_s \geq \beta \delta E_t \left(\phi_s^- \frac{y_{t+1}}{y_t} \right)$$

$$\phi_s \geq \beta^- \delta \phi_s^-, \text{ with } = \text{ if } \alpha_{ds(t+1)}^{\sim} > 0 \quad (A13)$$

From (A7) to (A14)

$$\phi_{mt} \geq (\beta_0 + \beta_1) E_t \left[\phi_{m(t+1)} + \tau \theta \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) y_{t+1} dG(\mathcal{E}) / p_{t+1} \right]$$

$$\frac{\phi_s^- y_t}{p_t} \geq \beta E_t \left[\frac{\phi_s^- y_{t+1}}{p_{t+1}} + \tau \theta \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) y_{t+1} dG(\mathcal{E}) / p_{t+1} \right]$$

$$\phi_s^- y_t \geq \beta p_t / p_{t+1} E_t \left[\phi_s^- y_{t+1} + \tau \theta \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) y_{t+1} dG(\mathcal{E}) \right]$$

$$1 \geq \frac{\beta}{\mu} E_t \left[y_{t+1} / y_t + \frac{y_{t+1} \tau \theta}{y_t \phi_s^-} \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) dG(\mathcal{E}) \right]$$

$$1 \geq \frac{\beta}{\mu} \gamma^- E_t \left[1 + \frac{\tau \theta}{\phi_s^-} \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) dG(\mathcal{E}) \right]$$

$$1 \geq \frac{\beta^-}{\mu} \left[1 + \frac{\tau \theta}{\phi_s^-} \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) dG(\mathcal{E}) \right]$$

$$1 \geq \frac{\beta^-}{\mu} \left[1 + \frac{\tau \theta}{\phi_s + \mathcal{E}^*} \int_{\mathcal{E}_{t+1}^*}^{\mathcal{E}_H} (\mathcal{E} - \mathcal{E}_{t+1}^*) dG(\mathcal{E}) \right], \text{ with } \neq \text{ if } \alpha_{im(t+1)}^{\sim} > 0 \quad (A14)$$

From (A8) to (A15)

$$\phi_{st} \geq (\beta_0 + \beta_1) \delta E_t \left[\varepsilon^- y_{t+1} + \phi_{s(t+1)}, + \tau \theta \int_{\mathcal{E}_L^*}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) y_{t+1} dG(\mathcal{E}) \right]$$

$$\phi_s y_t \geq \beta \delta E_t \left[\varepsilon^- y_{t+1} + \phi_s y_{t+1} + \tau \theta \int_{\mathcal{E}_L^*}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) y_{t+1} dG(\mathcal{E}) \right]$$

$$\begin{aligned}
\phi_s &\geq \beta \delta E_t [\varepsilon^- y_{t+1}/y_t + \phi_s y_{t+1}/y_t + \tau \theta \int_{\mathcal{E}_L}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) y_{t+1}/y_t dG(\mathcal{E})] \\
\phi_s &\geq \beta \delta \gamma^- [\varepsilon^- + \phi_s + \tau \theta \int_{\mathcal{E}_L}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) dG(\mathcal{E})] \\
\phi_s &\geq \beta^- \delta [\varepsilon^- + \phi_s + \tau \theta \int_{\mathcal{E}_L}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) dG(\mathcal{E})] \\
1 - \beta^- \delta \phi_s &\geq \beta^- \delta [\varepsilon^- + \tau \theta \int_{\mathcal{E}_L}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) dG(\mathcal{E})] \\
\phi_s &\geq \frac{\beta^- \delta}{1 - \beta^- \delta} [\varepsilon^- + \tau \theta \int_{\mathcal{E}_L}^{\mathcal{E}_{t+1}^*} (\mathcal{E}_{t+1}^* - \mathcal{E}) dG(\mathcal{E})], \quad \text{with } \neq \text{ if } \alpha_{is(t+1)}^- > 0 \quad (A15)
\end{aligned}$$

5.11 Appendix

Why (1) can be written as $\text{Max} [(\varepsilon_t^* - \varepsilon) (\alpha_{imt}^- - \alpha_{imt}) \frac{y_t}{p_t} - \varphi_t] (\theta) (\varphi_t) (1 - \theta) :$

Firstly, we consider

$$\varepsilon y_t \alpha_{ist}^- + W_{It} (\alpha_{it}^-, \varphi_t) - \varepsilon y_t \alpha_{ist} - W_{It} (\alpha_{it}, 0) \quad (M)$$

