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A Consumer Behavior Theory of Tutoring

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Abstract: The paper aims to explore the behavior of consumers in the tutoring market in China, in which consumers' perspectives would be divided into two parts: parents and students. Parents and students would consider and focus on different factors to maximize the utility based on their own standpoint, which is the reason why they usually make different decisions in reality. The paper would build a theory of consumer behavior in tutoring market based on the neoclassical consumer theory. There are several essential features that should be noticed in the paper: (1) for simplifying, the commodities consumed in the market are the average scores that are estimated to increase based on the original average scores; (2) the consumer's decisions are subject to two resource constraints of money and time; (3) the utility is not only a function of goods but also of the time that is required to "finish consuming" it, in other words, the time a student needs to spend for earning one more score.

Keywords: Lagrange function, value of time, intermediate goods, leisure goods

1. Introduction

In China, National College Entrance Examination is required for every high school student who are going to apply the university, except for whom have been recommended and get offer from the favored university before the examination. Students from everywhere in China would take the significant examination at the same time to compete. The grades would play the most important role in the decisions of giving offers by universities. Students who acquire more scores in the examination would be considered prior by the top universities. National College Entrance Examination is really cold and ruthless. Students from the whole country would be ranked only based on the final grade of the examination, regardless of how they perform outside of the examination, and only the top rank would get the better educational resource. Due to the huge population of China, it's not rare to see one score gap would imply a disparity with thousands of students. Sometimes earning one more score gives the opportunity to choose the top universities; in comparison, losing one more score can also increase the risking being rejected by the favored university. Acquiring as many scores as possible is the goal for every student and their parents to prepare for National College Entrance Examination. Most parents cannot help a lot with students' studying by themselves, but they are willing to pay for the extracurricular cram school to force students to acquire and review more knowledge, therefore the tutoring market is huge, and the tutoring industry is well developed in China. National College Entrance Examination is composed of several subjects, including literature, mathematics, foreign language and so on, the score of different subjects would be summed up to rank students, therefore parents and students need to make decisions of which subject tutoring; however, the decisions can

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vary between students' position and parents' position, which is why a lot of conflicts can happen in families before the important examination. This paper is inspired by the common phenomenon in China to explore a theory of consumer behavior from two different perspectives in tutoring market.

The paper would build a theory of consumer behavior in tutoring market based on the neoclassical consumer theory. There are several essential features that should be noticed in the paper: (1) for simplifying, the commodities consumed in the market are the average scores that are estimated to increase based on the original average scores; (2) the consumer's decisions are subject to two resource constraints of money and time; (3) the utility is not only a function of goods but also of the time that is required to "finish consuming" it, in other words, the time a student needs to spend for earning one more score. Starting from the several specifications, some new properties would be derived from the preserved implications of the neoclassical consumer theory.

2. Literature Review

Traditionally, scholars try to predict the pattern of consumers' behavior by assuming people aim to maximize their utilities, which implies people try to acquire as many products they prefer as possible, subject to only their budgets. However, people need to spend time in consuming, especially when they consume to enjoy some services. Besides, sometimes people don't enjoy the process of "consuming a good" but they can get a higher utility only when they get the good itself. For example, people buy a bus ticket doesn't mean they enjoy the experience of taking a bus, reversely, they may hate the time on a bus; nevertheless, people must take a bus as long as they want to arrive at the destination. In comparison, the time factor is especially important in the research of exploring the allocation of resource in transportation [1-4]. Bone used travel time runs to discover the relationship between miles per hour and miles per gallon, therefore he could estimate the gas consumption on congested streets from the traffic speed without a gasoline meter [5]. With a growing proportion of commuters who travel by car, Quarmby attempted to apply comparative statistics of time and cost to achieve a balanced degree between public and private transport, leading to a relief of rush-hour congestion [6]. Inspired by transportation economists, more scholars are considering the value of time in consumption. For instances, Lee et al. tried to present an outline of the level and variation in travel time values to estimate separately time gainers, who prefer a faster and more expensive traveling method, and time losers, who travel in a cheaper and slower way [7]. Based on the ideas from transportation and new consumer theories triggered by them, this paper is attempting to incorporate the time value in studying to explore how parents and students react with different valuation to different subjects, and further build model to predict the pattern of a tutor market.

