Proceedings of the 2nd International Conference on Business and Policy Studies
DOI: 10.54254/2754-1169/13/20230753

Select the Optimal Portfolio by Analyzing and Comparing the Better Performance of Markowitz Model and Index Model under 5 Different Constraints

Yating Wang[1,a,*]

[1]*Business School, The University of Kansas, Lawrence, Kansas, 66045, USA*
a. wangyating@ku.edu
**corresponding author*

Abstract: In the research of investment management and investment portfolio, how to choose the best investment portfolio and avoid risks is a topic worthy of attention and research for the field development and investors. This study is based on the most recent 20-year historical daily total return data for 10 stocks belonging to 3 different industries to estimate all appropriate optimization inputs for the full Markowitz model as well as the index model and use these optimization inputs for both models' areas that allow the portfolio will be required to deal with the additional constraints of five different situations.
Through a complete analysis and comparison, it turns out that the Markowitz model is more suitable for investors to obtain the optimal portfolio, and it performs better under these constraints. In addition, the results mainly analyze and compare the performance of Markowitz model and Index model in help investors to select the best portfolio by calculating the minimum variance, maximum Sharpe, and standard deviation respectively. In this study, the Markowitz model outperforms the Index model portfolio with lower risk and volatility and higher expected returns. An efficient frontier is produced by Markowitz portfolio analysis, where each portfolio represents the highest return portfolio for a specific degree of risk.

Keywords: markowitz model, index model, outperforms

1. Introduction

The study of risk diversification is becoming more and more important as portfolio theory advances. More and more people are paying attention to and researching the topic of how investors might choose or develop a stronger stock portfolio [1]. At present, which of the Markowitz model and the index model performs better in the optimal portfolio is still a topic of debate in the field. By looking for optimal inputs for various models, this study illustrates a comparison between the outcomes of the Markowitz model (MM) and the Index model (IM). The specific methods used include calculating the average annual return, annualized standard deviation, beta, annualized alpha and residual standard deviation, and correlations for 10 company stocks belonging to the technology, financial services, and industrial sectors. After correctly calculating the rate of return, standard deviation, and Sharpe ratio, add the corresponding 5 constraints to MM and IM respectively to find the allowable portfolio area, including efficient frontier, minimum risk portfolio, optimal portfolio, and minimum variance boundary. The results will be presented in tabular and graphical form in the paper, and the

DOI: 10.54254/2754-1169/13/20230753

goal is to obtain the optimal portfolio by comparing the set of constraints for each optimization issue and the comparison between the MM and IM. In addition, it is hoped that the conclusions of the study are expected to serve as a starting point for future data analysis and construction of portfolios.

2. Literature Review

This paper cites a total of 12 sources as support for the argument, and each cited source will be separately introduced to the main content of these source. This includes research and analysis of the Markowitz model versus the index model for portfolio selection, comparing the purpose of the MM and IM, risk, and the concept of optimizing product portfolios [1]. The definition of the technology sector, companies in the technology field, and introduces the top ten technology firms based on last 12-month (TTM) revenue, including company revenue, net profit, market capitalization and other information [2]. The definition of the financial services industry, the different types of companies it includes, the major brands in the financial services industry, common questions about financial services, and the bottom line [3]. The definition of the industrial sector, companies in the well-known industrial sector, and introduces the most valuable industrial stocks, the fastest growing industrial stocks, the most dynamic industrial stocks, and the advantages and risks of investing in industrial stocks [4]. Implication and calculations of expected returns and standard deviations, special considerations for investors making investment decisions, and frequently asked questions about calculations [5]. The concept of stock correlation, positive and negative stock correlation, how to calculate it, importance of stock correlation to investors and using correlation for a portfolio [6]. The introduction of Markowitz model, showing an efficient portfolio graph, a method for computing the efficient frontier under constraints - using Markowitz's Critical Line Algorithm (CLA). Also includes the practice of models and Markowitz-based trading strategies [7]. The motivation of index models, single factor models, factoring risk, single index regression, model benefits and costs, index model portfolio coefficients and risk, index model diversification, and examples of index models [8]. The definition and understanding of the capital allocation line, an example of using CAL to construct a portfolio, the calculation of the expected return (ER) of a portfolio and the formula for calculating the risk, and the calculation of the Sharpe ratio [9]. The definition and understanding of the efficient frontier, how to build an efficient frontier, how investors can use it, its importance, and the concept of an optimal portfolio. Also, there are critiques and special considerations for the efficient frontier, as the efficient frontier and the modern portfolio theory make a series of assumptions that might not accurately reflect reality. [10]. The concepts of minimum variance frontier and efficient frontier, global minimum variance portfolios are covered, and diagrams of these points and efficient frontier [11]. Fundamentals of portfolio analysis, the risk and return advantages of the Markowitz model, tax benefits, and its limitations [12].

