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Analysis of the Evolutionary Game Between Enterprises and Local Governments under Pollution Control

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Abstract: The strategic choices made by the government and enterprises, as the main participants in environmental governance, decide the direction of environmental governance. Meanwhile, the strategic choices of enterprises will also change the strategic layout of the government. Therefore, current research mainly focuses on what impact the government has on enterprises when it participates in the game, and how will the interested parties make optimal decisions. In this paper, from the perspective of the evolutionary game theory, the strategic choices of the interaction between pollutant-discharging enterprises and between local governments and pollutant-discharging enterprises are discussed; the model of the evolutionary game between pollutant-discharging enterprises and local governments is also built. According to the replicated dynamic equation, the evolutionarily stable strategy of the participants is obtained. The research results indicate that: the higher the return on pollution control is for an enterprise, the more inclined it will be towards pollution control and environmental protection. Environmental governance should reduce the expected benefits of enterprises without treatment of pollutants and should require local governments to strictly implement environmental protection policies, perform their duty, and identify and punish violators in a timely manner.

Keywords: environmental protection, government control, environmental protection tax, evolutionary game

1. Introduction

With the continuous development of society, environmental governance has become a key issue that cannot be ignored in economic development. Climate change, biodiversity loss, air pollution, and other environmental issues are major challenges facing the international community today. The main source of environmental pollution is the excessive emissions of enterprises. From an economic point of view, the main participants in environmental governance include the government and enterprises. Therefore, the decisions of government departments will to a certain extent influence the direction of enterprise development; the strategic choices of enterprises will in turn change the strategic layout of the government. Green development is now an important trend. Traditional manufacturing and power generating companies have begun to develop new technologies (green manufacturing) to change people's stereotype that traditional manufacturing must be a polluting industry. The green industrial development planning under the China's "14th Five-year Plan" sets the goal that by 2025, with the improvement of green manufacturing technology, the pollutant emission intensity will be significantly reduced, and the emission intensity of major pollutants in key industries will drop by 10%. For environmental governance, China is now promoting "treatment by all people". The government, enterprises, and the public shall each fulfill their responsibilities and make joint efforts. The government will take the lead; enterprises will take the initiative to assume the responsibilities as main participants in environmental governance; the public will consciously pursue a green way of life. The standards for environmental governance are uniformly developed by the central government and then implemented by local governments. The intensity of environmental governance mostly depends on the execution capacity of a local government and whether the pollutant-discharging enterprises can cooperate with the work of the local government. Local governments are mainly responsible for implementing the environmental protection policies of the central government and supervise the production of enterprises. Enterprises are required to make maximum profits on the premise that environmental protection requirements are met.

According to the existing literature, the game theory analysis of environmental issues focuses on the game between government regulation and enterprises' pollutant-discharging production; some scholars have introduced new subjects and variables of research. Wang and Bao studied the static game between local governments and enterprises under complete information, and concluded that: due to the information asymmetry, enterprises, as rational persons, wouldn't choose to emit pollutants at all, where the government should intervene and collect environmental protection tax to reduce the loss of government credibility. With further research, some scholars introduced new subjects in their game analysis so that their models would better agree with reality [1]. In the research by Liu and Meng, they introduced the variable of the public in the game between the government and enterprises and studied the strategic choices of both sides of the game for environmental protection with public participation. It further highlighted that in addition to the checks and balances between the government and enterprises, the parties' decision-making was also subject to third-party regulation [2]. Wang further broke government regulation down to policy making by the central government and enforcement by local governments; the two parts emphasized different benefits and might therefore see inconsistencies in policy formulation and implementation. He also concluded through analysis that local governments often focused more on maximizing short-term benefits and might take a negative attitude towards the policies [3]. Du proposed the effect of environmental protection tax on the decision-making of the government and enterprises and suggested that government departments fix and collect environmental protection tax appropriately, making sure that corporate profits, environmental conditions and government costs were all taken into account [4]. Zhang and Ye studied the tripartite dynamic game of the government, environmental protection departments and enterprises, and introduced the credibility mechanism to work out the credit changes that may be caused by different decisions made by environmental protection departments and the government; they also analyzed the benefits of different decisions and made suggestions to all parties for the balanced environmental protection [5]. Song, Liu and Yin expounded on how the government implemented pollution control and environmental protection as it achieved high-quality development with stable growth and secured employment. They studied the controlled evolutionary game between enterprises and people in the neighborhood under government regulation, from which they learned that the larger the production turnover of the enterprise, the larger the operable space for the government to implement environmental regulation policies [6]. Pan, Xi and Wang adopted the evolutionary game theory to analyze the decision-making evolution of the central government, local governments and pollutant-discharging enterprises during policy implementation. Whether the environmental issue could be effectively addressed depended on the implementation by local governments and the central

