

# ***Research on the Process of the Formation of an Optimal Investment Portfolio***

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**Abstract:** In the financial market, there are different types of financial instruments, such as equities, bonds, and mutual funds, and investors can build an investment portfolio through their buying and selling activity in the market. Limited-experienced individual investors are often attracted to stocks or bonds with high returns when trading in the financial market and building an investment portfolio. This could make them underestimate the adverse impact brought by investment risks on their expected returns for the portfolio. An optimal investment portfolio should be aimed at with a step-by-step process that considers risk tolerance, portfolio selection, and the monitoring of the portfolio after it has been formed. This theoretical process would first consider the way in which individual investors could assess the risk involved in the portfolio. The beta coefficient is one of the indicators and it can be estimated via the capital asset pricing model (CAPM). Then, a well-diversified portfolio would be favoured, not only across different financial instruments, but also across different industries, and the minimum desired number of instruments included in the portfolio is 30. Rebalancing portfolios is a crucial step in keeping the original structure, and Žilinskij's criterion helps individual investors to rebalance more efficiently. Although this process helps investors in building investment portfolios, it should include more areas of concern about investors' own interests and the plausibility should be further tested.

**Keywords:** investment, investment portfolio, risk, portfolio selection, investment strategy

## **1. Introduction**

Investment is the acquiring of assets through buying financial instruments, such as equities, bonds, derivatives, and commodities, for wealth growth; an investment portfolio consists of invested assets [1]. It can be commonly heard that people are buying stocks or bonds to earn money, and there are several different types of financial instruments common in investment, such as equities and bonds. Equity stands for ownership. Companies selling equities mean they are selling ownership for cash. These equities are regarded as common stock, and buyers have the right to vote on companies' significant decisions. Another type of equity is preferred stock. Preferred stock buyers are given dividends before common stock investors, which are higher for preferred stock owners [2]. Unlike equities, bonds, to a large extent, guarantee investors' earnings. Companies must pay the bond owners the correct amount when the bond matures. Otherwise, regulations and legislation would intervene. Many individual investors often disregard the importance of the systematic process of building an investment portfolio. Therefore, although they understand how and what to invest in, it is still highly

possible that investments without comprehensive management could fail and lose money. For instance, investors who fail to prepare for an economic recession may lose all their invested capital. Investors should also invest in assets whose value does not depreciate much or even appreciate during economic recessions. Another vital factor is market risk. Market movements are hard to predict, and stocks respond differently to these movements according to their beta. Therefore, this paper mainly focuses on the way that active investing individuals in the market manage their portfolios through three phases of the management process, investment preparation, portfolio selection, and portfolio rebalancing. By following this process, individuals, even with limited experience in investing, could have an optimal investment portfolio that enables them to maximise returns and minimise investment risks.

## 2. Literature Review

Thakur and Vaidya summarized the 6 most common investment types [1]. It includes investments in stocks, bonds, options, real estate, cryptocurrencies, and commodities. Moreover, Hall gave a clear difference between common stock and preferred stock in the article [2]. Common stock is the stock that gives investors ownership of the company, whereas preferred stock provides more dividends to investors. With this clear distinction between the two types of stocks in mind, investors could choose the more suitable stocks instead of randomly guessing whether the bought stock gives more ownership or monetary benefit. Also, Prayudha and Kuswanto proved that investment return is positively correlated with investment interest [11]. Therefore, according to Benson and Jackson, building of a good investment portfolio consists of the following steps: deciding whether to invest personally or with the help of professionals, choosing an account, choosing risk tolerance, determining the best asset allocation, and rebalancing the investment portfolio [3].

Dubinskas and Urbšienė constructed the genetic algorithms and built a portfolio based on the genetic algorithms [4]. Then, they built another portfolio using the deterministic and stochastic methods and compared the performance of the two portfolios. The result shows that a portfolio constructed by using the genetic algorithm-based method generates higher returns than a portfolio formed by means of deterministic and stochastic programming methods. However, as mentioned in the paper, the genetic algorithms-based method should be applied over a long period of time to draw a more reliable conclusion [4].