3. Model

In neoclassical consumer theory, a consumer is just an individual that play the roles of paying and enjoying the good at the same time. In the tutoring model, a consumer is a combination of parents and a student—parents would pay for the goods or service and students enjoy them. After all, a student has no income source, but it only makes sense when he or she take the cram courses, therefore later models with two different perspectives would be explored—one for parents and another one for students. In addition, it's essential to consider the case that a student has limited available time to study, and he or she can only take one course for studying only one subject at a time, which is why the time dimension is introduced.

3.1 Parents' Perspective

Let's start with the parents' perspective, which is relatively easier. Different from the specific goods that can be touched, it's hard to estimate the result a student can have after taking a cram course. In

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reality, parents would choose the tutoring courses based on the advertisement and reputation of tutoring center. It's common to see, at least in China, a student takes a practice test at the beginning, and then the tutoring teachers would estimate the potential improvement the student can have with the score for how long the tutoring is so that parents can make the final decision. For simplicity, the commodity in our model is the increased score a student can have in exams of the subject, and the corresponding price is the money need to be paid for earning one more score in the subject.

A commodity bundle for parents is set as

$$X = (X1, X2 \ldots Xn) \ldots \tag{1}$$

A fixed budget constraint (Y) is considered, which should equal to the total expenditure of the consumption, as

$$Y = \sum_{i=1}^{n} PiXi \ldots \tag{2}$$

where the variable X_i denotes some increased scores of ith subject, and $P_i \geq 0$ is the money paid to increase one more score on ith subject.

Similarly, a fixed amount of available time (T_a) is set for consumers as the time resource constraint

$$T_a = \sum_{i=1}^{n} Ti \ldots \tag{3}$$

where T_i denotes the amount of time allocated to the ith subjects. In reality, T_i can be interpreted as the expected studying time that are required to improve the grades; in other words, the tutoring teachers tell the parents the estimated required studying time for students and then parents make decisions based on these expectations. The sum of the allocated time to different subject should equal to the total time available.

Right now, a maximization of the Lagrange function with effectively allocating the time and money budget for parents is expressed as:

$$L = U(X_1 \ldots X_n) + \lambda(Y - \sum_{i=1}^{n} PiXi) + \mu(T_a - \sum_{i=1}^{n} Ti) \ldots \tag{4}$$

Furthermore, necessary conditions for maximization can be acquired:

$$\frac{\partial L}{\partial Xi} = \frac{\partial U}{\partial Xi} - \lambda Pi = 0 \ldots \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = Y - \sum_{i=1}^{n} PiXi = 0 \ldots \tag{6}$$

$$\frac{\partial L}{\partial \mu} = T\alpha - \sum_{i=1}^{n} Ti = 0 \ldots \tag{7}$$

λ is the shadow value of the marginal utility for money, and μ is the shadow value of the marginal utility for time.

As Johnson stated, the ratio of μ/λ is the marginal rate of the substitution between time and money, which may also be interpreted as the value of time [8].

Deriving a very simple but generally realistic utility function of parents is easy. In neoclassical consumer theory, consumers have a preference that implies diminishing marginal utility; however, parents won't be satisfied until students can get full scores, or at least, get the minimum passing score

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of the favored university. In addition, parents only care about the sum of the scores of all tutoring subjects; parents are indifferent to a single score of a specific subject, and their target is to maximize the total scores. Based on the properties above, set a linear function for parents' utility as

$$U(X_1...X_n) = X_1 + X_2 + ... + X_n \ldots \quad (8)$$

It's more clearly to see it from Fig.1. Assume the parents have only two subjects that need to be tutored—mathematics as X_1 and physics as X_2. It would be realized from the example that both of the money constraint and time constraint are binding. Parents would make the optimal choice to maximize their utility for scores only under the money and time constraints. where e_1 and e_2 refer the expected time for increasing one more unit of score for X_1 and X_2.

