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Game Theory in Companies: A Program-approach to Find All Nash Equilibria in Games with Two Players

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Abstract: Game theory is a famous mathematical formula for economic decision-making. When two companies launch competing games within a similar time frame in the game industry, they must choose wisely between launch dates, trailer release dates, and launch platforms to seek the highest return and lower risk. This report turns the three main strategies of the release time, the preview time, and the release platform into specific values and finds the optimal strategy choice using nash equilibria. Nash equilibrium is an essential concept in finding the optimal solution. Given the complexity of finding the equilibria, computational tools are used for the task. Software like *Gambit*, *Game Theory Explorer* and python programs are used for computing all nash equilibria. This paper quantifies the three main reasons that may affect the sales result: the timing and order of the trailer, the interval between launch days, and the release platform, and turning these factors into a pay-off matrix. According to the calculation results on those pay-off matrices, this paper formulates a set of well-founded game sales methods to maximize the company's profits.

Keywords: game theory, pay-off matrix, release date, trailer, game platform, software

1. Introduction

According to Game Informer, at the end of 2021, three FPS (first-person shooter) games, *Halo: Infinite*, *Call of Duty: Vanguard*, and *Battlefield 2042*, were launched in the game market in sequence [1]. As games with equal popularity, the sales results differ in the aggregate. In this case, we tried to maximize profits when two companies released the same game product in the same period. From literature *When to release a game: Ten things to consider when choosing a video game release date, what are the best platforms for video games?* And *The 9 do's and don'ts of Game Trailers* are three factors that can affect the launch of a game: the launch date of the game, the release date of the trailer, and the platform on which the game is released [2-4]. This paper will focus on finding the best choice for a company among launch dates, trailer release dates and launch platforms. Our goal is to use computing software to find the Nash equilibrium in the pay-off matrices. The significance of this research is that based on our calculation results, game companies can use these results to make the best decision for their game sales in order to maximize their profits.

Our scenario includes two players (*A* and *B*) who know each other's game release date (*RD*). The

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goal is to attract more gamers than other companies. The release date strategy tells us that if *B* decides first, *A* will decide based on *B*'s game release date. The trailer release date (*TRD*) can attract more attention from gamers before the official game release date. Therefore, the trailer must be released before the game's release date. Releasing trailers close to the game release date will have more benefits. Game platforms (*GP*) will affect the number of gamers and how many sales each company can make. We assume there are only two platforms: *Personal computer (PC)* and *Playstation* (*P*). According to Peppiatt's article, we assume the PC has a more significant number of gamers than the *P*. From this, we know that *P* will deliver a certain sign-on bonus [5].

Maschler states that game theory is good at formulating competitive relationships between one or more people and finding well-defined strategies [6]. In a *non-cooperative game*, *A* and *B* cannot form agreements. Through a set framework of *cooperative game theory*, it is seen that our situation is as opposed to the traditional cooperative game. Other models, such as Bayesian games, provide an approach where players are unaware of their opponents' attributes. However, *non-cooperative games are* more general and sufficient in analyzing companies' behaviors. With "*two*" players and "*n*" strategies (*n* being a finite number), we will first come up with a payoff matrix in each case with Microsoft Excel Solver®, then use programs to solve them.

2. Methodology

The payoff matrix formed by listing and arranging all the strategies can numerically quantify the advantages of each choice and calculate the optimal strategy for both companies. Based on the factor analysis, we may create the "*rules of the game*" by simulating the real situation. According to reality, each strategy choice can form a payoff matrix that shows the possible consequences, which can be formed by transforming these consequences into a reasonable number. Finally, a flow of rules is applied for each factor, and scoring results are recorded in tables.

Figure 1: Flow of the game.

2.1. Launch Dates Strategy

According to the calculation of the rules made above, the payoff matrix can be drawn, and the corresponding value of each strategy can be obtained, as shown in Table 1. In Figure 2, once the game release date is less than one month, both sides will have minus 20 points. Releasing games earlier per month will help the company earn 10 points. However, releasing games late per month will let the company to lose 10 points.