Moreover, Rajendran reviewed that analyzing asset allocation should through different aspects, such as determination of asset allocation, portfolio management process, and return objectives and investment constraints [5]. It comprehensively analysed asset allocation and allowed investors to better understand the core determinates that influence a portfolio the most: risk tolerance, time horizon, and situations of finance. Investors need to use the beta coefficient to evaluate the systematic risks of an investment. Beta is an indicator of the risk level (variance) of a stock. [6] It can be estimated using the CAPM model and statistical techniques. Moreover, Sukrianingrum and Manda investigated the effects of systematic and unsystematic risk on expected return and concluded that systematic risk is negatively correlated with an expected return, whereas unsystematic risk is positively correlated with an expected return [7]. This may imply that keeping some unsystematic risks in the portfolio is beneficial, and investors should try to minimise systematic risks.

One of the core foundations of Modern Portfolio Theory (MPT) is Markowitz's Portfolio Selection [8]. It mentioned the 'expected return-variance of return' rule that investors should follow to maximise expected return and minimise variance. Markowitz strongly suggested diversification, which involves not only regards a large number of securities in the portfolio, but related industries should also be diversified [8]. Moreover, Statman provides significant data and mathematical analysis in article that investigates the number of stocks required to construct a well-diversified portfolio [9] and concludes that at least 30 securities are required to well-diversify portfolios.

Rebalancing a portfolio helps investors keep the original structure after some price movement of invested securities. However, actively managing portfolios incurs transition costs. If the cost of rebalancing exceeds the benefits, it would be unnecessary to rebalance a portfolio. Zilinskij introduced a criterion that helps investors decide whether they should rebalance portfolios [10].

### 3. Analysis

#### 3.1. Investment Preparation

Before investments happen, there are two factors to consider: expected return and risk [5]. The predicted return is the total amount of money they expect to gain or lose from an investment portfolio. People's priority aim in investing is to accumulate wealth. Hence, they should have a clear view of the expected return when an investment is 'finished'. For example, a stock's risk premium is 12.7%. When the risk-free rate of this stock is 2%, the expected return of this stock would be 14.7%, assuming the beta of this stock is 1. The expected return of a portfolio, then, is the sum of the weighted expected return of each stock or bond investment.

Regarding risk, there are two aspects to consider – risk tolerance and market risk. Firstly, investors need to determine their investment risk tolerance. Risk tolerance is the acceptance of investment losses in exchange for high returns. In other words, risk tolerance refers to the proportion of money investors can accept to invest in equities and bonds. For example, equities and bonds are classic combinations. Equities, in contrast to bonds, are generally riskier since they do not have the contractual property of re-payment. On the other hand, equities usually provide higher returns than bonds, and investors have to choose between this risk-return trade-off. If an investor is conservative, a 70% bonds and 30% equity portfolio is suitable. A portfolio consisting of 20% bonds and 80% equities is ideal for investors who are not risk averse. Investors generally need to consider their expected returns and related risks if they want to include more financial instruments in their portfolios. This is where beta becomes an essential index for investors to refer to.

After deciding on risk tolerance, investors should know how to calculate their portfolio risks. Market risk is associated with the beta index. Beta measures the volatility of a stock or portfolio relative to the whole market. Positive beta means that stock prices change in line with market movements and vice versa. Beta is estimated using the Capital Asset Pricing Model (CAPM), which is established below [6]:

$$ER_i = R_f + \beta_i * (ER_m - R_f) \quad (1)$$

$ER_i$  is the expected return on investment.  $R_f$  is the risk-free rate.  $\beta_i$  is the beta of the investment. And,  $(ER_m - R_f)$  is the market risk premium

With the technique of OLS estimation, the beta level could be calculated. Beta is significant, especially to people who have invested a large amount of money in various fields. It allows investors to have a good combination of investments according to the overall market situation. When the economy is performing well, and the market is in a growth stage, people could invest in more equities for more considerable gains. Suppose the economy is in a recession and the value of goods decreases dramatically. In that case, people could choose those financial instruments with a beta close to zero or even negative because these instruments' values vary less volatile than market movements, and there might be potential gains. A portfolio may have equities for primary earning, bonds for 'safe' appreciation of assets, and metal commodities, like silver, whose value stays stable for high liquidity. This is the diversification of portfolios.

### 3.2. Portfolio Selection

The next step in the process is selecting the financial instruments to build the investment portfolio based on an investors' risk tolerance. This helps investors build an optimal portfolio.

