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Markowitz and Index Model in Capital Markets

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Abstract: Investment has become an essential subject in this age, and the Markowitz Model is one of the most classical theories. However, after this, a new model is based on it borne. Nevertheless, it is similar to the Markowitz, so we want to figure out why this new model is better than M. In this work, we use excel to make two models called Markowitz and Index and displaying them. As a result, we found that the Index Model does better than the M because of the higher slope. With this conclusion, investors can use them to choose their best combination.

Keywords: markowitz, index, minvar, maxsharpe, CAL

1. Introduction

The return and risk of investments have always been an essential part of the financial investment markets and an important reference point for decision-makers. As the economy continues to grow and new financial instruments are created, capital markets have become central to the U.S. financial market. In 1952, Harry Markowitz published his infamous "Portfolio Selection." There are three parts to Markowitz's Portfolio Selection: 1. To determine the permissible portfolios region using the risky investments available, which in particular includes determining the minimum variance frontier of allowed combinations of risk and return 2. To determine the optimal risky portfolio, and 3. Select a suitable complete portfolio by mixing the optimal risky portfolio with the risk-free asset maximizing the investor's utility function, given the level of the investor's risk-aversion. Modern portfolio theory, therefore, is a risk-reducing approach to portfolio management that achieves a certain level of portfolio's expected return subject to some possible additional constraints by optimally diversifying across all given investment options. However, Markowitz's theory requires comprehensive estimation of various expected quantities (returns, standard deviations, and correlation coefficients), and the model does not specify how to estimate them; the estimates obtained by historical data sample averaging may not be reliable. Despite these shortcomings of the Markowitz model, it is undeniable that the field of finance has undergone a revolutionary change before it and after it as a result. To remedy some of these shortcomings, a simplified model of optimal portfolio allocation was proposed and instantly became widely used in the financial industry is; the Index model developed by William Sharpe about ten years after the Markowitz publication in 1963. This model simplifies the measurement of individual security and portfolio risk and return. It enhances the analysis of expected returns and allows measuring the risk of a given portfolio, in addition to simplifying the estimation of covariance matrix problems.

Our research here focuses on ten well-known stocks using their historical daily total return data over the last twenty years. We then aggregate the daily total return data into the monthly total return data and use the Markowitz Model and Index Model to calculate all the required model estimates based on the monthly total return data running the models with those estimates; we find the efficient frontiers, the global minimum risk portfolio, the optimal risky portfolio, and the minimum return portfolio frontiers for our portfolio.

#	Group #4	Full Name	Sector (Yahoo!finance)
1	QCOM	QUALCOMM Incorporated	Technology
2	AKAM	Akamai Technologies, Inc.	Technology
3	ORCL	Oracle Corporation	Technology
4	MSFT	Microsoft Corporation	Technology
5	CVX	Chevron Corporation	Energy
6	XOM	Exxon Mobil Corporation	Energy
7	IMO	Imperial Oil Limited	Energy
8	KO	The Coca-Cola Company	Consumer Defensive
9	PEP	PepsiCo, Inc.	Consumer Defensive
10	MCD	McDonald's Corporation	Consumer Cyclical

Figure 1: The company that we choose.

The Markowitz Method formulas we used are as follows.

The Markowitz Model (MM) expected portfolio return:

$$r_p = \vec{w} * \vec{\mu}^T \tag{1}$$

Markowitz Model (MM) investment portfolio expected standard deviation:

$$\sigma_p = \sqrt{\vec{v} P \vec{v}^T} \tag{2}$$

The Index Model (IM) formulas we used are as follows.

The expected portfolio return following the Index Model:

$$r_p = \vec{w} * \vec{\mu}^T \tag{3}$$

The Index Model investment portfolio expected standard deviation:

$$\sigma_p = \sqrt{\left(\sigma_M \beta_p\right)^2 + \sum_{i=1}^{n} w_i^2 \, \sigma^2(\varepsilon_i)}, \ \beta_p = \vec{w} * \vec{\beta}^T \tag{4}$$

Some of the shortcomings of the Markowitz Model are 1. For large portfolios, the number of estimates required is large, which effectively prohibits the application of MM to large portfolios. 2. The MM does not tell exactly where to get the expected estimates; if one naively assumes that we need to get them from historical data estimation, this introduces a dependency on the sample size and sampling frequency. None of this is discussed in the original MM. This is why the IM has made such progress and become popular with users.