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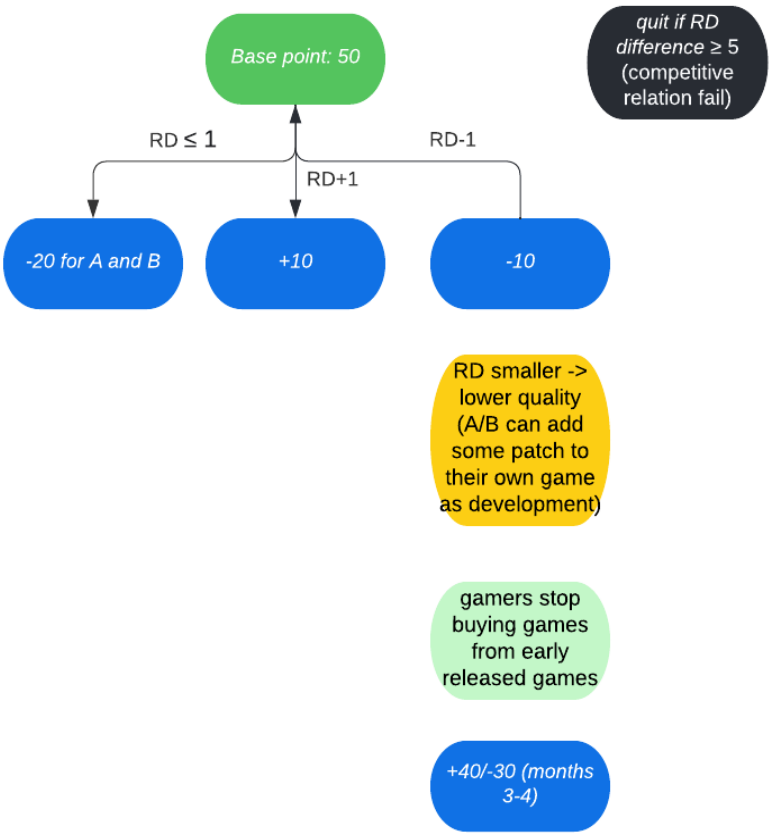

Figure 2: Rules for the release date (*RD*) in two players.

Table 1: Payoff matrix within six launch months.

		A					
		1	2	3	4	5	6
B	1	30/30	40/60	30/70	60/50	50/60	50/50
	2	60/40	30/30	40/60	30/70	60/50	50/60
	3	70/30	60/40	30/30	40/60	30/70	60/50
	4	50/60	70/30	60/40	30/30	40/60	30/70
	5	60/50	50/60	70/30	60/40	30/30	40/60
	6	50/50	60/50	50/60	70/30	60/40	30/30

2.2. Trailer Release Strategy

According to the rules above, we can get a simple 2x2 payoff matrix, as shown in Table 2. In Figure 3, if both companies choose early release, they will get 20 points. If both of them choose the late release, they will get no points. If one company chooses early release and another chooses late release, early release will bring 45 points to the company. However, the late release will only bring 15 points to the company.

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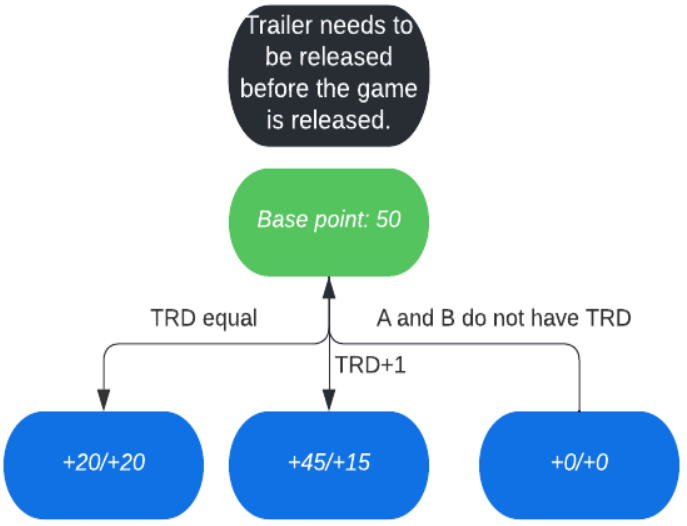

Figure 3: Rules for the trailer release date (*TRD*) in two players.