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government's supervision thereof [7]. According to Lu, to reduce or put an end to environmental pollution, measures must be taken to reduce the returns or expected returns of pollutant-discharging enterprises with no treatment for pollution. If an enterprise found more benefits in not dealing with pollutants, it would choose not to treat pollution, causing serious environmental problems [8]. Zhang and Zhong pointed out that most existing game analyses ignored the loss of credibility brought by environmental pollution to government departments and pollutant-discharging enterprises, and that increasing the loss of credibility for pollutant-discharging enterprises and neglecting local governments would improve environmental governance [9]. According to the analysis of Wang, Li, Zuo and Gao, the probability that the government would supervise pollutant-discharging enterprises and the probability that such enterprises would purify the wastewater would influence each other; the keys to green development were government regulation and public supervision [10]. Shi, Wang & Yao, noticed that after introducing punishment by the central government to the model, the evolution path between local governments and enterprises would be closer to the spiral shape with a stable focus. It was concluded that being punished by the central government played a positive role in treating industrial pollution [11].

In agreement with the literature review, most existing research focuses on the game between the government and enterprises, without paying much attention to subdivision of the enterprises. This paper studies the effect of the environmental policy of the local government on the strategic choices of Enterprise 1 and Enterprise 2, which fills the gap in relevant research. At the same time, the increasingly serious environmental problems have caused greater pressure on government finance. Therefore, it is necessary to further study the game strategy of the government and enterprises and find the equilibrium of decision and the optimal strategy of each party.

This paper introduces parameters to build a tripartite evolutionary game model of the local government, Enterprise 1 and Enterprise 2. At the same time, its expected utility under different decisions is analyzed. In addition, through literature review, the game research on environmental protection in the existing literature is analyzed, on which basis the optimal strategy for the parties and the corresponding suggestions are made.

2. Basic Assumptions

Assumption 1: The three-game subjects of environmental governance considered in this paper are enterprise 1, enterprise 2, and local government. All have certain learning abilities and behavioral choice power.

Assumption 2: Enterprise 1, as the main body of the market, chooses pollution control and emission reduction or maintains the original production without pollution control according to the principle of maximizing benefits, the probability of pollution control is x, and the probability of non-pollution control is $1-x$. Similarly, the probability of enterprise 2 pollution control is y, and the probability of choosing not to control pollution is $1-y$. Since the environment is a public good, the local government will formulate strategies for corporate pollution, such as pollution tax, the probability z of the local government choosing a loose environmental protection policy, and the probability of choosing a strict environmental protection policy is $1-z$.

Assumption 3: Suppose enterprise 1 maintains the original production net income as R_1 and the amount of pollutant discharge is Q_1, the net income after emission reduction is R_2, the amount of pollutant discharge is Q_2, $Q_1 > Q_2$, and the emission reduction cost is C_1, $R_1 > R_2$. Enterprise 2 maintains the original production Net income is P_1, and pollutant discharge is H_1. The net income after emission reduction is P_2, the amount of pollutants discharged is H_2,$H_1 > H_2$,and the emission reduction cost is C_2, $P_1 > P_2$. If one company chooses to control pollution and the other company chooses not to control pollution, the latter will receive additional revenue from sales of goods S. If both companies choose not to control pollution, then both companies will receive sales revenue of G, and $G < S$.