2. Markowitz Model's Description

We define $\vec{\mu} = \{\ \mu1, \mu2, \mu3, \ldots\ldots. \mu n\ \}^T$ is the set of instruments' average returns; $\vec{w} = \{w1,w2,w2,\ldots..wn\ \}^T$ is the unknown set of instruments' weights; $\vec{\sigma} = \{\sigma1, \sigma2, \sigma3, \ldots\ldots \sigma n\ \}^T$ is the set of instruments' standard deviations; $\vec{\beta} = \{\beta1, \beta2, \beta3, \ldots. \beta n\ \}^T$ is the set of instruments' betas; $\{\sigma(\varepsilon1), \sigma(\varepsilon2), \sigma(\varepsilon3), \ldots.. \sigma(\varepsilon n)\ \}^T$ is the set of the residuals' standard deviations; $\vec{v}=\{\overrightarrow{v1},\overrightarrow{v2},\overrightarrow{v3},\ldots\ldots\overrightarrow{vn}\}^{T}$ is an auxiliary vector; and $\begin{cases}\rho 11 & \rho 12 & \ldots. & \rho 1n \\ \rho 21 & \rho 22 & \ldots\ldots & \rho 2n \\ \rho n1 & \rho n2 & \ldots\ldots. & \rho nn\end{cases}$ p= is the matrix of instruments' cross-correlation coefficients.

The Markowitz Model can be used in the practical applications of asset allocation. By obtaining the optimal asset portfolio, the investment proportion of securities can be changed so that the portfolio can diversify the investment risk and achieve the objective of maximizing the investment utility. Moreover, such a model implies a unique property: the risk reduction due to diversification by choosing the assets in a portfolio with low correlation.

3. Data Processing Steps

Step1:

First, we choose data for all these ten companies. Then we copy them into our specific model, which can run a particular procedure to help us calculate our outputs. We need to get the results of Markowitz's global MinVar and MaxsSharpe. For Minvar, we chose this data.

Table 1: Variant data.

	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD	Return	StDev	Sharpe	20.0%	Dummy Variable	
MM	173.4%	-17.8%	-11.2%	-15.1%	-33.5%	-46.0%	71.9%	-27.3%	32.7%	21.4%	-48.5%	-4.2%	20.0%	-0.21	498.9%	With Regularization	
IM	0.0%	10.9%	20.3%	8.2%	44.6%	-12.7%	-57.8%	3.8%	-16.3%	5.2%	93.8%	22.0%	28.0%	0.79	273.7%	Without Regularization	
	173.4%	17.8%	11.2%	15.1%	33.5%	46.0%	71.9%	27.3%	32.7%	21.4%	48.5%						

Moreover, we use an Excel tool called Solver; we choose the range of weights from SPX to PEP, then click "min" and then click ok; after a few minutes, the results will show off. The MaxSharpe is similar to the global Minver, but the difference is that we used the minimum for the global Minvar and the maximum for the Maxsharpe, as the exhibits below show.

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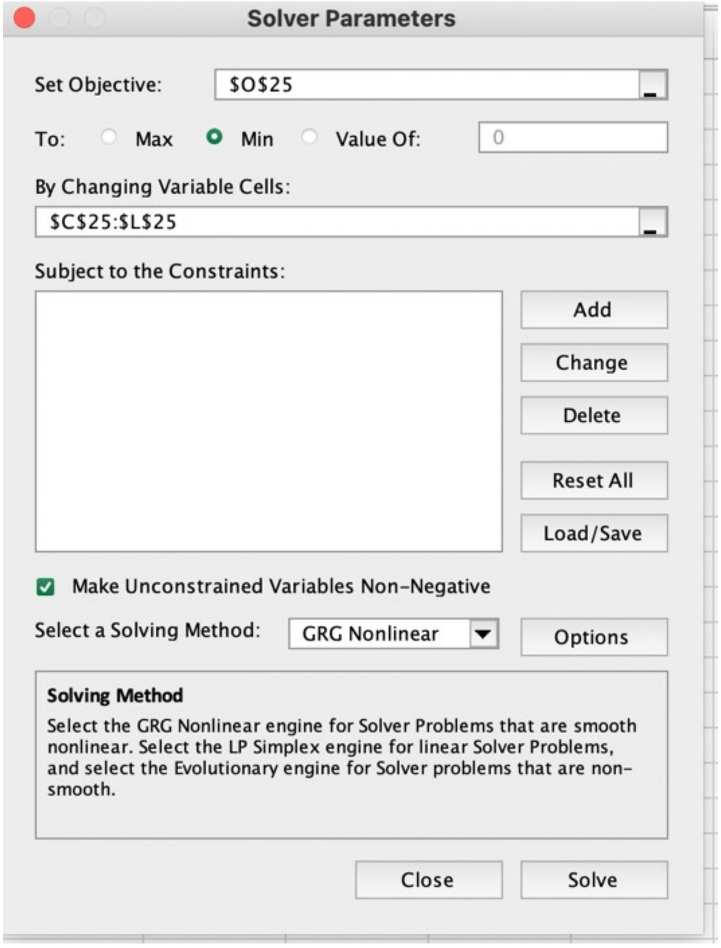

Figure 2: Markowitz's global Minvar(constra1).