Table 2: Payoff matrix with early/late launch strategies.

Company A			
Company B		Early release	Late release
	Early release	20/20	15/45
	Late release	45/15	0/0

2.3. Game Platform Strategy

As shown in Figure 4, there are only two platforms for game companies to choose from, PC and P. Platform PC has a more significant number of gamers than P. However, if one company decides to publish on P, the P company will give the game company a certain amount of sign-on bonus. Also, game companies cannot publish their games on both platforms. If both companies choose the PC platform, both of them will get 20 points. If both of them choose the P platform, they will get t points. If one company chooses the PC platform and another chooses the P platform, the PC platform will bring 50 points to the company. However, the P platform will only bring 30 points to the company. After formulating the rules and assumptions of the three main strategies, players can combine them into a payoff matrix that contains all the strategies. Using the Excel program, they could list the numerical influence of each strategy, draw the payoff matrix quickly, and calculate the payoff value of each strategy correctly.

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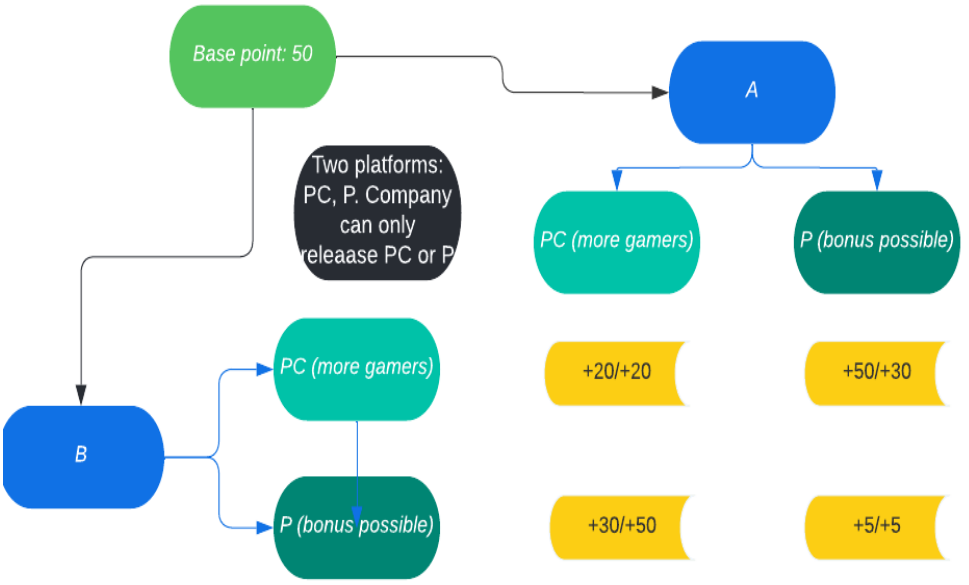

Figure 4: Rules for the game platform (*GP*) in two players.

TP = Basic points + Platform Points + Trailer Points + Δ month points + early/late profits						
x		1 month	2 month	3 month	4 month	
Basic points	50					
						+
both PC platform	20					
both Sony platform	5					
sony platform only	50					
PC platform only	30					
						+
Trailer both early release	20					
Trailer both late release	0					
trailer late release	45					
trailer early release	15					
						+
same month discount	-20					
						+
early profits/month		10	20	30	40	
late release lose/month		-10	-20	-30	-40	
						+
better quality profits (months 3-4)					40	
lower quality lose (months 3-4)					-30	

Figure 5: Rules formulated in microsoft excel.