3. Introduction of Three Industries

This paper investigates the performance of portfolio under different model. In the portfolio, it covers three main sectors: technology, financial services, and industrial, including 10 stocks. The 10 stocks are ADBE: Adobe Inc, IBM: International Business Machines Corporation, SAP SE, which are technology area. BAC: Bank of America Corporation, C: Citigroup Inc, WFC: Wells Fargo & Company, TRV: The Travelers Companies, Inc, they belong to the financial services sector. LUV: Southwest Airlines Co, ALK: Alaska Air Group, Inc, HA: Hawaiian Holdings, Inc, which belong to the industrials. Companies in the technology sector engage in the research, develop, and produce products and services with a technological foundation. They design and produce computers, mobile devices, and home appliances in addition to producing software. Additionally, they offer items and services associated with information technology. [2]. The most famous companies in the industry

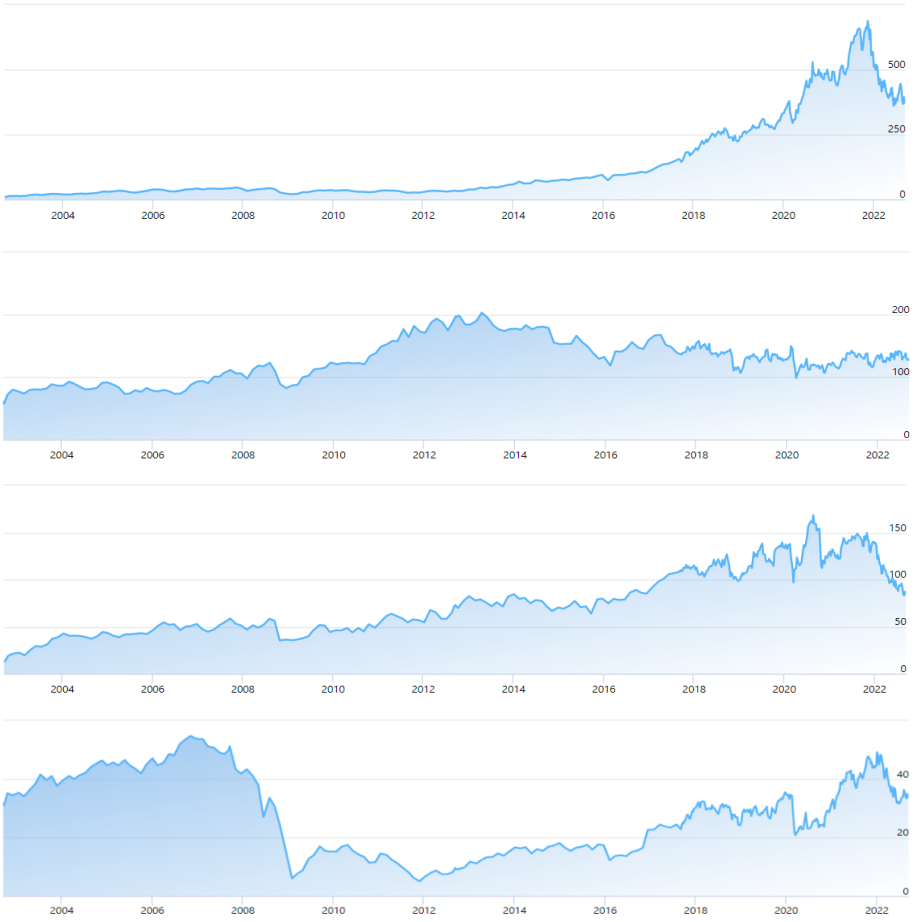
DOI: 10.54254/2754-1169/13/20230753

are Apple Inc., Apple designs, manufactures and sells a broad range of consumer technology products, including smartphones, PCs, tablets, wearables and more. Samsung Electronics Co. Ltd, it's engaged in a broad range of businesses, including consumer electronics, information technology, and communications. The Hon Hai Precision Industry Co. Ltd, also known as Foxconn, is a multinational electronics manufacturer based in Taiwan, China.

Financial services companies operate within the finance industry, includes banks, investment banks, insurance providers, credit card issuers, and brokerage firms, etc. A major brand in financial services is Berkshire Hathaway, which is led by Warren Buffett. The company's shares are among the most expensive in the world and trade on the New York Stock Exchange (NYSE). American Express, a Fortune 100 company and a component of the Dow Jones Industrial Average, initially provided freight forwarding services. Subsequently, it started offering travel services. It became one of the first companies in the world to offer charge cards [3].

The industrial sector consists of businesses that manufacture tools, materials, and supplies needed in manufacturing and construction as well as those that offer related services. Lockheed Martin Corp. (LMT), Honeywell International Inc. (HON), and 3M Company are well-known industrial sector businesses (MMM). Companies that offer air transportation services, like United Airlines Holdings Inc., are also included in the industrial sector (UAL) [4].

In this section, the price trend chart of 10 stocks in the past 20 years will be displayed in order.