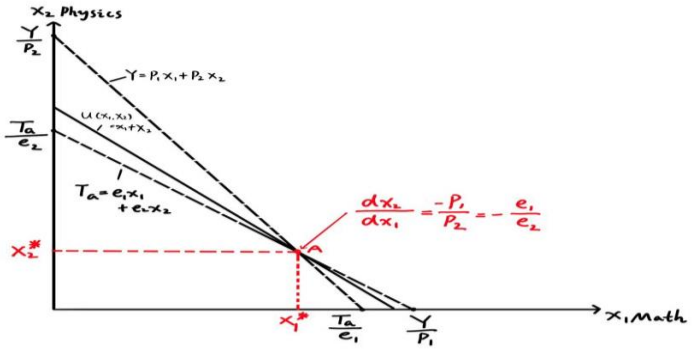

Figure 1: Parents' utility.

3.2 Students' Perspective

Next step is to see how students make decisions for tutoring. Noticeably, even though scores are regarded as the commodities bundle in our model, scores are not the direct commodities, but the tutoring service is. Parents can pay for the cram courses and get the result instantly and concurrently, which is estimated and promised by the tutoring teachers with their reputation, but students have to spend time taking the cram course. Different from parents, the time spent on the cram courses should be included in students' utility function besides the increased scores to get a utility function for students as

$$U = U(X_1...X_n, T_1...T_n) \ldots \quad (9)$$

Different from their parents, students also want increasing scores but they may dislike a long time studying. The utility of students can decrease with a longer time of studying, which is also related to increasing scores, thus the properties of diminishing marginal utility appear here.

As mentioned before, the tutoring teachers would estimate the improved scores under some tutoring time; obviously, the estimation can have differences from the reality. Generally, it's reasonable to believe the teachers would give parents the expected studying time that is longer than the real required one because they don't want to break the reputation, thus there is another relationship in our model like

$$T_i \geq a_i X_i,\ i = 1, 2...n \ldots \quad (10)$$

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where a_i implies the technologically required minimum amount time to improve one more score, in other words, the difficulty of studying the subject for a student. Higher a_i implies the student need to spend more time to get one more unit of X_i. The vector of a_i's is certainty for simplicity.

DeSerpa has pointed out that referring the above n inequalities as the time consumption constraint, which is distinguished from the time resource constraint [9].

By considering the time required to consume in people's utility, as combining that of Oort and that of Atici with reasonable adjustments, a maximization of the Lagrange function for students is expressed as:

$$L = U\ (X1...Xn, T1...Tn) + \lambda(Y - \sum_{i=1}^{n} PiXi) + \mu(T\alpha + \sum_{i=1}^{n} Ti) + \sum_{i=1}^{n} Ki(Ti - \alpha iXi) \ldots (11)$$

where the income constraint is not included. Students don't have income sources in reality, although their parents definitely would sponsor them with all strengths, thus they only need to consider how to get as many scores as possible in a limited time. In contrast, leisure time plays a bigger time in student's utility, as the non-working activity in the allocation of time [10-12].

And then the first-order condition is as follows:

$$\frac{\partial L}{\partial Xi} = \frac{\partial U}{\partial Xi} - \lambda Pi - \alpha i Ki = 0 \ldots \quad (12)$$

$$\frac{\partial L}{\partial Ti} = \frac{\partial U}{\partial Ti} - \mu + Ki = 0 \quad \ldots \quad (13)$$

$$\frac{\partial L}{\partial \mu} = T\alpha - \sum_{i=1}^{n} Ti = 0 \ldots \quad (14)$$

$$Ki(Ti - \alpha iXi) = 0 \ \ldots \quad (15)$$

where either $T_i = a_i X_1$ or $K_i = 0$

Fortunately, the concept of ratio μ/λ also works here. Rearrange the equation (13) and dividing it by λ can get

$$\frac{UTi}{\lambda} = \frac{\mu}{\lambda} - \frac{Ki}{\lambda} \ldots \quad (16)$$

where $U_{T\boldsymbol{i}}$ is the partial derivative of the utility function with respect to T_i.