Figure 5 shows that we listed all the policies and their corresponding values in excel files. In this way, the final value of each strategy can be obtained by adding together the corresponding values for each strategy during calculation. For example, in Microsoft Excel, the numerical calculation method can be AA5+AA7+AA12+AA17&"/"&AA5+AA7+AA12+AA17. This sample equation shows that both companies chose the PC platform and RD early in the first month. By adding the cells containing the corresponding values shown in Figure 5, we obtained the final number of the corresponding strategy. We can quickly calculate all the values of the overall payoff matrix, and the results are shown in Figure 6.

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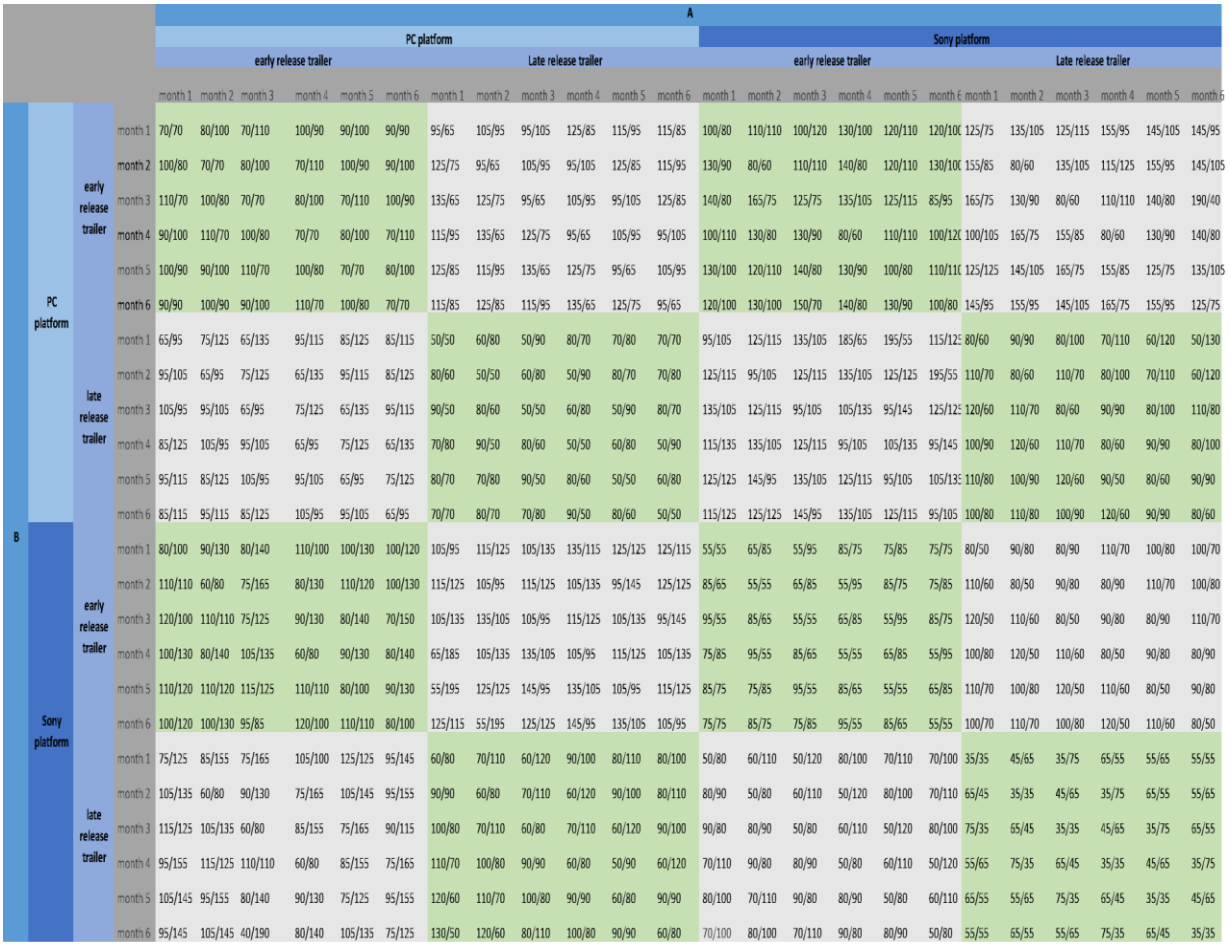

Figure 6: Payoff matrix with 24 strategies.