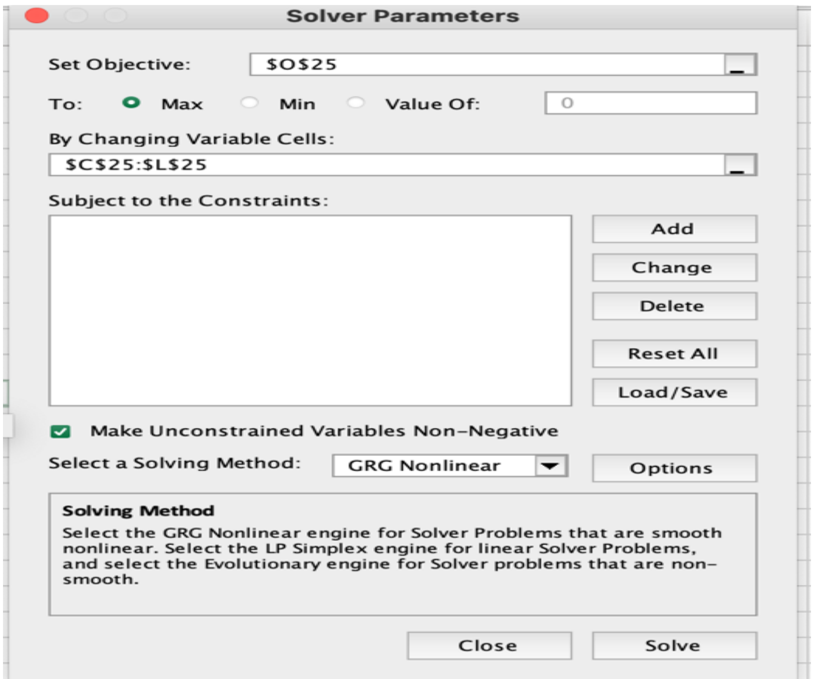

Figure 3: Markowitz's global MaxsSharpe(constra1).

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However, these are just MM; hawse also needs to estimate the IM - Index Model, and the Index Model's global Minvar and MaxSharpe points can be different from Markowitz Model ones. We want to get to know why IM is better than MM. So firstly, we made a model similar to the MM to find their differences.

The target function is now different, the weights are different, and the standard deviation is also different. So the target function needs to be changed into IM's standard deviation, and the weights need to be changed into the ones on the line corresponding to the IM.

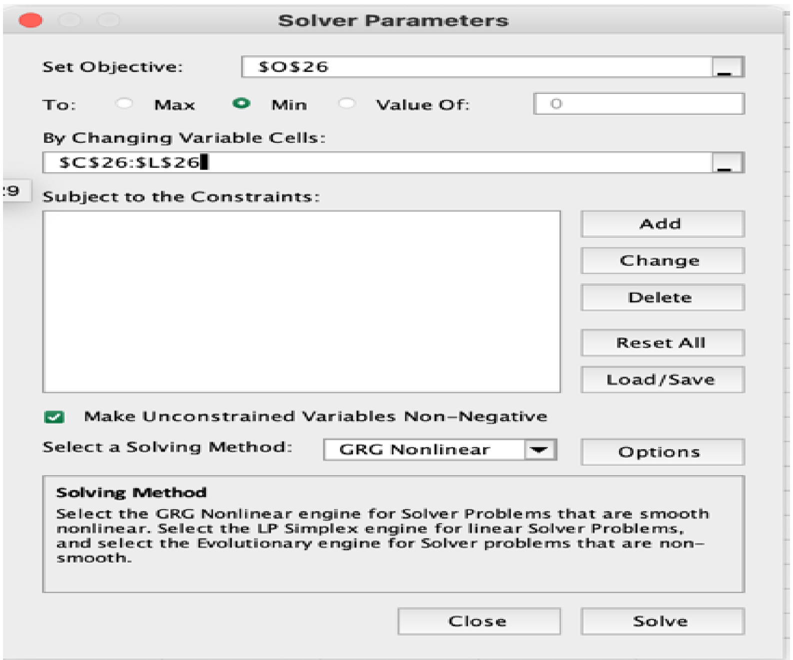

Figure 4: Index model Minvar(constra1).