Assumption 4: The environmental protection tax rate is t, the cost of the local government's strict environmental protection policy is M, the tax reduction and exemption ratio for pollution control enterprises is α; the amount of fines charged for non-pollution enterprises is F. When local governments adopt loose environmental protection policy, β represents the degree of easing, $0 < \beta < 1$, which means that the cost becomes βM, the tax reduction, and exemption ratio is $\beta\alpha$, and the amount of fines is also reduced to βF. To facilitate the discussion, according to the research of Zhang et al., it is assumed that the economic benefits of local governments can be quantified as specific income $v_1 R$ or $v_1 P$, and v_1 represents the local government's preference for the economy. Similarly, environmental benefits are quantified as $v_2 H$ or $v_2 Q$,where v_2 represents the local government's preference for the environment.

Table 1: Mixed strategy game matrix of enterprise 1, enterprise 2 and local government.

		local government			
		strict policy z		loose policy $1-z$	
		enterprise 2		enterprise 2	
		pollution control y	No pollution control $1-y$	pollution control y	No pollution control $1-y$
enterprise 1	pollution control x	$R_2-(1-\alpha)tQ_2-C_1$, $P_2-(1-\alpha)tH_2-C_2$, $v_1(R_2+P_2)-v_2(Q_2+H_2)-M$	$R_2-(1-\alpha)tQ_2-C_1$, P_1-tH_1-F+S, $v_1(R_2+P_1)-v_2(Q_2+H_1)-M+F$	$R_2-(1-\beta\alpha)tQ_2-C_1$, $P_2-(1-\beta\alpha)tH_2-C_2$, $v_1(R_2+P_2)-v_2(Q_2+H_2)-\beta M$	$R_2-(1-\beta\alpha)tQ_2-C_1$, $P_1-tH_1-\beta F+S$, $v_1(R_2+P_1)-v_2(Q_2+H_1)-\beta M+\beta F$
	No pollution control $1-x$	R_1-tQ_1-F+S, $P_2-(1-\alpha)tH_2-C_2$, $v_1(R_1+P_2)-v_2(Q_1+H_2)-M+F$	R_1-tQ_1-F+G, P_1-tH_1-F+G, $v_1(R_1+P_1)-v_2(Q_1+H_1)-M+2F$	$R_1-tQ_1-\beta F+S$, $P_2-(1-\beta\alpha)tH_2-C_2$, $v_1(R_1+P_2)-v_2(Q_1+H_2)-\beta M+\beta F$	$R_1-tQ_1-\beta F+G$, $P_1-tH_1-\beta F+G$, $v_1(R_1+P_1)-v_2(Q_1+H_1)-\beta M+2\beta F$

3. Results and Discussion

3.1. Evolutionary Game Model Analysis

3.1.1. Enterprise 1 Evolutionary Stability Strategy

Enterprise 1 pollution control income:

$$\pi_{11} = yz[R_2-(1-\alpha)tQ_2-C_1]+(1-y)z[R_2-(1-\alpha)tQ_2-C_1]+y(1-z)[R_2-(1-\beta\alpha)tQ_2-C_1]+(1-y)(1-z)[R_2-(1-\beta\alpha)tQ_2-C_1] \quad (1)$$

Enterprise 1's non-pollution income:

$$\pi_{12} = yz[R_1 - (1-t)Q_1 - F + S] + (1-y)z[R_1 - (1-t)Q_1 - F + G] + y(1-z)[R_1 - (1-t)Q_1 - \beta F + S] + (1-y)(1-z)[R_1 - (1-t)Q_1 - \beta F + G] \quad (2)$$