Market risks can be divided into two groups – systematic risks and unsystematic risks. The difference between these two is that systematic risks, measured by beta, are associated with market changes, whereas unsystematic risks are related to changes in companies or industries. Systematic and unsystematic risks have different impacts on expected returns, which are worth investors considering before investing in. According to Sukrianingrum and Manda, the effects of systematic and unsystematic risks on the expected return of an optimal portfolio consisting of the most liquid 45 company stocks on the Jakarta Stock Exchange, known as LQ45, during the period 2015–2019 [7]. After a series of analyses, they concluded that systematic risks have statistically significant negative impacts on the portfolio's expected return. It means that the higher the systematic risk, the lower the expected return. Unsystematic risks have exactly the opposite effect on the expected return. Moreover, systematic and unsystematic risks simultaneously influence expected returns positively. Although it seems that taking risks while selecting stocks is beneficial in terms of expected returns, having a superior level of risk would increase the degree of uncertainty in the portfolio. It is still necessary to control the risks, and risk diversification is a no effective way.

Numerically, the risk of stocks can be understood as the dispersion (variance) of the returns. Markowitz mentioned the 'expected return – variance of return' rule about investment portfolio selection, assuming that there is a portfolio that provides both maximum expected return and minimum variance [8].

Suppose Figure 1 represents all possible combinations of expected returns (E) and variances (V) within the circle, and the expected return–variance of return rule suggests that investors should choose or diversify a portfolio towards the efficient combination of expected returns and variances. Using statistical calculations, the mean returns and the standard deviation, the square root of the variance, can be determined, which are then adjusted according to other factors outside the scope of the analysis.

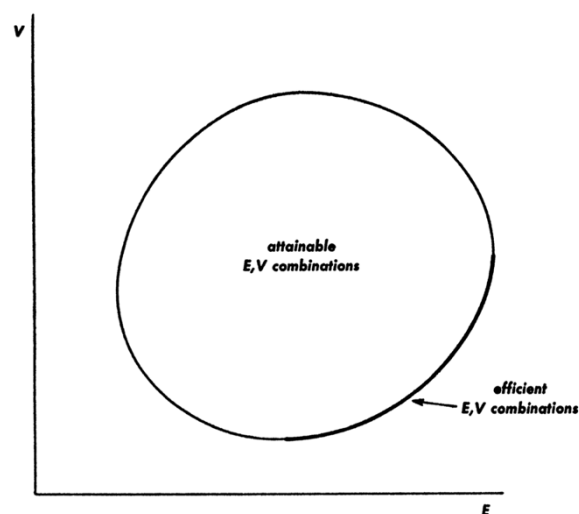


Figure 1: Possible combinations of expected returns (E) and variances (V) [8].

A critical fact about the diversification of portfolios is that it can not be defined only based on the number of securities in the portfolio. As Markowitz illustrated that a portfolio of sixty different railway securities would still be rated risky [8]. This is because all of the securities are from the same industry. Investors will likely lose money if the railway industry suffers from a severe decline. A

well-diversified portfolio is a same-size portfolio containing various industries' securities. Each pair of securities should also have low covariance. In other words, it would be best for every two securities to have a little relationship to minimise the variance of the portfolio return.

However, Markowitz assessed the theory based on only three securities in the expected return, variance of return' model, which could regard the plausibility of the rule with suspicion [8]. The number of stocks is insufficient to imply the rule's usefulness. It is necessary to investigate the minimum number of stocks that investors should invest in to fully diversify their portfolios. To do this, mathematical analysis is required. Statman compared the performance of a randomly chosen portfolio consisting of 10 stocks, G(10), and a portfolio of 10 stocks from the Standard and Poor's (S&P) 500 Index, P(10) [9]. The equation used is below:

$$E[RP(n)] = (RF + \alpha) + \left\{ \frac{E[RP(500)] - (RF + \alpha)}{\sigma_{P(500)}} \sigma_{P(n)} \right\} \quad (2)$$

Where  $p(n)$  is a portfolio with  $n$  stocks, and  $P(500)$  is a portfolio with 500 stocks.  $E[R]$  is the expected return.  $RF$  is the risk-free rate.  $\sigma$  is the standard deviation of the portfolio, and  $\alpha$  is the excess of the borrowing rate over the lending or risk-free rate. Based on previous studies, the risk premium,  $E[RP(500)] - RF$ , of the S&P 500 portfolio was approximately 8.2 from 1926 to 1984.  $\alpha$  was estimated to be 2. The  $\sigma$  of G(10) was 23.932, and the  $\sigma$  of P(10) was 19.265.