These two results also do not require any constraints.

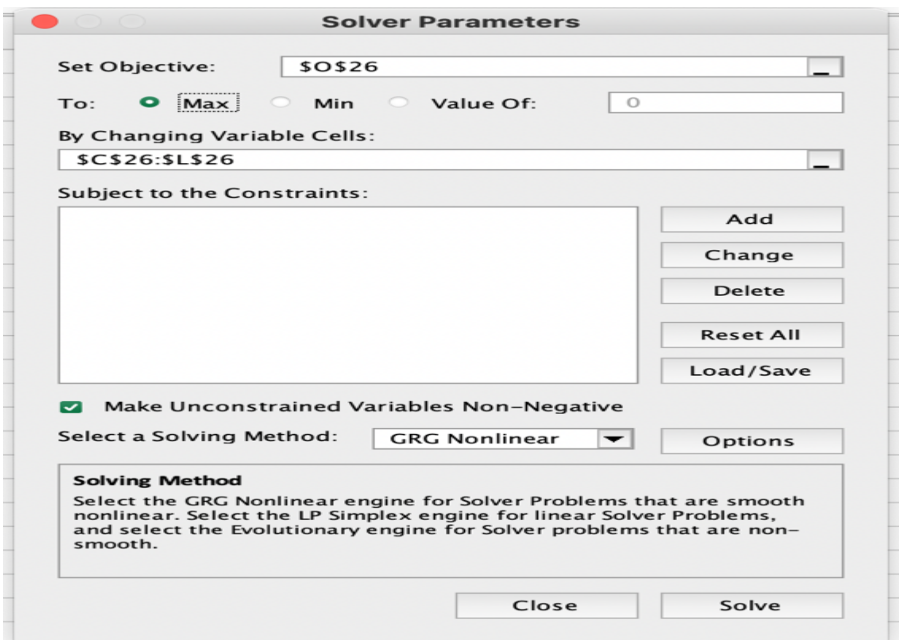

Figure 5: Index model MaxsSharp.

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As a part of this project, we also need to make similar calculations with an additional constraint, MM constr2, and IM constr2. The additional constraint, which we call constr2, corresponds to the weight of SPX being equal to zero, and everything else remains the same as described above for constr1. Once we set up all these models, we run the other Excel tool called SolverTable, which allows up to run the Solver recursively multiple times. For that, we need to make some additional steps before we run the SolverTable. For MM constr1, we add a new limit: making the standard deviation equal to the Dummy Variable and changing the target function to the Expected Return, after which we must save the Excel spreadsheet.

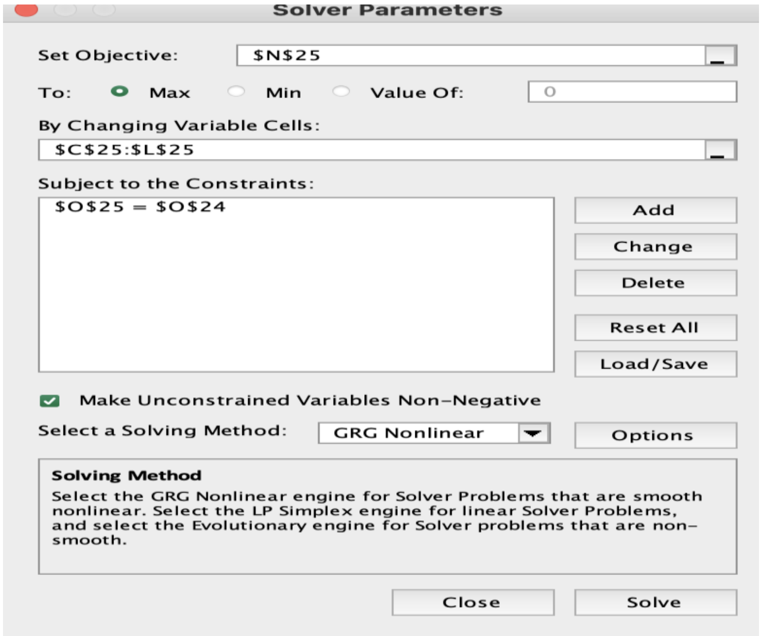

Figure 6: Markowitz model MaxsSharpe(constra2).