DOI: 10.54254/2754-1169/13/20230753

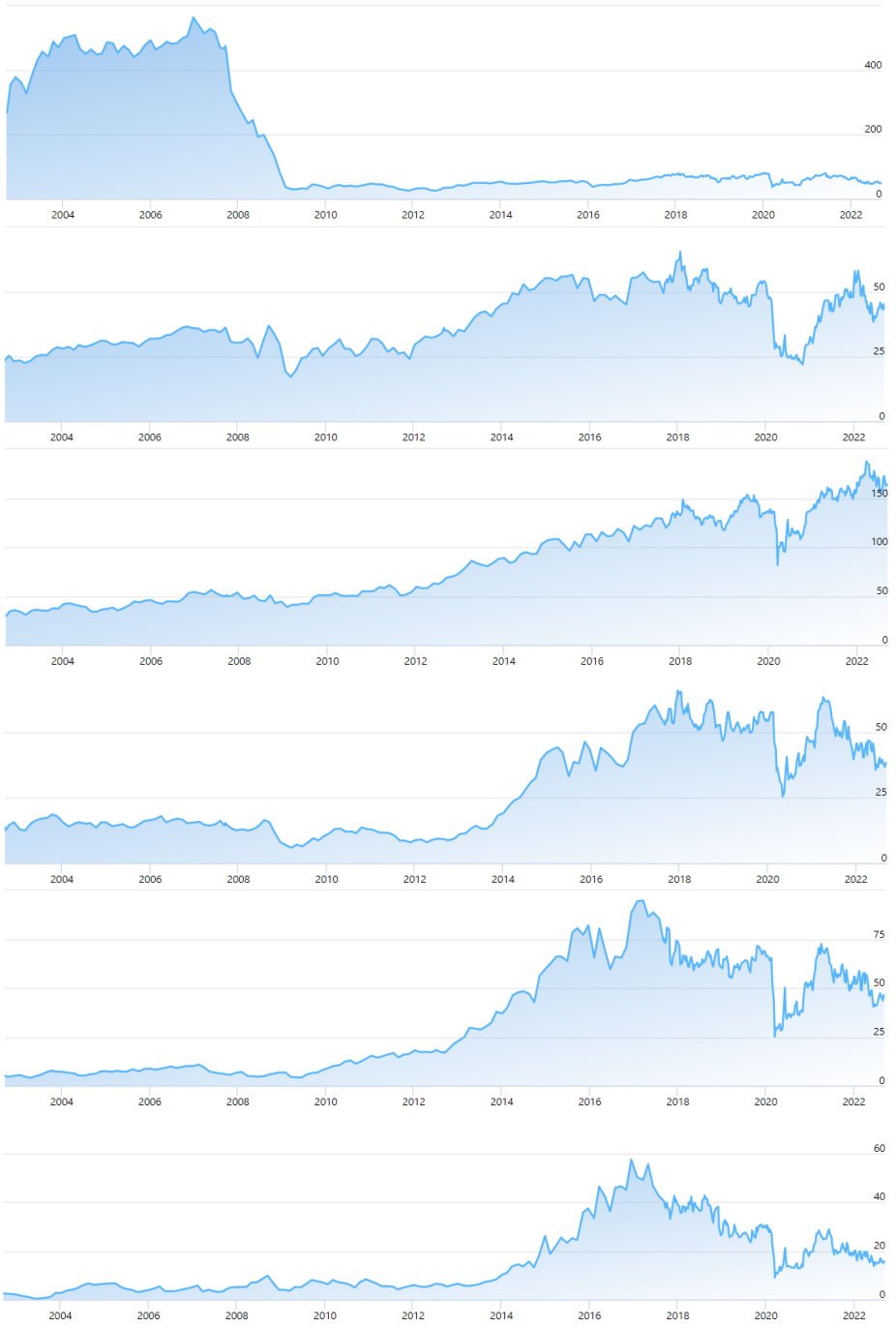


Figure 1: Stock price of individual stock.

Source: Hawaiian Holdings (HA) Stock - Price | Charts | Financials (NASDAQ: HA) (financecharts.com)

Shown below are the results of 10 stocks calculated and processed by formulas, and the data added to the constraints calculated later will also be based on these results (see Table 1 and Table 2).

DOI: 10.54254/2754-1169/13/20230753

Table 1: The calculated data of 10 stocks.

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
Annualized Average Return	7.5%	19.6%	4.8%	12.0%	11.1%	1.0%	8.9%	9.1%	9.8%	17.4%	26.9%
Annualized StDev	14.9%	31.8%	23.2%	33.9%	39.3%	42.5%	28.1%	20.0%	31.8%	37.7%	62.1%
beta	1.00	1.42	1.01	1.48	1.60	2.01	1.05	0.80	1.15	1.18	1.63
Annualized alpha	0.00	0.09	-0.03	0.01	-0.01	-0.14	0.01	0.03	0.01	0.09	0.15
Residual StDev	0.0%	23.8%	17.6%	25.8%	31.4%	30.3%	23.4%	16.0%	26.8%	33.4%	57.2%

This table displays the annualized average return, annualized standard deviation, beta, annualized alpha, and residual standard deviation of 10 stocks. Annualized standard deviation is used as a risk measure. It is used by CAPM and Sharpe ratio and information ratio. The higher the standard deviation, the higher the variance between the price and mean. In other words, a more volatile investment means a higher standard deviation and therefore more risk and reward [5].

Table 2: The correlation of 10 stocks.