The expected return of enterprise 1 is:

$$\pi_1 = x\pi_{11} + (1-x)\pi_{12} \quad (3)$$

To sum up, the replication dynamic equation of enterprise 1 when making strategy selection is:

$$F(x) = \frac{dx}{dt} = x(\pi_{11} - \pi_1) = x(1-x)(\pi_{11} - \pi_{12}) = x(x-1)(C_1 + G - Q_1 + R_1 - R_2 - \beta F - zF - Gy + Q_1 t + Q_2 t + Sy + F\beta z - Q_2\alpha\beta t - Q_2\alpha t z + Q_2\alpha\beta t z) \quad (4)$$

Suppose $z_0 = \frac{-\alpha\beta t Q_2 + y(S-G) + R_1 - R_2 - \beta F - Q1 + C_1 + G + t(Q_1+Q_2)}{(1-\beta)(\alpha t Q_2 + F)}$, when $z = z_0$, $0 < \frac{-\alpha\beta t Q_2 + y(S-G) + R_1 - R_2 - \beta F - Q_1 + C_1 + G + t(Q_1+Q_2)}{(1-\beta)(\alpha t Q_2 + F)} < 1$, replicator dynamics equation $F(x) \equiv 0$, no matter what the value of x is at this time, the strategy selection of enterprise 1 is in a stable state.

When $0 < z < \frac{-\alpha\beta t Q_2 + y(S-G) + R_1 - R_2 - \beta F - Q_1 + C_1 + G + t(Q_1+Q_2)}{(1-\beta)(\alpha t Q_2 + F)} < 1, \left.\frac{dF(x)}{dx}\right|_{x=1} < 0, \left.\frac{dF(x)}{dx}\right|_{x=1} > 0, x = 0$ is the evolutionary stable point at this time, so enterprise 1 chooses not to control pollution.

When $0 < \frac{-\alpha\beta t Q_2 + y(S-G) + R_1 - R_2 - \beta F - Q_1 + C_1 + G + t(Q_1+Q_2)}{(1-\beta)(\alpha t Q_2 + F)} < z < 1, \left.\frac{dF(x)}{dx}\right|_{x=1} > 0, \left.\frac{dF(x)}{dx}\right|_{x=1} < 0, x = 1$ is the evolutionary stable point at this time, so enterprise 1 chooses to control pollution. According to the evolutionary stability strategy of enterprise 1, the following conclusions can be drawn:

Conclusion 1: The smaller the net income difference $(R_1 - R_2)$ between pollution control and non-pollution control of enterprise 1, the lower the cost of pollution control C_1, and the smaller the pollution discharge Q_2 after pollution control, the greater the probability x of the enterprise's pollution control.

Conclusion 2: The higher the degree β of the local government's implementation of environmental policies, the higher the fine F for violating enterprises, the higher the environmental protection tax rate t and the higher the tax deduction ratio α, the greater the probability x of enterprise 1 pollution control.

Conclusion 3: When enterprise 1 chooses not to control pollution, enterprise 2 chooses to control pollution, and enterprise 1 obtains additional income S, at this time, if enterprise 2 also chooses not to control pollution, enterprise 1 obtains additional revenue G, when the difference between $(S - G)$ is smaller, the probability x of enterprise 1 pollution control is larger.

3.1.2. Enterprise 2 Evolutionary Stability Strategy

Enterprise 2’s emission reduction and pollution control benefits:

$$\pi_{21} = z[P_2 - (1-\alpha)tH_2 - C_2] + (1-z)[P_2 - (1-\beta\alpha)tH_2 - C_2] \quad (5)$$

Enterprise 2's non-pollution benefits:

$$\pi_{22} = xz(P_1 - tH_1 - F + S) + (1-x)z(P_1 - tH_1 - F + G) + x(1-z)(P_1 - tH_1 - \beta F + S) + (1-x)(1-z)(P_1 - tH_1 - \beta F + G) \quad (6)$$