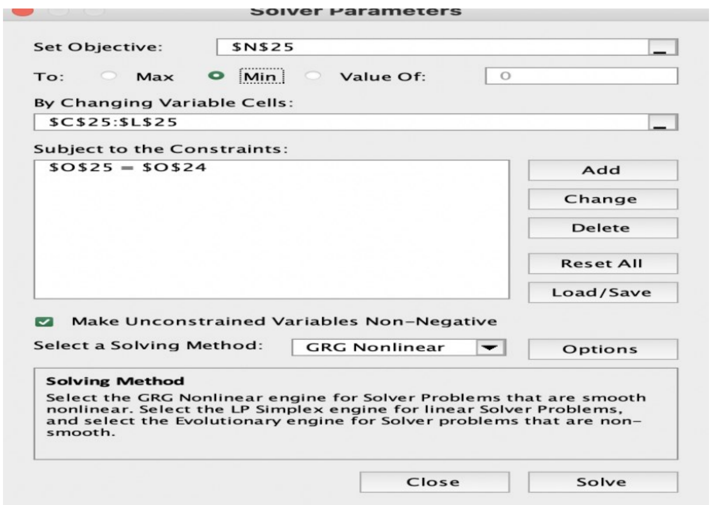

Figure 7: Markowitz model Minvar.

Once we have set up all these steps, we can run the SolverTable, and we need to pass on the Dummy Variable to it and set up the output range from the SPX to the Sharpe. Once these steps are done, we click ok, and once it shows us the last menu, we must click no because if we press yes, it will go into an infinite loop. So as the result of running the SolverTable, we calculate the MM

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Efficient Frontier constr1, and if we changed the target into the expected return and the weights into the IM, when we again run the SolverTable, and we produce the inefficient frontier for the MM with constr1. Moreover, we can produce the MM Minimal Variance Frontier with constr1 similarly to the efficient frontier above, with the only difference between them being that the variance frontier's minimum value is -0.1. Moreover, this is how we obtain three constr1 results.

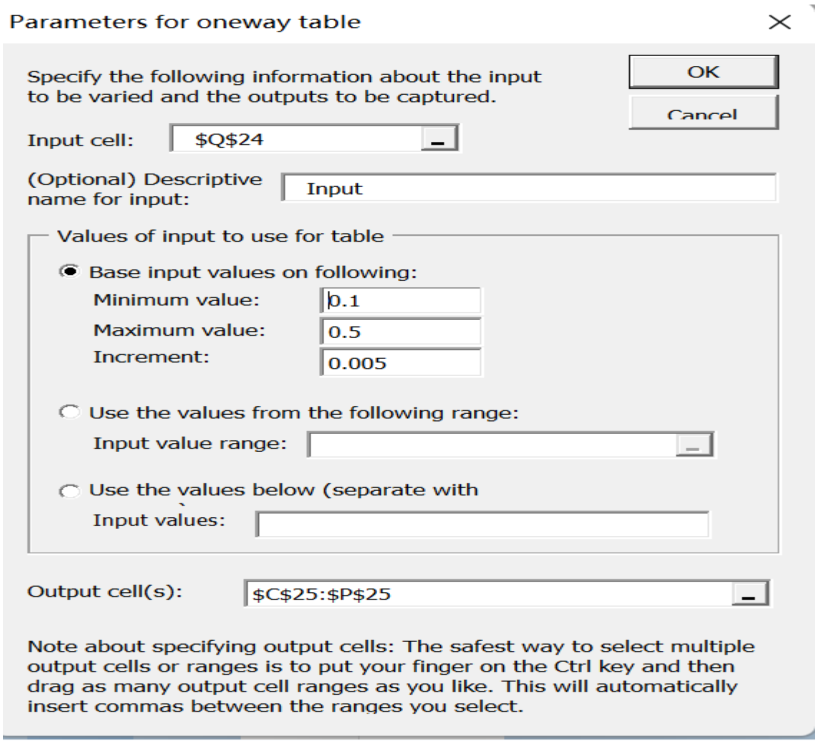

Figure 8: Solvertable.