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
SPX	100.0%	66.5%	64.9%	64.9%	60.2%	70.2%	55.5%	59.8%	53.7%	46.4%	39.0%
ADBE	66.5%	100.0%	45.5%	53.4%	42.3%	46.3%	29.8%	45.2%	38.8%	23.3%	18.0%
IBM	64.9%	45.5%	100.0%	58.5%	31.3%	42.0%	26.7%	38.2%	34.7%	35.7%	24.6%
SAP	64.9%	53.4%	58.5%	100.0%	33.1%	43.4%	29.8%	37.5%	31.8%	28.2%	14.4%
BAC	60.2%	42.3%	31.3%	33.1%	100.0%	82.6%	76.1%	39.3%	42.8%	27.5%	33.8%
C	70.2%	46.3%	42.0%	43.4%	82.6%	100.0%	70.3%	51.2%	42.8%	30.4%	34.3%
WFC	55.5%	29.8%	26.7%	29.8%	76.1%	70.3%	100.0%	34.5%	40.6%	34.7%	35.8%
TRV	59.8%	45.2%	38.2%	37.5%	39.3%	51.2%	34.5%	100.0%	40.7%	36.0%	24.0%
LUV	53.7%	38.8%	34.7%	31.8%	42.8%	42.8%	40.6%	40.7%	100.0%	51.9%	42.2%
ALK	46.4%	23.3%	35.7%	28.2%	27.5%	30.4%	34.7%	36.0%	51.9%	100.0%	40.4%
HA	39.0%	18.0%	24.6%	14.4%	33.8%	34.3%	35.8%	24.0%	42.2%	40.4%	100.0%

The relationship between two stocks' relative price movements is referred to as stock correlation. It can also be used to describe how equities are related to other asset classes include bonds or real

DOI: 10.54254/2754-1169/13/20230753

estate. Investors could determine the correlation of two stocks by investigating how each stock performs relative to its historical average return [6]. When stocks rise or fall at the same time, they can be positively correlated. A correlation value of 1 or 100% indicates that the positive correlation between the two equities. From this table, these stocks are positively correlated. Understanding stock correlations can help investors manage risk by diversifying and shielding their portfolios from inevitable market volatility.

4. Markowitz Model and Index Model

In this section, two models will be introduced.

Harry Markowitz first presented the Markowitz model in 1952. It is also known as the mean-variance model and is a portfolio optimization technique that analyzes alternative portfolio investments based on the expected return (mean) and standard deviation (variance) of the asset rate portfolio in order to create the riskiest return. [7]. Typically, three important variables (return, standard deviation, and correlation coefficient) are needed to determine an efficient portfolio set for investors. An investor must take two steps to find the best risk-return balance within various potential portfolios. The first is to identify the efficient portfolio set. The second approach involves choosing a specific final portfolio from an efficient set, given an investor's preferred return, acceptable degree of risk, or preference for the best return-to-risk ratio. Investors will select the portfolio with the lowest risk when deciding which portfolio to invest in out of those with similar returns. However, investors will pick the portfolio with the highest return out of those with similar levels of risk.

The expected return of the portfolio is:

$$E_{(rp)} = w_D \times E_{rD} + w_E \times E_{rE} \tag{1}$$

The Standard deviation (return volatility) of the portfolio is:

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{\sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}} \tag{2}$$

A statistical model of securities returns known as an Index model assumes that correlations with a common index are the main source of covariance among stock returns, which simplifies the estimation of covariance matrix issues. Therefore, fewer estimations are made. But still, it oversimplifies how the world really is. For instance, it divides security risk into market and asset-specific components [8].

On the other hand, the index model improves the analysis of predicted security returns (risk premiums), explicitly breaks down risk into systematic and firm-specific components and enables measurement of these risk components for specific securities and portfolios.

5. Five Constraints and Formulas

For each of the following five scenarios of the additional constraints, determine the areas of acceptable portfolios (efficient frontier, minimal risk portfolio, optimal portfolio, and minimal return portfolios frontier):

1.This additional optimization constraint simulates the FINRA Regulation T, which permits broker-dealers to permit their clients to hold positions that are funded to at least 50% by the client's account equity:

$$\sum_{i=1}^{11} |wi| \leq 2 \tag{3}$$

DOI: 10.54254/2754-1169/13/20230753

2.This additional optimization restriction is created to emulate some arbitrarily specified "box" constraints on weights, which might be given by the customer:

$$|wi| \leq 1, for\ \forall i; \tag{4}$$

3.A "free" problem to demonstrate how the efficient frontier and the area of permitted portfolios in general look when there are no further optimization constraints.

4.This additional optimization constraint simulates typical restrictions on the U.S. mutual fund market, such as the prohibition on short positions for U.S. open-ended mutual funds.

$$wi \geq 0, for\ \ \forall \mathrm{I} \tag{5}$$

5.Finally, a further optimization restriction will be taken into account depending on whether the inclusion of broad indices in our portfolio has a favorable or negative impact:

$$w1 = 0 \tag{6}$$

In the five constraints, 10 stocks exhibit different values of minimum variance and maximum Sharp in MM and IM.

Table 3: Minimum variance and max sharpe of MM in 5 constraints.