2.4. Python Program for Pure Strategy

There is a simplified version of the matrix with four strategies for simplicity. Now we will code a python program to find its nash equilibria.

			Company A			
			PC		P	
			month 1	month 2	month 1	month 2
Company B	PC	month 1	**50/50**	**60/80**	**80/60**	**90/90**
		month 2	**80/60**	**50/50**	**110/70**	**80/60**
	P	month 1	**60/80**	**70/110**	**35/35**	**45/65**
		month 2	**90/90**	**60/80**	**65/45**	**35/35**

Figure 7: Payoff matrix with four strategies.

2.5. Algorithm for Exhaustively Finding All Pure Nash Equilibria in a Two-Player Game

In this section, we describe an algorithm to solve a Two-Player game. Pseudo-code for the algorithm is given below:
Begin

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```
        Step (1):
set best payouts = {}
set row number = length of payout grid
set column number = length of payout grid [0]
        Step (2)
                For j = 0 to column num
                        max payout = max([payout grid] for r in range(row num))
                        For j = 0 to row num:
                                if payout grid[row][col][p1] == max payout:
                                best payouts[(row, col)] = (row_labels[r], col_labels[c])
                best payouts labels = []
                For n = 0 to row num:
max payout = max([payout grid] for r in range(col num))
                        For m = 0 to row num:
                                if payout grid[row][col][p2] == max payout:
                                        if (row, col) in best payouts:
                                           bestpayoutlabels.append(best_payouts[(row,col)])
        return best payouts labels
```

2.6. Available Software for Mixed Strategy

There are numerous algorithms for finding nash equilibria, but they are time demanding for finding nash equilibria with multiple strategies. In this way, using software is efficient in applying the developed algorithm.

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s) \quad (1)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j) \quad (2)$$

2.6.1. Gambit

Gambit (http://www.gambit-project.org/) is an open-source software whose purpose is to find Nash equilibria. Python is used for applications. The application can do various kinds of games, including non-cooperative games. As shown in Figure, this software efficiently solves games with a larger number of strategies. After entering our scores table, the software asks for a method to return the results. We choose "compute as many nash equilibria as possible" and "use Gambit's recommended method".