The expected return of enterprise 2 is:

$$\pi_2 = y\pi_{21} + (1-y)\pi_{22} \tag{7}$$

To sum up, the replication dynamic equation of enterprise 2 when making strategy selection is:

$$F(y) = \frac{dy}{dt} = y(\pi_{21} - \pi_2) = y(1-y)(\pi_{21} - \pi_{22}) = y(y-1)(C_2 + G + P_1 - P_2 - F\beta - H_1 t + H_2 t - xG - zF + xS + F\beta z - H_2\alpha\beta t - H_2\alpha tz + H_2\alpha\beta tz) \tag{8}$$

Suppose:

$$z_1 = \frac{-\alpha\beta tH_2 + x(S-G) + P_1 - P_2 - \beta F + C_2 + G - t(H_1 - H_2)}{(1-\beta)(\alpha tH_2 + F)}$$

when $z = z_1$,$0 < \frac{-\alpha\beta tH_2 + x(S-G) + P_1 - P_2 - \beta F + C_2 + G - t(H_1 - H_2)}{(1-\beta)(\alpha tH_2 + F)} < 1$, replicator dynamics equation $F(y) \equiv 0$, no matter what the value of y is at this time, the strategy selection of enterprise 2 is in a stable state.

when $0 < z < \frac{-\alpha\beta tH_2 + x(S-G) + P_1 - P_2 - \beta F + C_2 + G - t(H_1 - H_2)}{(1-\beta)(\alpha tH_2 + F)} < 1$, $\left.\frac{dF(y)}{dy}\right|_{y=0} < 0$, $\left.\frac{dF(y)}{dy}\right|_{y=1} > 0$, $y = 0$ is the evolutionary stable point at this time, so enterprise 2 chooses not to control pollution.

when $0 < \frac{-\alpha\beta tH_2 + x(S-G) + P_1 - P_2 - \beta F + C_2 + G - t(H_1 - H_2)}{(1-\beta)(\alpha tH_2 + F)} < z < 1$, $\left.\frac{dF(y)}{dy}\right|_{y=0} > 0$, $\left.\frac{dF(y)}{dy}\right|_{y=1} < 0$, $y = 1$ is the evolutionary stable point at this time, so enterprise 2 chooses to control pollution.

According to the evolutionary stability strategy of Enterprise 2, the following conclusions can be drawn:

Conclusion 4: The smaller the net income difference $(P_1 - P_2)$ between pollution control and no pollution control of enterprise 2, the lower the cost C2 of pollution control, and the greater the probability y of the company to conduct pollution control.

Conclusion 5: The higher the degree β of the local government's implementation of environmental policies, the higher the fine F for non-compliance enterprises, the higher the environmental protection tax rate t and the higher the tax deduction ratio α, the greater the probability y of enterprise 2 pollution control.

Conclusion 6: When company 2 chooses not to control pollution, company 1 chooses to control pollution, and company 2 obtains additional income S. At this time, if company 1 also chooses not to control pollution, company 2 obtains additional revenue G. The difference of (S-G) is smaller, the probability $y$of enterprise 2 pollution control is larger.

3.1.3. Evolutionary Stability Strategies of Local Governments

The benefits of local governments strictly implementing environmental protection policies are:

$$\pi_{31} = xy[v_1(R_2 + P_2) - v_2(Q_2 + H_2) - M] + x(1-y)[v_1(R_2 + P_1) - v_2(Q_2 + H_1) - M + F] + (1-x)y[v_1(R_1 + P_2) - v_2(Q_1 + H_2) - M + F] + (1-x)(1-y)[v_1(R_1 + P_1) - v_2(Q_1 + H_1) - M + 2F] \tag{9}$$

The benefits of the loose implementation of environmental protection policies by local governments are:

$\pi_{32} = xy[v_1(R_2 + P_2) - v_2(Q_2 + H_2) - \beta M] + x(1-y)[v_1(R_2 + P_1) - v_2(Q_2 + H_1) - \beta M + \beta F] + (1-x)y[v_1(R_1 + P_2) - v_2(Q_1 + H_2) - \beta M + \beta F] + (1-x)(1-y)[v_1(R_1 + P_1) - v_2(Q_1 + H_1) - \beta M + 2\beta F]$ (10)