The term $\frac{UTi}{\lambda}$ is the marginal rate of substitution of T_i. for money, implying how students value the time allocated to increase the score of its subject. Here the time is regarded as a commodity instead of a resource. It's crucial to distinguish the two conditions by figuring out whether students are willing to spend more time than the required amount on studying ith subject, in other words, whether students are interested and enjoying studying the subject.

3.3 Value of Time

The model from students' perspective can help to explore much more properties about time consumption and the economic intuitions behind.

It's reasonable to believe the allocated time for studying some subjects is much more than the required amount because the teachers and parents want to make sure students maintain a stable increasing level. As for students, once they spend the minimum required time on studying and attain the minimum required grade level, they may regard the left time from T_i as the opportunity cost

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because they can do something else that can increase their utility more. For example, a student is asked to take a 45 minutes class and 1 hour self-studying for reviewing the content in the class in a cram school; however, the student has fully understood the content after 20 minutes self-studying, and then he may prefer to play computer games instead of continuing to stay in the cram school, but he has to stay in the cram school, and the left 40 minutes is his opportunity time. This example shows the why time spending can be regarded as commodity consumption in the model of student.

The distinction between time consumption constraint and time resource constraint interprets K_i as the marginal utility of saving time and the ratio $\frac{Ki}{\lambda}$ as the value of saving time [13,14]. The value of saving time can be gotten easily by rearranging the equation (16)

$$\frac{Ki}{\lambda} = \frac{\mu}{\lambda} - \frac{UTi}{\lambda} \ \ldots \tag{17}$$

The equation (17) determines the value of saving time by figuring out the algebraic difference between the value of time that can be spent on doing something alternative and the value of time doing some specific thing ($\frac{UTi}{\lambda}$). Since the value of saving time can also be interpreted as the opportunity cost, therefore it is always positive.

3.4 Leisure Goods

In neoclassical theory, consumers can purchase commodities instantly and they only enjoy the commodities themselves. However, in reality, people always take time to consume, and they may not enjoy the time for consuming, which is the concept of travel time introduced by M. Bruce Johnson [8]. The goods that people need to spend travel time on consuming are called intermediate goods. Generally, students may not enjoy studying and the incentive to studying is to improve their scores. In this case, the time allocated on studying ith subject (T_i) is the travel time for students, and the commodities--improved scores for ith subject--are intermediate goods for them.

Leisure is defined as the free time used for rest by economists. People escape from restriction and responsibility in leisure time. People have responsibilities when they are working, which is why working time is incompatible with leisure time. The same logic can also apply in studying. Students have the duty of getting a good grade therefore studying time is not leisure time for them. However, there are also some students enjoy studying some subjects themselves, besides improving grades. Students decides to study for ith subject in the expected allocated time T_i before they truly realize the difficulty of studying ith subject for them. Once they realize they have attained the basic goal and they have left time, but they decide to continue to study, which means they are interested in studying the subject and their interests are in accordance with their responsibility. In this case, the allocated time (T_i) is the leisure time and the increased score (X_i) is the leisure good for students. With respect to the model of students, leisure goods mean the time consumption constraint is ineffective, and students have no opportunity cost for saving time [15], thus

$$\frac{Ki}{\lambda} = 0 = \frac{\mu}{\lambda} - \frac{UTi}{\lambda} \ldots \tag{18}$$

$$Ki = 0 \ldots \tag{19}$$

$$\mu = UTi \ldots \tag{20}$$

Once $K_i = 0$, the condition of equation (15) would be satisfied.

