

Application of Game Theory in Different Auction Forms

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Abstract: Auctions are employed in many areas all across the world. It is a key topic of study in game theory. This article primarily summarizes the winning strategies in auctions from a game theory perspective. To examine this issue, this paper reviews the past literature from three auction angles. The winning strategies applied in English auctions, Dutch auctions, and first-price auctions will differ depending on the form of auction. During a customary English auction, the winning strategy is to bid at their genius willingness. However, when multiple goods and deadlines are added to the auction, the winning strategy becomes to bid lower than the valuation. In Dutch auctions, bidders should bid at the beginning at a lower price. In first-price auctions, bidders should bid for all combinations when there are many items, and overbid when there is standard information feedback. This article is a review of previous relevant articles and provides references to related articles.

Keywords: game theory, English auction, Dutch auction, first-price sealed-bid auction

1. Introduction

1.1. Game Theory

Economy is used to refer to methods of controlling resources and wielding power [1]. Economic concerns are a top priority for modern people and governments, and everyone's life is impacted by economics [2]. Game theory is a branch of economics, which is a common tool in economics, and it is a study of behavior [3]. It provides a conceptual foundation that is utilized to envision social events involving rival participants [3]. Game theory is, in some ways, the study of strategy, or at the very least it studies how autonomous, competitive players might decide wisely in a strategic context [3]. It is widely utilized in numerous domains, including economics, mathematics, and political science, and can be used to represent a wide range of real-world circumstances like auctions [3].

1.2. Auctions

Auctions are the main object of study in game theory, which can be explored primarily via one of the perspectives of game theory [4]. At an auction, goods are put up for sale, bids are received, and then either the item is sold to the player who bid the highest or purchased from the auctioneer who bid the lowest [4]. The auction sets a classic example of modern economics research, i.e., the research problem is derived from and fed back into practice, fully reflecting the reciprocal advancement of theory and practice, and demonstrating modern economics' characteristic

progression [4]. English auction, Dutch auction, and first-price sealed-bid auction are the three fundamental types of auction models. An English auction reveals the equilibrium between buyers [5]. A simple illustration demonstrates that equilibrium fails to exist in English auctions when bidder valuations include both common and private components and private information is held on both dimensions [5]. The Dutch auction demonstrates the bidding behavior of players [6]. Bidders in Dutch auctions tend to shade their bids in order to increase their expected surplus, as they don't know other bidders' valuation [6]. Individual bidding behavior is the primary demonstration of the first price auction [7]. Auction experiments have been shown to foster overbidding. Since players are unaware of one another's bid price, first price auctions have usually shown bidding over valuation in compared to a risk-neutral equilibrium [7]. There have been many reviews of auction pricing and of the economic theory of auctions, but no review of the use of game theory in auctions has been undertaken. The purpose of this paper is to summarize all developments in the application of game theory in three different types of auctions. This article will help the reader to gain a deeper grasp of the application of game theory in auctions.

2. English Auction

The most often used sort of auction is the English auction. It takes the following form: during the auction, the bidding for the auction item climbs up and up in accordance with the ladder of bidding, and the highest bidder is declared the auction's winner [8]. Many scholars have studied the models of English auctions and the strategies used to win them.

In 1998, some scholars examined what strategies are more likely to win an English auction. The authors used a computer simulation of the conduct of an auction to assign different bidding strategies to different players. In this simulated auction, every bidder placed a bid on what they were actually willing to pay. In the end, they found that the player's probability of winning is the greatest when they bid at their true willingness to pay [9].

Later on, scholars added items to the conventional English auction model to make the number of auctions increase and discussed winning strategies in this auction model. They carried out theoretical analyses, derivations, and computations based on the classical model to obtain a stable solution. Ultimately, their calculations demonstrated that bidders should bid lower than their valuation, regardless of how their rivals may behave to win an English auction with multiple auctions [10].

Some academics studied the strategies used by bidders to win the auction, which incorporated the influence factor-deadlines. It was modified based on the original English auction model by adding a new deadline to prevent endless auctions. They used a computerized mock auction to assign different bidding strategies to different players where all auctioneers bid lower than their genius willingness, maximizing the utility of all bidders. In the end, they provided bidder agents with the most effective and reliable bidding strategies: bidding lower than what they were actually willing to pay when combined with the deadline rule [11].

In 2005, academics investigated bidders' strategies to maximize their expected utility. They gathered six auctioneers to conduct a human-to-human experiment in which each bidder was randomly assigned a bidding strategy. The authors then took a contrasting approach, comparing five popular bidding strategies—bidding higher, bidding lower, bidding truth, last-bidding, and first-bidding to the proposed one-bidding after planning. Finally, this study found that the average expected utility of agents after making a plan to write a persuasive, winning proposal that sets them apart from their competitors is greatest in an auction [12].