The expected return of the local government is:

$$\pi_3 = z\pi_{31} + (1-z)\pi_{32} \tag{11}$$

To sum up, the replication dynamic equation when the local government makes policy selection is:

$$F(z) = \frac{dz}{dt} = z(\pi_{31} - \pi_3) = z(1-z)(\pi_{31} - \pi_{32}) = -z(\beta - 1)(z-1)(M - 2F + xF + yF) \tag{12}$$

Suppose: $x_0 = \frac{(2-y)F-M}{F}$, when $x = x_0$, $0 < \frac{(2-y)F-M}{F} < 1$, replicator dynamics equation $F(z) \equiv 0$, no matter what the value of z is at this time, the strategy choice of the local government is in a stable state.

When $0 < x < \frac{(2-y)F-M}{F} < 1$, $\left.\frac{dF(z)}{dz}\right|_{z=0} < 0$, $\left.\frac{dF(z)}{dz}\right|_{z=1} > 0$, $z = 0$ is the evolutionary stable point at this time, so the local government chooses to implement loose environmental protection policies.

When $0 < \frac{(2-y)F-M}{F} < x < 1$, $\left.\frac{dF(z)}{dz}\right|_{z=0} > 0$, $\left.\frac{dF(z)}{dz}\right|_{z=1} < 0$, $z = 1$ is the evolutionary stable point at this time, so the local government chooses to implement strict environmental protection policies.

According to the evolutionary stabilization strategy of local government, the following conclusions can be drawn:

Conclusion 7: The higher the fine F imposed by the local government on violating enterprises and the lower the inspection cost M, the higher the probability z of implementing strict environmental protection policy.

3.2. Stability Analysis of Evolutionary Equilibrium

In an asymmetric game, the evolutionary game equilibrium E is an evolutionary stable equilibrium, and it must be a strict Nash equilibrium, and the strict Nash equilibrium is a pure strategy equilibrium, that is, the mixed strategy equilibrium in an asymmetric game must not be an evolutionary stable equilibrium. Therefore, according to Ritzberger and Weibull, this paper only discusses pure strategy equilibria E1(0,0,0), E2(0,0,1), E3(0,1,0), E4(1,0,0), E5(1,1,0), E6(1,0,1), E7(0,1,1), E8(1,1,1).

According to Friedman's research, the evolutionary stable strategy (ESS) of the game can be judged by the eigenvalue of the Jacobian matrix. According to the Lyapunov method, if the eigenvalues are all greater than zero, the equilibrium point is unstable and is the source point; the eigenvalues are positive or negative, and the equilibrium point is the saddle point; if the eigenvalues are all less than zero, the equilibrium point is the sink point, which is also evolutionary. Be stable. The Jacobian matrix of the three-party evolutionary game in this paper is:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix} = \begin{pmatrix} \partial F(x)/\partial x & \partial F(x)/\partial y & \partial F(x)/\partial z \\ \partial F(y)/\partial x & \partial F(y)/\partial y & \partial F(y)/\partial z \\ \partial F(z)/\partial x & \partial F(z)/\partial y & \partial F(z)/\partial z \end{pmatrix} \tag{13}$$

$$J_{11} = (2x-1)(C_1 + G - Q_1 + R_1 - R_2 - F\beta - Fz - Gy + Q_1 t + Q_2 t + Sy + F\beta z - Q_2\alpha\beta t - Q_2\alpha tz + Q_2\alpha\beta tz)$$

$$J_{12} = -x(G-S)(x-1)$$

$$J_{13} = -x(x-1)(F - F\beta + Q_2\alpha t - Q_2\alpha\beta t)$$

$$J_{21} = -y(G-S)(y-1)$$

$$J_{22} = (2y-1)(C_2 + G + P_1 - P_2 - F\beta - H_1 t + H_2 t - xG - zF + xS + F\beta z - H_2\alpha\beta t - H_2\alpha tz + H_2\alpha\beta tz)$$

$$J_{23} = -y(y-1)(F - F\beta + H_2\alpha t - H_2\alpha\beta t)$$

$$J_{31} = -zF(\beta-1)(z-1)$$

$$J_{32} = -zF(\beta-1)(z-1)$$

$$J_{33} = -z(\beta-1)(M - 2F + xF + yF) - (\beta-1)(z-1)(M - 2F + xF + yF) \quad (14)$$