Where

$$W_{It} (\alpha_{it}^-, \varphi_t) = \phi_t \alpha_{It}^- - \varphi_t + W_{It}^-$$

$$W_{It} (\alpha_{it}, 0) = \phi_t \alpha_{It} + W_{It}^-$$

$$\begin{aligned}
(M) &= \varepsilon y_t \alpha_{ist}^- + \phi_t \alpha_{It}^- - \varphi_t + W_{It}^- - \varepsilon y_t \alpha_{ist} - \phi_t \alpha_{It} - W_{It}^- \\
&= \varepsilon y_t \alpha_{ist}^- - \alpha_{ist} + \phi_t \alpha_{it}^- - \alpha_{it} - \varphi_t \\
&= \varepsilon y_t \alpha_{ist}^- - \alpha_{ist} + \phi_{mt} \alpha_{imt}^- + \phi_{st} \alpha_{ist}^- - \phi_{mt} \alpha_{imt} - \phi_{st} \alpha_{ist} - \varphi_t \\
&= \varepsilon y_t \alpha_{ist}^- - \alpha_{ist} + \phi_{mt} (\alpha_{imt}^- - \alpha_{imt}) + \phi_{st} (\alpha_{ist}^- - \alpha_{ist}) - \varphi_t \\
&= (\varepsilon y_t + \phi_{st}) \alpha_{ist}^- - \alpha_{ist} + \phi_{mt} (\alpha_{imt}^- - \alpha_{imt}) - \varphi_t
\end{aligned}$$

With the budget constraint binding condition

$$\begin{aligned}
\alpha_{imt}^- + p_t \alpha_{ist}^- &= \alpha_{imt} + p_t \alpha_{ist} \\
\alpha_{ist}^- - \alpha_{ist} &= -1/p_t (\alpha_{imt}^- - \alpha_{imt})
\end{aligned}$$

Then,

$$(M) = (\varepsilon y_t + \phi_{st}) (1/p_t) (\alpha_{imt}^- - \alpha_{imt}) + \phi_{mt} (\alpha_{imt}^- - \alpha_{imt}) - \varphi_t$$

$$\text{Let } \varepsilon_t^* = \frac{p_t \phi_{mt} - \phi_{st}}{y_t}$$

$$\begin{aligned}
(M) &= [\phi_{mt} - (\varepsilon y_t + \phi_{st}) \frac{1}{p_t}] (\alpha_{imt}^- - \alpha_{imt}) - \varphi_t \\
&= [\frac{p_t \phi_{mt} - \phi_{st}}{p_t} - \frac{\varepsilon y_t}{p_t}] (\alpha_{imt}^- - \alpha_{imt}) - \varphi_t \\
&= [\frac{p_t \phi_{mt} - \phi_{st}}{y_t} - \varepsilon] \frac{y_t}{p_t} (\alpha_{imt}^- - \alpha_{imt}) - \varphi_t \\
&= (\varepsilon_t^* - \varepsilon) (\alpha_{imt}^- - \alpha_{imt}) \frac{y_t}{p_t} - \varphi_t
\end{aligned}$$

Secondly, we consider

$$W_{Dt}^\wedge (\alpha_{dt}, \varphi_t) - W_{Dt}^\wedge (\alpha_{dt}, 0) \quad (N)$$

$$W_{Dt}^\wedge (\alpha_{dt}, \varphi_t) = \phi_t \alpha_{dt} + \varphi_t + W_{Dt}^-$$

$$W_{Dt}^{\wedge}(\alpha_{dt}, 0) = \phi_t \alpha_{dt} + W_{Dt}^{-}$$

$$(N) = \phi_t$$

6. Conclusion

We develop a dynamic OTC asset-market model in which monetary policy operates through a turnover-liquidity channel. Trading frictions and bargaining generate a valuation cutoff ε^* : conditional on meeting a dealer, investors with valuations above the cutoff rebalance into equity while others hold money. Because the cutoff is pinned by the money–equity wedge, an increase in money growth μ lowers ε^* , raises the trader mass $\tau[1 - G(\varepsilon^*)]$, and thereby increases turnover and the price–dividend ratio. Two modeling ingredients sharpen both economic content and empirical discipline relative to existing work [6]. First, we decouple payout timing from survival by allowing temporary dividend suspensions with probability ι and embed a trend-adoption parameter Ω in dividend dynamics; this separates liquidity-driven trading from cash-flow news and yields cross-sectional variation in policy pass-through. Second, a two-speed intertemporal structure with a real rate r and a delisting hazard p clarifies existence of monetary equilibria and delivers closed-form money-growth thresholds that bound regions where money is valued [8]. The framework implies stronger transmission when meeting intensity τ and investor surplus share θ are high, and it organizes measurement around sufficient statistics such as trader mass and standardized OTC turnover. These features make the model tractable for calibration and suggest event-study tests using heterogeneity in meeting frictions, payout suspension risk, and adoption intensity to identify the turnover-liquidity channel [9].

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