Using the above equation,  $E[RP(10)]$  equals  $RF + 9.702$ . The difference between  $E[RP(10)]$  and  $E[RP(500)]$  would be  $(RF + 9.702) - (RF + 8.2) = 1.502$ . This 1.502% difference represents an investor's benefit from increasing the number of stocks in the portfolio from 10 to 500 in terms of expected returns. With this concept in mind, if the difference in the expected return of real portfolios of 10 stocks and 500 stocks can be estimated, the number of stocks required to form a fully diversified portfolio can be calculated with the help of the following table.

Table 1: Difference between the expected returns of G(n) and the expected returns of P(n), varying as the number of stocks (n) changes [9].

**Difference between Expected Annual Return of a Portfolio of  $n$  Stocks,  $G(n)$ , and Expected Annual Return of a Portfolio  $P(n)$  That Levers a 500-Stock Portfolio Such That Standard Deviations of Returns of Portfolios  $G(n)$  and  $P(n)$  Are Equal<sup>a</sup>**

Number of Stocks in Portfolio ( $n$ )	Return Differences for Borrowing and Lending Investors	
	Borrowing Investor	Lending Investor
10	1.502	1.986
20	0.776	1.027
30	0.517	0.683
40	0.383	0.507
50	0.302	0.399
100	0.135	0.179

Statman used Vanguard Index Trust, a no-load index fund that mimics the S&P 500 Index. Compared with the S&P 500 Index, the average annual difference return during the period 1979 to 1984 was 0.49. Regarding the table above, 30 stocks would be enough for individual investors to build a diversified portfolio [9].



### 3.3. Portfolio Rebalancing

The last step of this process is to rebalance the portfolio. This could allow investors to keep their ideal portfolio structure against the rapid-changing market after the formation of original portfolio. With conservative investors as an example, the change in stock prices could influence the portion of stocks among the invested assets. Conservatives may wish to keep their original portfolio structure by selling and buying securities.

The transaction cost of these selling and buying actions needs to be considered when deciding whether to rebalance portfolios or not. If the gain of rebalancing portfolios only exceeds the cost of portfolio rebalancing by a small amount, investors may not be willing to do it [10]. Zilinskij proposed a portfolio rebalancing decision-making strategy that uses two main investment characteristics, expected returns and risks, to rebalance the portfolio for higher returns.

‘P0’ represents the initial portfolio, and ‘P’ represents the rebalanced portfolio. Let C denote the transition cost of rebalancing. The decision-making method would follow the conditions below:

$$\frac{E[R_P] - E[R_{P_0}]}{C} \geq K \quad (3)$$

$E[R]$  is the expected return of the corresponding portfolio. K is a decision-making criterion. The inequality condition above could be interpreted as investors choosing to rebalance their portfolios only when the benefit (difference between the expected return of the two portfolios) of rebalancing exceeds the transition cost by K times. This K value varies between each investor. For example, if an investor decides to let K be 2, then only when the difference between expected returns is at least twice the transition costs, the investor will choose to rebalance. Otherwise, the investor prefers to keep the portfolio unchanged. This condition allows investors to stop rebalancing portfolios every period, but only in a certain period when rebalancing is necessary for investors' desire.

## 4. Conclusion

The aforementioned strategy of portfolio construction involves preparation, portfolio selection and rebalancing.

The application of this process allows individual investors to build an optimal portfolio that maximises expected return and minimises variance. At the preparation stage, investors need to decide their investment strategy in terms of risk tolerance and consider the required expected return and risk level. Then, when selecting securities for the portfolios, diversification is important. In no case is a diversified portfolio preferred to all non-diversified portfolios. It has been testified that a portfolio with at least 30 securities is considered well-diversified. The ideal scenario is that every two stocks in the portfolio are not correlated. However, this is very hard to realise and therefore, diversification hardly removes all variance of returns of portfolios. In the last stage, portfolio rebalancing is introduced with a criterion. The criterion helps investors make decisions about whether to rebalance the portfolio or not.

This method of building an optimal portfolio is limited since most portfolio formation involves only two investment characteristics: expected return and risk. To improve this process, it needs to be more considerate. There are many other reasons for people to invest, such as pleasure, work, and hobbies [11]. Different investment purposes would lead to completely different portfolio structures, leading to different results. Possible further research on this topic could add the impact of investors' interests as a factor that influences the construction of portfolios. Another limitation of this process would be that the plausibility of the process is uncertain. There is no real case where an investor successively builds an optimal investment portfolio with the use of the process suggested in this entire paper. The value of this issue is also worth investigating.

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