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
Weights	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.5
Minvar 1	111.46%	-9.91%	5.27%	-10.07%	0.60%	-22.81%	14.06%	19.53%	-0.26%	-4.98%	-2.88%
Minvar 2	100.00%	-8.53%	8.25%	-9.24%	0.66%	-23.03%	16.19%	22.93%	0.28%	-4.76%	-2.75%
Minvar 3	111.46%	-9.91%	5.27%	-10.07%	0.60%	-22.81%	14.06%	19.53%	-0.26%	-4.98%	-2.88%
Min-var 4	83.61%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.39%	0.00%	0.00%	0.00%
Min-var 5	0.00%	3.53%	34.20%	-1.99%	1.18%	-24.94%	34.79%	52.68%	5.03%	-2.86%	-1.61%
Max Sharpe 1	33.88%	24.22%	-15.10%	4.86%	20.11%	-44.73%	13.91%	24.06%	-10.03%	9.10%	7.36%
Max Sharpe 2	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%

DOI: 10.54254/2754-1169/13/20230753

Table 3: (continued).

Max Sharpe 3	50.09%	35.81%	-22.32%	7.19%	29.74%	-66.14%	20.56%	35.57%	-14.83%	13.45%	10.87%
Max Sharpe 4	0.00%	50.27%	0.00%	0.00%	0.00%	0.00%	0.00%	17.56%	0.00%	19.52%	12.66%
Max Sharpe 5	0.00%	50.82%	-19.51%	13.81%	36.74%	-76.96%	28.73%	49.95%	-16.54%	18.45%	14.50%

Table 4: Minimum variance and max Sharpe of IM in 5 Constraints.

	SPX	ADBE	IBM	SAP	BAC	C	WFC	TRV	LUV	ALK	HA
Weights	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.5
Minvar 1	138.68%	-9.81%	0.04%	-9.65%	-8.17%	-15.37%	-0.47%	11.24%	-2.13%	-1.75%	-2.62%
Minvar 2	100.00%	-6.58%	8.50%	-6.99%	-6.49%	-13.97%	3.74%	23.09%	0.74%	0.08%	-2.12%
Minvar 3	144.39%	-10.70%	-0.63%	-10.37%	-8.62%	-15.69%	-1.35%	10.96%	-2.97%	-2.27%	-2.75%
Min-var 4	38.23%	16.70%	0.00%	0.00%	0.00%	0.00%	1.72%	27.64%	0.26%	10.47%	4.98%
Min-var 5	0.00%	2.68%	29.08%	0.61%	-1.69%	-10.08%	15.22%	50.40%	9.09%	5.38%	-0.70%
Max Sharpe 1	48.18%	35.88%	-14.54%	2.04%	-0.61%	-34.85%	3.97%	28.36%	3.60%	17.73%	10.24%
Max Sharpe 2	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%
Max Sharpe 3	54.94%	38.26%	-22.71%	2.98%	-2.31%	-37.58%	4.25%	28.63%	4.00%	18.66%	10.89%
Max Sharpe 4	0.00%	40.75%	0.00%	0.00%	0.00%	0.00%	0.00%	26.17%	0.24%	20.49%	12.36%
Max Sharpe 5	0.00%	52.54%	-20.56%	8.64%	0.79%	-41.18%	9.50%	42.11%	8.47%	25.23%	14.46%

Note: Minvar stands for minimum variance

DOI: 10.54254/2754-1169/13/20230753

Table 5: Return, Standard deviation, Sharpe of MM in 5 Constraints.

	Return	StDev	Sharpe
	18.498%	36.997%	0.500
Constraint 1	6.69%	11.75%	0.570
	14.93%	14.43%	1.035
Constraint 2	6.97%	11.79%	0.591
	22.07%	21.33%	1.035
Constraint 3	6.69%	11.75%	0.570
	22.07%	21.33%	1.035
Constraint 4	7.79%	14.61%	0.533
	18.24%	24.75%	0.737
Constraint 5	9.38%	15.45%	0.607
	26.53%	25.98%	1.021

Table 6: Return, Standard deviation, Sharpe of IM in 5 Constraints.

	Return	StDev	Sharpe
	18.498%	35.996%	0.514
Constraint 1	6.08%	11.96%	0.508
	18.90%	21.04%	0.898
Constraint 2	6.90%	12.47%	0.553
	19.81%	21.99%	0.901
Constraint 3	5.85%	11.95%	0.490
	19.81%	21.99%	0.901
Constraint 4	12.00%	17.52%	0.6847
	17.26%	23.34%	0.7398
Constraint 5	9.26%	16.64%	0.556
	23.79%	26.68%	0.892

In Table 3 and 4, the minimum variance, and maximum Sharpe of the Markowitz model and Index model for 5 different constraints are shown. In Table 5 and 6, it shown the corresponding return, standard deviation, and Sharpe for each constraint. Among the results for these 5 constraints, as in constraint 3, the Markowitz model yields returns and Sharpe's higher than index model, while standard deviations are lower than IM. The risk-adjusted return is more appealing the higher the Sharpe ratio. A lower standard deviation indicates that most values are less different from their mean, with less volatility and less risk. Under constraint 1 and constraint 3, the Markowitz model consistently performed better at predicting minimum variance portfolios, while exponential models were particularly good at predicting maximum Sharpe ratio portfolios.