The setup for constr2 is very similar to the setup for constr1, but we need to add a new constraint to make SPX weight equal to 0. However, after we ran the SolverTable with this setup, the results did not converge well, so we changed our Dummy Variable's value and reran the SolverTable to produce the well convergent results after this change. The next step is to implement the IM for constr1 and constr2. The target function and the weights for constr2 need to be changed into IM's, and the constraint also needs to be changed into IM's. Moreover, constr2 is the same as cosnstr1; one just needs to change its data into IM's data. We are almost finished. What remains is to make two charts of frontiers the Markowitz model and the Index model. We use points and lines to show our data and distinguish different frontiers. Finally, we added a new part called Monte-Carlo, for which we used about 50,000 randomly generated instrument weights using an Excel function similar to (RAND()-0.5)*CH1. Moreover, we have also used the Excel formula SUMPRODUCT() for the Index model.

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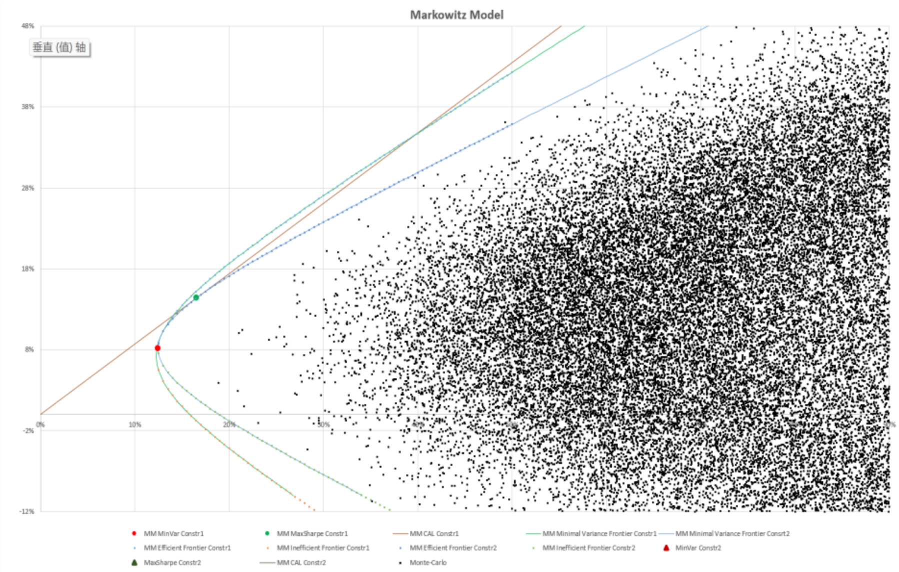

Figure 9: Markowitz model's results.

As a result of calculating the models, we found that IM and MM results have different limits, which obviously indicates differences between their results. The meaning of the Markowitz Model is that although the individual asset's risk is entirely determined by its standard deviation, the portfolio risk, in addition to being determined by the individual assets' standard deviations, is also determined by the asset's cross-correlations. According to the Markowitz Model, the best investment portfolios lie on the Efficient Frontier in the two-dimensional space of risk and return. The Efficient Frontier of the investment opportunity set in the two-dimensional space of "mean-standard deviation" is given below [1]. The Index Model was proposed as a simplification of the Markowitz Model, which is supposed to produce results close enough to the Markowitz Model, and it remedies several significant drawbacks of the Markowitz Model, such as a too large number of estimates required for its calculation for large portfolios [2]. Moreover, the Index Model has found widespread use for practical portfolio optimization in all countries. However, this is only at the theoretical level; for our specific comparisons, we put the two results in a graph together to compare our findings:

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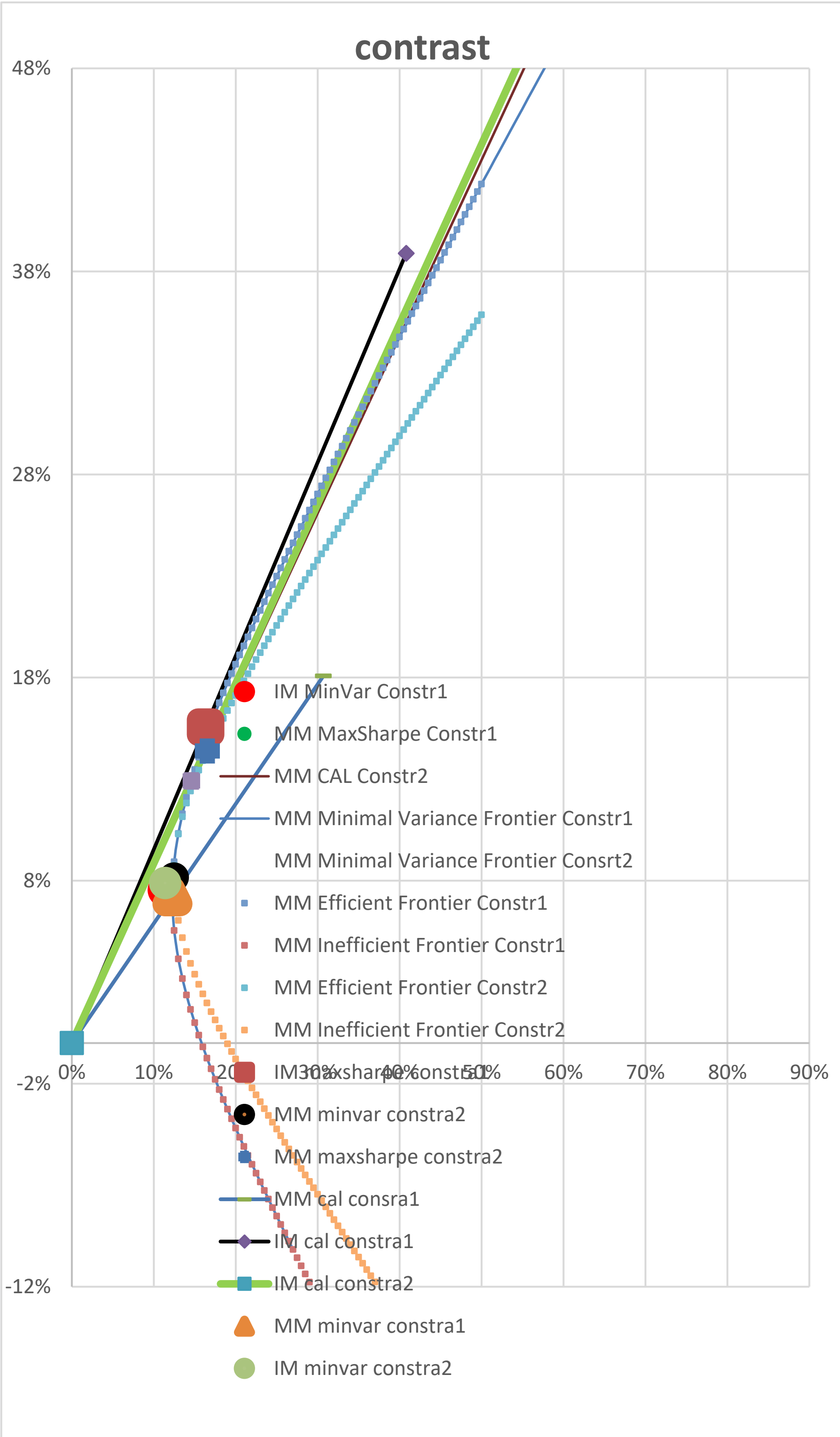

Figure 10: The result.

As one can see from this chart, whether we are trying to resolve the global minvar or maxsharpe points, with constr1 or constr2, the Index Model's performance is better than Markowitz Model's. Moreover, if we compare the CALs, we can see that the Index Model's CAL, whether with constr1 or with constr2, has a higher slope than that for the Markowitz Model's capital allocation lines. Therefore, we need to add the riskless asset to the mix to form a complete portfolio.

Let us denote the risk-free rate of return as RF, and since its risk (standard deviation) is zero, the riskless asset corresponds to a point located on the vertical axis. It can be shown that point Y corresponds to the maximal utility, where the riskless asset F is combined with the optimal risky portfolio P laying on the Efficient Frontier located between P and F [3]. The CAL line describes all possible allocations between the optimal risky portfolio and the riskless asset [3]. The slope of the CAL line is the Sharpe Ratio, which means that the portfolio risk premium is measured in terms of the risk such portfolio is bearing. The CAL line reflects the linear relationship between expected return and risk [1][3]. The higher the slope (or the higher the Sharpe Ratio is), the higher the expected return for a given level of risk, or the better the investment is [4]. These two results show that Index Model is doing better than Markowitz Model in our specific case. In addition to the Markowitz model and the Index model, various similar in-spirit models involve more factors than just one – the broad index – multi-factor models that are also widely used in practice. However, these two models are the basis of the portfolio optimization theory and still have much practical use.