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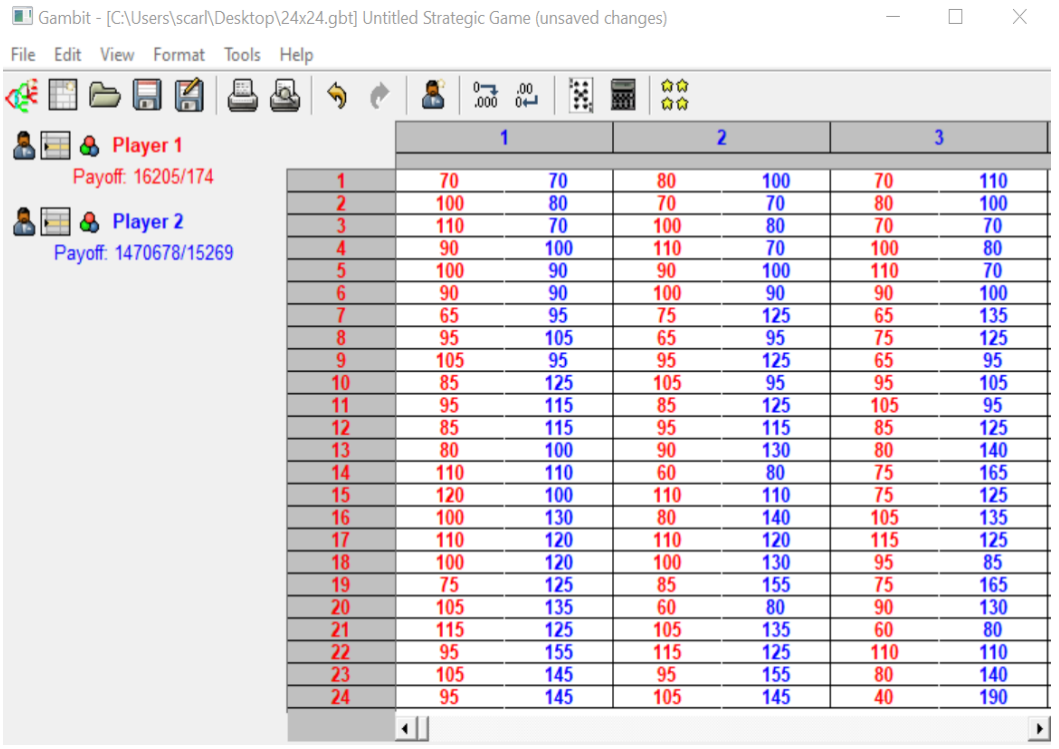

	1		2		3	
1	70	70	80	100	70	110
2	100	80	70	70	80	100
3	110	70	100	80	70	70
4	90	100	110	70	100	80
5	100	90	90	100	110	70
6	90	90	100	90	90	100
7	65	95	75	125	65	135
8	95	105	65	95	75	125
9	105	95	95	125	65	95
10	85	125	105	95	95	105
11	95	115	85	125	105	95
12	85	115	95	115	85	125
13	80	100	90	130	80	140
14	110	110	60	80	75	165
15	120	100	110	110	75	125
16	100	130	80	140	105	135
17	110	120	110	120	115	125
18	100	120	100	130	95	85
19	75	125	85	155	75	165
20	105	135	60	80	90	130
21	115	125	105	135	60	80
22	95	155	115	125	110	110
23	105	145	95	155	80	140
24	95	145	105	145	40	190

Figure 8: Gambit's screen with a game mode for two players and two strategies and the results found by the software.

2.6.2. Game Theory Explorer

Game Theory Explorer, GTE (http://www.gametheoryexplorer.org) was developed with the objective of analyzing games as models of strategic interaction.

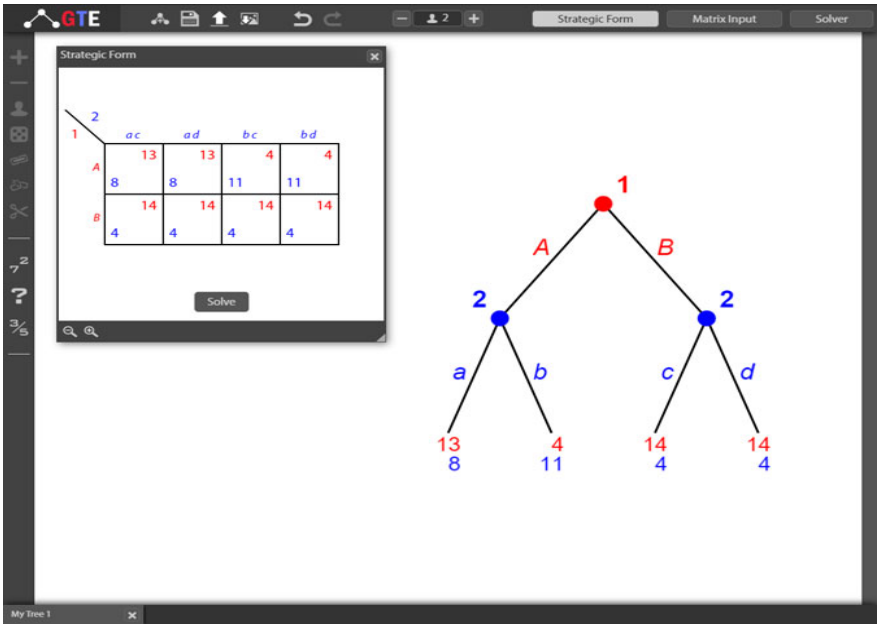

Figure 9: GTE's screen with a game mode for two players and two strategies and the results found by the software.

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Other Approaches

3. Results and Conclusion

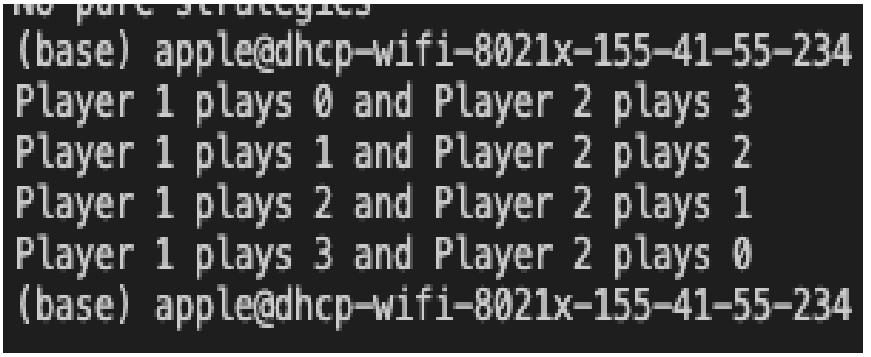

Figure 10: Terminal results for payoff matrix with four strategies.

3.1. Pure Strategy

Among four best choices in the table, which can be divided into two cases. In the first case, one company releases on *PC* in the first month, whereas the other company releases on *P* in the second month. The payoff for both sides is relatively maximized and equal to 90 in this case. In the second case, one company chooses *PC* and releases the game in the second month, while the other company chooses *P* and releases the game in the first month. The company that releases *P* gets the most benefit, while the other gets a poorer but good benefit.

Among the four best plans, there is no duplication in the choice of strategy between the two companies. When one company launched on *PC*, another company launched on *P*; When one company releases a game in the first month, another company releases a game in the second. It is concluded that when two game companies release competing products, it is best to stagger the release platform and release time with each other to maximize profits or minimize losses. Also, if you can make sure the platform is *P* and you can ship it in the first month, then you can maximize the revenues in this case.