6. Comparison of MM and IM

In this section, results, and comparison of capital allocation line (CAL) and 3 frontiers will be introduced.

DOI: 10.54254/2754-1169/13/20230753

Table 7: Comparison of CAL for MM and IM in 5 Constraints.

	Constraint 1	Constraint 2	Constraint 3	Constraint 4	Constraint 5
CAL	0.0%, 0.0%	0.0%, 0.0%	0.0%, 0.0%	0.0%, 0.0%	0.0%, 0.0%
(MM) 2.5	37.3%,36.1%	55.2%,53.3%	55.2%,53.3%	45.6%,61.9%	66.3%,65.0%
(IM) 2.5	47.3%,52.6%	49.5%,55.0%	49.5%,55.0%	43.2%,58.3%	59.5%,66.7%

Investors can determine how much money they should put into risky and risk-free assets by looking at the capital allocation line. The trade-off between risk and return is represented by the CAL's slope. A higher slope means investors earn higher expected returns in exchange for taking more risk. This calculated value is called the Sharpe ratio [9]. Therefore, the higher the Sharpe ratio of a venture portfolio, the more attractive a portfolio an investor can choose from. In the plots of these two models, the slope of the capital allocation line for Constraint 2 in MM is steeper than the capital allocation line for Constraint 2 in IM, and investors will prefer the steeper one.

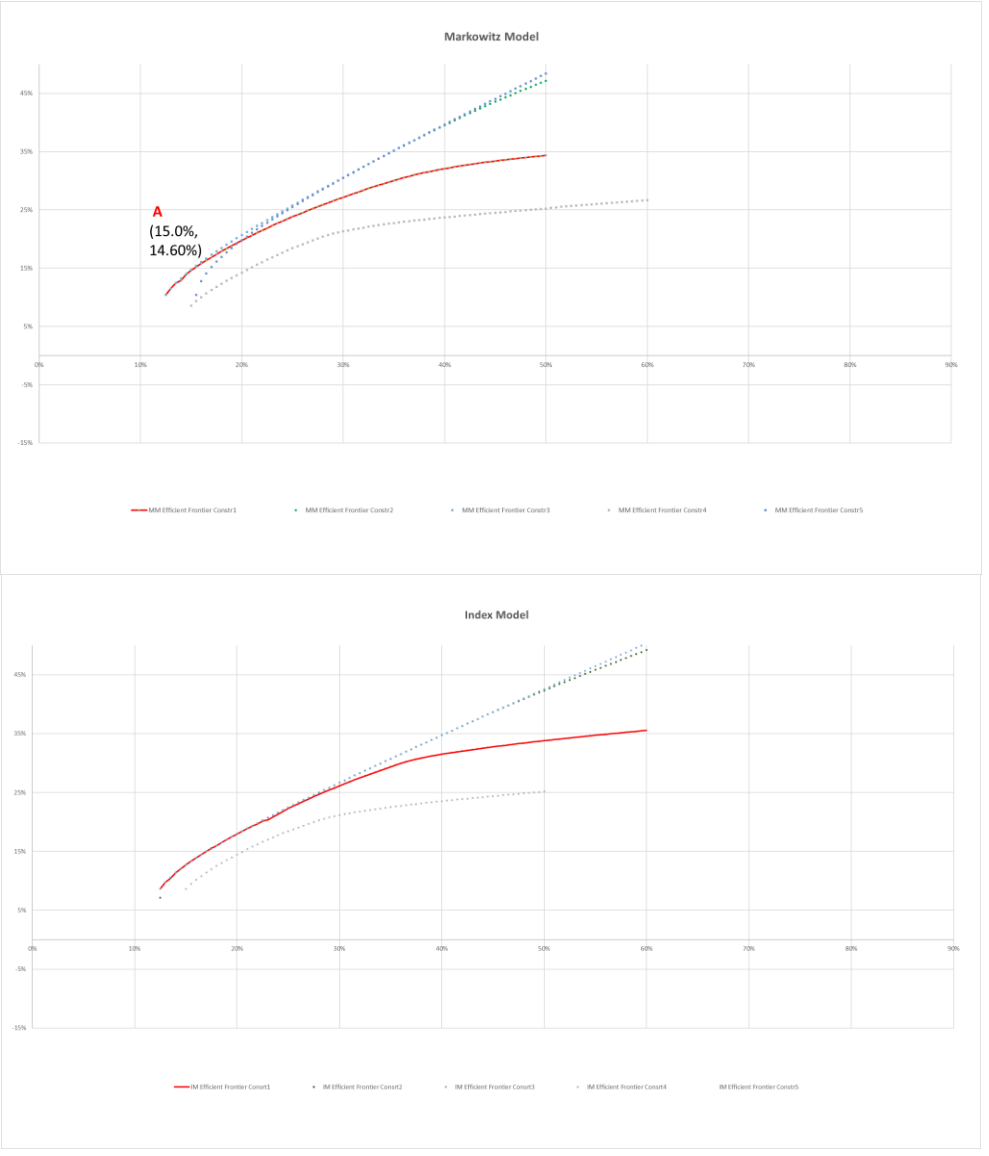


Figure 2: Efficient frontier of Markowitz Model and Index Model in 5 constraints.