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Noticeably, the time consumption constraint is not considered in parents model is inevitable. Parents make estimation, with the help of tutoring teachers, for how long the student should study for some subjects, it's reasonable to hold $T_i = a_i X_i$ in parents model. Even though there are some left time, parents still expect students to study consistently. As for parents, there are no opportunity cost for saving time, and the time has value only as the studying time, thus s $\frac{Ki}{\lambda} = 0$and $\frac{Ki}{\lambda} = 0$ and it's not required to consider the time consumption constraint.

3.5 Substitution Between Two Commodities in Students' Model

Two different goods for students are disclosed---intermediate goods and leisure goods---by referring the time dimension. Now, suppose students can make decisions of tutoring by themselves, and they know exactly what the difficult level of a subject is for them to improve the grade. In this case, let's try to explore how they decide the optimal choice in two-commodity condition and how it differs with that of parents.

The two-commodity case is shown diagrammatically below with Fig.2, which is extracted from reasonable adjustment of that of DeSerpa [9].

Suppose X_1 is the leisure good in which student enjoy studying the subject 1 so that the time consumption constraint does not work, and X_2 is the intermediate good in which students only want to attain the basic target for subject 2 so that the time consumption constraint is binding between X_2 and T_2. Here all of the substitution of two commodities, the substitution between the allocated time for each commodity, and the substitution between goods and the allocated time for purchasing---or studying---it are considered.

At first, let's rearrange equation (12) (13) to get some useful specifications

$$UXi = \lambda Pi + aiKi \ldots \tag{21}$$

$$UTi = \mu - Ki \ldots \tag{22}$$

where U_{Xi} and U_{Ti} are the partial derivatives of the utility function with respect to X_i and T_i.

Since the time consumption constraint is binding between X_2 and T_2, it's required to find their equilibrium condition at first. The budget for X_2 and T_2 is the marginal utility they can maximize, therefore

$$C_1 = X_2(\lambda P_2 + a_2 K_2) + T_2(\mu - K_2) \ldots \tag{23}$$

where C_1 is arbitrary constant.

The slope between X_2 and T_2 is$\frac{-(\mu - K2)}{\lambda P2 + \alpha 2K2}$,and the optimal choice appears once the utility level is tangent to the budget line, as the equilibrium point, where $\frac{\partial U/\partial T2}{\partial U/\partial X2} = \frac{(\mu - K2)}{\lambda P2 + \alpha 2K2}$ and T_2^* , as showing in Fig.2 b.

It should be noticed that the equilibrium point is also at the line of $T_2 = a_2 X_2$, because students only want to reach the basic goal for subject 2.

Next step turns to Fig.2 d to see how students decide the substitution between T_1 and T_2. $T_a \geq T_1 + T_2$, but for getting the optimal choice, it's required to consider the marginal utility they can maximize as

$$T1 = \frac{-(\mu - K2)}{\mu} T2 + C2 \ldots \tag{24}$$

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where C_2 is arbitrary constant.

T_2' is a fixed proportional time with T_2^*. It's not hard to calculate $T_a - T_2'$ to find the left time available of T_1^* for studying subject 1. However, T_1^* is at the point where the utility level is not tangent. It can be explained that students are only willing to spend the minimum required time to studying subject 2, thus the allocated time T_2 would be proportional with X_2 by a_2. In comparison, students are willing to spend more time on studying subject 1 besides the minimum required time, resulting in the allocated time T_1 can be chosen more freely in the model [3,4].

Looking at Fig.2 c, X_1 is the leisure good for students so that $K_1 = 0$. The budget for X_1 and T_1 is

$$C_3 = X_1\lambda P_1 + T_1\mu\ldots \tag{25}$$

where C_3 is arbitrary constant, and the slope between T_1 and X_1 is $\frac{-\lambda P1}{\mu}$

Finding the equilibrium point is as similar as the process to find out X_2 and T_2, in which the substitution of marginal utility with respect to X_1 and T_1 equals to their slope. Something different from the substitution between T_2 and X_2 is the equilibrium point can stay away with the line of $T_1 = a_1X_1$,, which is also due to students are willing to engage in studying subject 1 for a longer time than the minimum required amount of time.