Based on Nash Equilibria's calculation with a 4 x 4 matrix, when two game companies release competing products, *postponing the release platform and release time from each other is the best choice to maximize profits or minimize losses*. *If B chooses to release a game on the P, the best strategy for A is to maximize profit is to releasing the game one month after on PC*. If you can ensure that the release platform is *P* and released in the first month, the revenue can be maximized. A literature review made it possible to identify available programs to find Nash equilibria.

3.2. Mixed Strategy

No pure strategy was found for the 24 x 24 payoff matrix; thus, a mixed strategy must exist. To calculate it, we may change the criteria of the original 4 x 4 matrix. In mixed strategy games, we found the probability distribution of nash equilibria through tools like *Gambit* and *GTE*. Our heatmap shows that strategy 1-6 is maximized for *A*, and strategy 1-8 is maximized for *B*.
Given the limitations of modeling and how it differs from reality, we will update the model and develop a better algorithm in the future.

3.3. Conclusion

In this report, we try to figure out when two companies in the gaming industry are about to launch

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new games of similar types and how they can resort to a mathematical formula to make profitable decisions. Game theory, particularly Nash equilibria, helps companies make wise choices among three factors: time to release and preview and platform to release. Companies could turn these elements into a pay-off matrix and formulate and execute sales strategies accordingly to achieve optimal company profits. To make the research process more intuitive for target audiences, we created two players, A, and B, in the virtual scenario and focused on exploring how it worked under the cooperative other than the traditional uncooperative mode. We actively resorted to tools, including Gambit, Game Theory Explorer, and Python, for the calculations.

Additionally, we applied the listing, arranging, and calculating method in the Microsoft Excel Solver to make the analysis developed based on more accurate and intuitive data. We then produced and presented corresponding figures and algorithms accordingly. Finally, we draw the results by deploying the pure strategy and mixed strategy simultaneously. From the Nash Equilibria calculation with a 4 x 4 matrix, one of the two companies is suggested to delay its release time when the other one releases similar products earlier. By doing so, both can achieve their maximized profits as anticipated.

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Sijia Li, Zhexiang Li and Zhuang Liu contributed equally to this work and should be considered co-first authors.

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