DOI: 10.54254/2754-1169/13/20230753

The efficient frontier shows the portfolio of assets with the highest level of expected return for a given level of risk. It is represented by plotting the expected return of a portfolio and the standard deviation of the return [10].

In efficient frontier theory, investors will prefer portfolio A if its expected return of portfolio A is greater than portfolio B, and its standard deviation of portfolio A is smaller than portfolio B. There are points A and B representing two portfolios. At point A with a standard deviation of 15%, its expected return is 14.6%; at point B, with a standard deviation of 16%, its expected return is 13.9%. Therefore, Portfolio A is a better investment than Portfolio B because it provides a higher expected return with a lower risk.

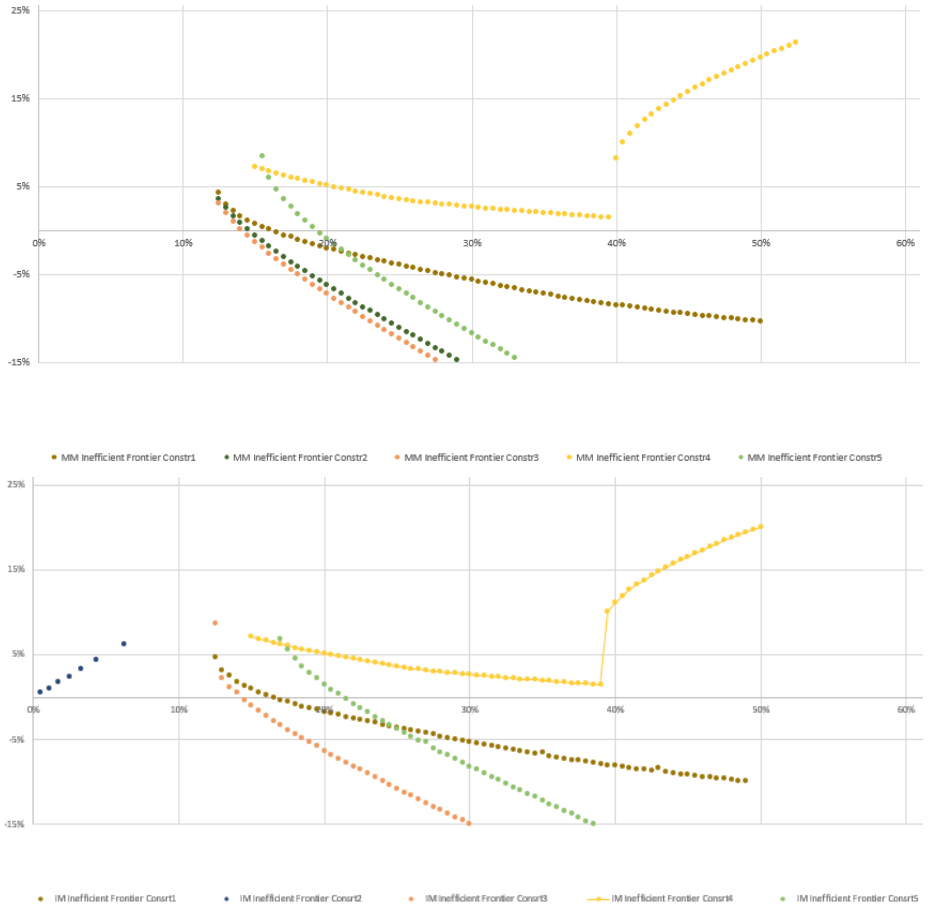


Figure 3: Inefficient frontier of Markowitz Model and Index Model in 5 constraints.

For the inefficient frontier, my point is the opposite of the efficient frontier. The comparison plot of the two models looks similar but based on the results from the analysis in the efficient frontier, Index model performs is less efficient than Markowitz model.