After finding out X_1^* and X_2^* in Fig.2 c and Fig.2 b, as showing in Fig.2 a, the equilibrium point is at the money budget line but the time constraint is not binding. It can be interpreted that students are willing to pay money to improve their grades, but they may not be happy to spend time on studying. As for a favorite subject, students can enjoy studying it and the time spent on it can also increase their utility; as for a subject they are not interested, the only incentive to studying it is pursuing a good grade, thus less time spent on it would make them happier, if they get the minimum required grade.

Take a look back to Fig.1. If parents have a precise estimation, which means $e_1 = a_1$ and $e_2 = a_2$, and then the Fig.1 would be under a similar situation of Fig.2 a. Fig.1 shows parents can sponsor student on studying with all their economic strength and they also expect students spend all of the available time on improving the scores. In contrast, Fig.2 a implies students are not keen to pursue grade as their parents, after all, studying is not easy and even making people painful. The bigger gap between parents' optimal decision and students' one is more likely to stimulate conflict in families. The best situation making everybody happy is students enjoying studying all subjects so that they don't need to consider time consumption constraint as their parents, but it is absolutely rare in reality.

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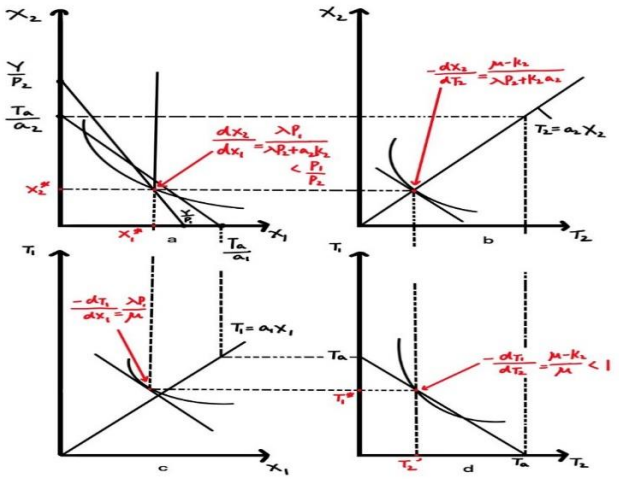

Figure 2: Substitution between commodities and time [2].

4. Drawback and Extension

In reality, it's not effective to evaluate the studying abilities of students consistently. Studying is a process of accumulating. It's common to see students sustain a long time of reviewing and numerous practices again and again, and their grades don't move forward at all during the painful period. Only when they get over the period, they can see a great improvement, therefore it's too idea to set the improved scores in a short unit time in the model. To make the model more reasonable, it is necessary to highlight more about the expectation for improving grades at first, and then contrast it with the final result later. In this case, dynamics optimization is a useful tool.

In addition, the difficulty level for ith subject, ai, can vary through long time studying. Even though students may feel difficult to study some subjects at the beginning, they won't be troubled by them after consistent accumulation. If ai is flexible, it can be connected with the time allocated on studying ith subject and create a dynamic optimization model to determine when students would decide to study some subjects with only the incentive of improving grades, which fits better in reality [16,17].

5. Conclusion

The paper has tried to explore the consumer behavior theory for tutoring by deriving some specifications from the neoclassical theory and plugging in the time dimension. Models are created based on parents' perspective and students' perspective. From parents' perspective, the equilibrium condition is same as that in neoclassical theory except there is an additional time constraint. From students' perspective, as the consumers who directly engage in the service of tutoring, the time spent on studying is also a commodity besides the increased scores. Students may not enjoy some subjects therefore they would maximize their utility by eliminating the time spent on the subjects as much as possible once they get the least acceptable grade. If they enjoy studying some subjects, then studying itself would become a kind of leisure, thus they would pursue to studying even without the incentive of improving grade. The situation follows parents' expectations, but which happens rarely in reality.

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