According to the Jacobian matrix, the eigenvalues of the partial equilibrium of the tripartite evolutionary game are shown in Table 2;

Suppose: $a = C_1 - Q_1 + R_1 - R_2 + Q_1 t + Q_2 t$, $b = G$, $c = -F\beta - Q_2\alpha\beta t < 0$, $d = -F - Q_2\alpha t < 0$, $e = C_2 + P_1 - P_2 - H_1 t + H_2 t$, $f = -F - H_2\alpha t < 0$, $g = -F\beta - H_2\alpha\beta t < 0$, $h = \beta - 1 < 0$.

Table 2: Eigenvalues and evolutionary stability conditions of partial equilibrium in tripartite evolutionary games.

Equilibrium	Eigenvalues			Sink Stability Condition
	λ_1	λ_2	λ_3	
E1(0,0,0)	$a+b+c$	$e+b-F-H_2\alpha\beta t$	$h(M-2F)$	$-(a+b+c)<0, e+b-F-H_2\alpha\beta t<0, h(M-2F)<0$
E2(0,0,1)	$a+b+d$	$e+b+f$	$h(M-2F)$	$-(a+b+d)<0, -(e+b+f)<0, -h(M-2F)<0$
E3(0,1,0)	$a+d+S$	$e+b+g$	$h(M-F)$	$-(a+d+S)<0, e+b+g<0, h(M-F)<0$
E4(1,0,0)	$a+b+c$	$e+g+S$	$h(M-F)$	$a+b+c<0, -(e+g+S)<0, h(M-F)<0$
E5(1,1,0)	$a+c+S$	$e+g+S$	hM	$a+c+S<0, e+g+S<0, hM<0$
E6(1,0,1)	$a+b+d$	$e+f+S$	$h(M-F)$	$a+b+d<0, -(e+f+S)<0, -h(M-F)<0$
E7(0,1,1)	$a+d+S$	$e+b+f$	$h(M-F)$	$-(a+d+S)<0, e+b+f<0, -h(M-F)<0$
E8(1,1,1)	$a+d+S$	$e+f+S$	$-hM$	unstable

By analyzing the asymptotic stability conditions of the equilibrium point, it is obtained that the existence of the eigenvalue of the equilibrium point E8 is positive, so it cannot be an evolutionarily stable strategy.

Case 1: When $-(a+b+c)<0, e+b-F-H_2\alpha\beta t<0$ and $h(M-2F)<0$,the equilibrium point E1 (0, 0, 0) is the evolutionary stable point of the system. At this time, the income of enterprise 1 during pollution control is lower than the cost and cannot obtain additional sales income S, so it chooses not to control pollution; the income of enterprise 2 during pollution control is also lower than the cost and cannot obtain additional sales income, so Choose not to control pollution; the benefits (including economic benefits, environmental benefits and fines) of local governments implementing loose environmental protection policies are lower than the costs, so they choose to implement strict environmental protection policies.

Case 2: When $-(a+b+\mathrm{d})<0, -(e+b+f)<0$ and $-h(M-2F)<0$,the equilibrium point E2(0,0,1) is the evolutionary stable point of the system. At this time, the income of enterprise 1 in pollution control is lower than the cost and can obtain additional sales income G, so it chooses not to control pollution; the income of enterprise 2 in pollution control is also lower than the cost and can obtain additional sales income G, Therefore, they choose not to control pollution; the benefits (including economic benefits, environmental benefits and fines) of local governments implementing loose environmental protection policies are greater than the costs, so they choose to implement loose environmental protection policies.