4. Conclusion

In 1952, Markowitz, an American economist, wrote in his seminal academic paper "Portfolio Selection“: "In Effective Diversification, two quantitative indexes, mean and variance of portfolio returns, are applied for the first time to define investors' preferences mathematically, explain the principle of investment diversification mathematically, and systematically elaborate the problem of portfolio and selection," which marks the beginning of the Modern Portfolio Theory. The theory is that combining individual securities into portfolios reduces unsystematic risk (https://baike.baidu.com/item/). The theory of, Markowitz is instrumental in helping investors make investment choices. In Markowitz's time, people bought stocks based on broker recommendations or other non-systematic reasons. Markowitz uses variance (or standard deviation) as the only risk measure to help investors make more rational choices. Before Markowitz, there was some recognition that diversification could reduce investment risk， but it has not been systematized in theory。

Every model has its area of applicability and assumptions, is not perfect, and the Markowitz model is not an exception – it has some drawbacks. It is well known that in the Markowitz model, we need to calculate the variance and covariance of all the stocks, so if we have multiple stocks, the process of estimation of the model calculation may be prohibitively extensive to the extent that the model cannot be used. For example, even in our case of the 11-asset portfolio, we had to do many calculations before we ultimately got the result. It is a long process that quickly gets ever more complicated for more extensive portfolios, so some people do not want to use the Markowitz model. One another drawback of the Markowitz model is that the standard deviation does not reflect all the risks well. For example, if we take two stocks or securities and calculate their standard variances, and they turn out to be the same in the Markowitz world, their risk is the same. However, one can quickly come up with an example where one of these stocks is a riskier investment than the other, even though they have precisely the same standard deviations. One company may be actually near default because it owes too much debt, which it cannot pay the interest on. Such investments may have a slight standard variance but very high actual risk. However, the advantage of the Markowitz model is straightforward: if your estimation is accurate, then your Markowitz model is more accurate than the other models. The Markowitz model is relatively straightforward. If you have calculated all the required estimates, you run the model, and you obtain the allocations into each

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stock, and you can judge if the proposed portfolio is meaningful. I can bring a good example from my personal life illustrating this.

My brother was ready to invest, but he had difficulty choosing among several stocks. He used this Markowitz method to calculate the variance and standard deviation of the stocks. Although he had to spend some time estimating the model, the results it produced were beneficial to him, allowing him to correctly select the correct stocks and amounts to invest in. Not all investors know how to use the Markowitz model to invest in stocks. In reality, we can say that the Markowitz model has already played a role in China's stock market. It has helped some investors analyze investment returns and risks and promoted the development of the stock market. However, we should not rely on it entirely in the applications and let it just be one of the tools that assist us in rationalizing our investments. In summary, the Markowitz model quantified risk for the first time and greatly contributed to portfolio management and risk management.

After the Markowitz model was introduced and its drawbacks and limitations were understood, the Index model was introduced. The advantages and disadvantages of the Index model from an investment perspective, as well as its practical application, will be attempted to analyze in this work. The single Index model has an obvious advantage when we come to investing. Its application leads to a significant reduction in the number of estimates needed for portfolio optimization, which makes optimizing large portfolios possible. When we use a single Index model, it is simply not as computationally intensive. This is an obvious advantage. The above analysis shows that the Index model divides the risk premium of individual securities into the market and non-market parts, which significantly simplifies the investment analysis. The simplification of the Index model is significant for analysts to do securities analysis. It is challenging for security analysts to estimate security covariances across industries. In our view, the Index model gives an easier way to calculate the covariance because the covariance of the security is derived from the influence of the same market index. Of course, everything has its advantages and disadvantages. Let us analyze the disadvantages of this model. When applying the Index model to investment analysis, this model has the limitation of asset return uncertainty structure. For example, we did not consider industry factors in our investment analysis. For example, they will affect some industries linked to world events and politics. In reality, the Index model has been used extensively in hedge fund portfolios to help investors maximize returns and minimize risks; it helps investors correctly grasp the direction of obtaining better returns or reducing risk. After the above analysis, we can conclude this. The Markowitz model and the Index model have a strong connection; these two models have their characteristics, advantages, and disadvantages. The Markowitz model is challenging to calculate, and all the stocks' variance and covariance need to be calculated. This disadvantage makes the Markowitz model not widely used in practical applications. The advantage of the Markowitz model is that it is straightforward, and if you get your estimates right, it is pretty accurate. Compared with the Index model, the Markowitz model does not ignore some seemingly unimportant factors; the Markowitz model can give better results than the Index model. Compared with the Markowitz model, the Index model requires less computation under the same conditions; it can be used more in actual applications for large portfolios. The results from the Index model may not be as accurate as those from the Markowitz model. However, it is much easier to calculate, and the Markowitz model gave accurate results, but a large amount of data caused estimation errors that may offset the accuracy benefit. Both models are good models with their advantages. In daily investment, other methods can also be integrated for investment analysis.

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