3. Dutch Auction

The situation with the Dutch auction is more complicated, and historically. A dutch auction starts out high and gradually lowers it until a bidder accepts the auctioneer's offer or it reaches a specified reserve price [13]. Many academics have examined strategies, revenue, and even multi-items in Dutch auctions.

In 1982, three academics determined the optimal time for bidders to place their bids in a Dutch auction through mathematical derivation and theoretical analysis. They calculated the bidders' expected utility and utility income related to bidding time to obtain a stable solution. Their computations illustrated that placing bids at the beginning is the best strategy in a Dutch auction [14].

Later on, in a realistic online Dutch auction, scholars found that the auction yielded higher expected revenue than the traditional theoretical expectation. To investigate the causes of this phenomenon, the authors added an additional bidding cost to the bidders on top of the classical game theoretical model and solved the new model theoretically. Their results found that the bidding cost does account for the phenomenon of higher expected revenue than the traditional theoretical solution [15].

In 2009, two scholars discovered the bidding strategies in auctions with descending prices compared to those with ascending prices counterparts. The authors used two computerized mock auctions: a descending price auction and an ascending price auction, where players in various auctions adopted different bidding strategies. Other variables, such as the number of bidders, were all the same. In descending auctions, bidders placed bids lower than their true willingness, whereas in ascending auctions, bidders bid at their valuation. In the end, they came to the conclusion that bidders bid lower in descending price auctions than in ascending price auctions [16].

Following that, researchers investigated how the Dutch auction's participant count affected the auctioneer's bidding strategies. This article approached the issue by modeling the auctioning process using a computer simulation to conduct several auctions with varying numbers of bidders where various players had various bidding methods as a process of resolving an optimization issue, in which bidders shaded more bids when there were more bidders. They showed that bidders should place far lower bids than what they are truly ready to pay, as there are more bidders and bid levels [17].

4. First-price Sealed-bid Auction

One such common kind of auction is the first-price auction. It's also referred to as the blind auction. In an auction of this nature, the auctioneer (seller) sells an indivisible item to potential bidders (buyers) in a first-price auction. The rule is straightforward: the auctioneer receives simultaneous bids from all bidders, each of which is hidden from the other bidders; the object is won by the bidder who offers the highest bid [18].

In 1996, one academic studied bidders' tactics to win first-price auctions that incorporated the influence factor—an interval of messages. He conducted a computerized mock modified auction based on the traditional model by adding a closed interval of messages to be used in the resolution of the ties. All bidders in this auction must send a message with the bid they offer. After comparing bidders' expected utility, they proved that the best strategy for bidders is to adhere to the course of their initial strategy regardless of how other players behave [19].

Later on, two academics studied bidders' bidding strategies in first-price auctions with the standard information feedback. They used an individual choice experiment by gathering ten people, only one of whom received standard information feedback as the treatment variable. In the auction,

the one with the standard information overbid the most. The outcome demonstrated that when there is standard information feedback in auctions, bidders will overbid [20].

In 2007, two academics found bidders' optimal strategies in the first-price auction. They did a computer-simulated auction where each bidder utilized a valuation model that was unique and with an even distribution—one model was highly broad, requiring bids for all combinations, and the other was restricted, requiring single bids for the remaining items. After comparing payout and revenue, the results showed bidders should submit bids for all combinations [21].

This year, some scholars studied the efficiency of a successful bid for a single bid in first-price auctions. Through mathematical derivation and theoretical analysis, they calculated the expected utility of bidders using the integral formula to measure the efficiency of a successful bid owing to the self-serving behavior of bidders. Finally, according to the indicator, they proved that there is a high efficiency of a successful bid for a single bid in first-price auctions [22].

5. Conclusion

This article summarizes the winning strategies in auctions from a game theory perspective. To examine this issue, this paper reviews the previous literature from three auction angles. Initially, consider the English auction. Depending on the format of the auction, different winning strategies will be used. In a traditional English auction, the winning strategy is to bid at their genius willingness. However, when multiple items are included, the winning strategy becomes to bid lower than the valuation, and when the deadline rules are applied, the winning strategy is still to bid below the valuation. By comparison with other tactics, the best strategy is to bid after making a plan. As for Dutch auctions, the winning tactics will also vary depending on the form of the auction. In traditional Dutch auctions, the winning strategy of the player is to bid when the auction first starts. As more people participate in auctions, bidders should bid much lower, and when there are bidding costs, bidders can expect to earn more revenue. Similarly, the winning strategy for different forms of first-price auctions will change depending on the format of the auction. In the presence of an information interval, the winning strategy is to maintain the original strategy. When there is standard information feedback, the winning strategy becomes overbidding, and when there are many auction items, bidders should submit bids for all combinations.

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