Case 3: When $-(a+\mathrm{d}+S)<0, e+b+\mathrm{g}<0$ and $h(M-F)<0$,the equilibrium point E3(0,1,0) is the evolutionary stable point of the system. At this time, the income of enterprise 1 is lower than the cost of pollution control and can obtain additional sales income S, so it chooses not to control pollution; the income of enterprise 2 when it chooses not to control pollution is lower than the cost, so it chooses pollution control; the local government implements The benefits of loose environmental protection policies (including economic benefits, environmental benefits and fines) are lower than the costs, so they choose to implement strict environmental protection policies.

Case 4: When $a+b+\mathrm{c}<0, -(e+\mathrm{g}+S)<0$ and $h(M-F)<0$,the equilibrium point E4(1,0,0) is the evolutionary stable point of the system. At this time, when enterprise 1 chooses not to control pollution, the income is lower than the cost, so it chooses pollution control; when enterprise 2 chooses to control pollution, the income is lower than the cost and can obtain additional sales income S, so it chooses not to control pollution; the local government implements The benefits of loose environmental protection policies (including economic benefits, environmental benefits and fines) are lower than the costs, so they choose to implement strict environmental protection policies.

Case 5: When $a+\mathrm{c}+S<0, e+\mathrm{g}+S<0$ and $h\,M<0$, the equilibrium point E5(1,1,0) is the evolutionary stable point of the system. At this time, the income of enterprise 1 is lower than the cost when it does not control pollution, so it chooses pollution control; the income of enterprise 2 when it does not control pollution is also lower than the cost, so it chooses pollution control; the local government implements the benefits of loose environmental protection policies (including Economic benefits, environmental benefits, and fines) are lower than costs, so they choose to implement strict environmental protection policies.

Case 6: When $a+b+\mathrm{d}<0, -(e+f+S)<0$ and $-h(M-F)<0$,the equilibrium point E6(1,0,1) is the evolutionary stable point of the system. At this time, when enterprise 1 chooses not to control pollution, the income is lower than the cost, so it chooses pollution control; when enterprise 2 chooses to control pollution, the income is lower than the cost and can obtain additional sales income S, so it chooses not to control pollution; the local government implements The benefits of strict environmental protection policies (including economic benefits, environmental benefits and fines) are lower than the costs, so they choose to implement loose environmental protection policies.

Case 7: When $-(a+\mathrm{d}+S)<0, e+b+f<0$ and $-h(M-F)<0$, the equilibrium point E7(0,1,1) is the evolutionary stable point of the system. At this time, when enterprise 1 chooses to control pollution, the income is lower than the cost and can obtain additional sales income S, so it chooses not to control pollution; when enterprise 2 chooses not to control pollution, the income is

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lower than the cost, so it chooses pollution control; the local government implements The benefits of strict environmental protection policies (including economic benefits, environmental benefits and fines) are lower than the costs, so they choose to implement loose environmental protection policies.

4. Conclusion

The main reason behind the serious environmental pollution in some places today lies in that the pollutant-discharging enterprises see more benefits in not controlling pollution than those from controlling pollution. Therefore, it is necessary that effective measures be taken to reduce the benefits for enterprises not treating pollution so that they will be unprofitable when they left the pollutants unhandled. Under the circumstance that local governments can strictly implement environmental protection policies, perform their duty and identify violators in a timely manner, enterprises violating the regulations will face heavy fines and severe punishments; the punishment for enterprises that do not control pollution will be greater than the costs of pollution control. No matter whether one enterprise treats its pollution, the benefits of pollution control for the other enterprise will always be greater than the benefits of not controlling pollution. The higher the benefits of pollution control are, the more inclined the enterprises will be towards pollution control and environmental protection. Therefore, to reduce environmental pollution. At first, the government should provide subsidies to encourage enterprises' research on green production. Secondly, the costs for "discharging and no treatment" enterprises should be increased while their benefits are reduced. Besides, the costs for local governments' monitoring of pollution-producing enterprises should be reduced.

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