DOI: 10.54254/2754-1169/13/20230753

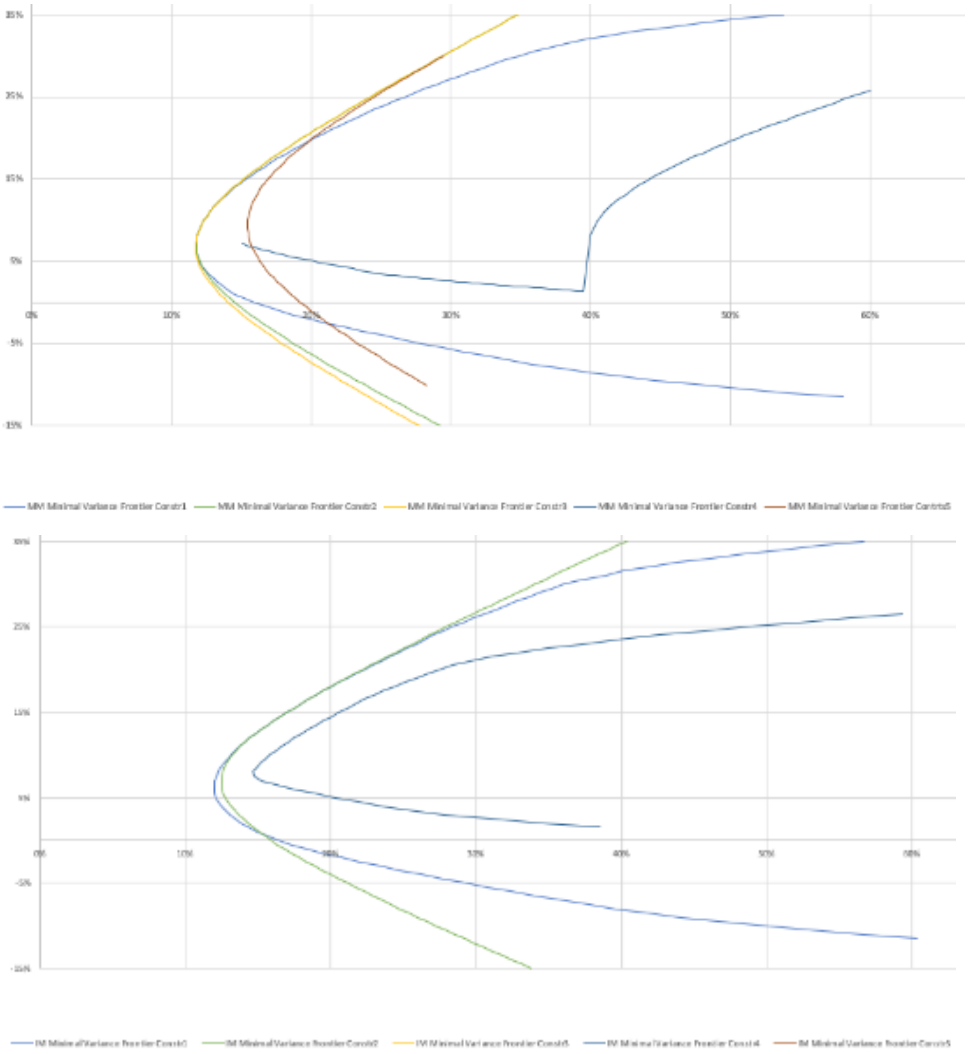


Figure 4: Minimum variance frontier of Markowitz Model and Index Model in 5 constraints.

All minimum variance portfolios are joined together to form the minimum variance frontier. The global minimal variance portfolio is the point on the minimum variance border that is closest to the y-axis, or the point with the least risk [11]. From the comparison of these two graphs, the minimum variance boundary of Markowitz model tends to be more closed, which indicates that the data distribution of this group is more concentrated, the variance is smaller, and the data is more stable. The boundary area displayed by Index model is more scattered, and the variance is higher than MM, so the data fluctuation is larger. Therefore, MM performs better.

7. Conclusion

This study compares the performance of the Markowitz model versus the Index approach to create the ideal portfolio, mainly by calculating all appropriate optimization inputs for both models and adding constraints to determine the portfolio area. The Markowitz model is an optimization tool that examines multiple portfolios based on the expected returns and standard deviations of assets to produce the optimum risk-return efficient portfolio. [7]. From the results presented, it can be concluded that the Markowitz model performs better in selecting the best portfolio because it has lower variance and standard deviation and lower risk. In the efficient frontier, it also results in a higher expected return with less risk. The "optimized" portfolio produced by the Markowitz model will either

DOI: 10.54254/2754-1169/13/20230753

have a greater expected return with the same level of risk, or a lower expected return with the same level of risk. The advantage of portfolio analysis is that it minimizes the impact of taxes on portfolio returns in addition to optimizing returns for a given level of risk [12]. However, its limitations include that past performance is no guarantee of future results, and investment objectives may change over time. For index model, it improves the evaluation of stocks' expected returns and makes the estimate of the covariance matrix problem simpler, however, it oversimplifies the true nature of the world.

References

[1] *Jin, Mingzhou. Research and Analysis on Markowitz Model and Index Model of Portfolio Selection, https://www.atlantis-press.com/ (2021).*
[2] *Johnston, Matthew. 10 Biggest Technology Companies, https://www.investopedia.com/ (2022).*
[3] *J.B.Maverick. What Companies Are in the Financial Services Sector? https://www.investopedia.com (2022).*
[4] *Reiff, Nathan. Top Industrial Stocks for September 2022, https://www.investopedia.com/ (2022).*
[5] *Nickolas, Steven. Expected Return vs. Standard Deviation: What's the Difference? https://www.investopedia.com/ (2021).*
[6] *Lake, Rebecca. What Is Stock Correlation, and How Do You Find It? https://smartasset.com (2022).*
[7] *Hanicova, Daniela and Quant Analyst. Markowitz Model. https://quantpedia.com/ (2021).*
[8] *Aldrich, Eric M. Index Model. https://ealdrich.github.io/ (2014).*
[9] *Chen, James. Capital Allocation Line (CAL). https://www.investopedia.com (2020).*
[10] *Ganti, Akhilesh. Efficient Frontier: What It Is and How Investors Use It. https://www.investopedia.com (2022).*
[11] *Concept 65: Minimum-Variance and Efficient Frontiers. n.d. https://ift.world (2022).*
[12] *Thorne, Douglas. Advantages & Limitations of a Portfolio Analysis. https://www.sapling.com (2022).*