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Research on the Financial Model Selection Between Capital Asset Pricing Model, Arbitrage Pricing Model, and Fama-French Model

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Abstract: Financial modeling magnifies the importance of various systematic and unsystematic risks occurring in modern finance, which elicits the functionality of the Capital Asset Pricing Model (CAPM) introduced by Sharpe, Lintner, and Treynor. A considerable amount of societal support for CAPM has been triggered by its simplicity and precision in terms of making decisions for assets based on absolute risk rather than total risk. Objectively speaking, CAPM is also deeply limited by its unique prerequisites or assumptions. One pivotal point for investors to ponder is the selection of different methods and judge the individual applicability before implementation and this article aims to offer investors some financial advice derived from this process. Methods applied in this article involve information-gathering from empirical examples and tests, as well as financial analysis by comparison and selection among several models. This paper concludes that the Arbitrage Pricing Model (APM) and Fama French factor(s) Model (FFM) are differentiated and should be applied selectively and accordingly to cater to the imminent situations that investors are about to tackle.

Keywords: financial markets, Capital Asset Pricing Model, Arbitrage Pricing Model, model analysis, Fama-French Model

1 Introduction

The biggest advantage of CAPM is its conciseness, which categorizes risk securities into three elements: risk-free return, price under risk, and the mathematical unit of risk calculation, and then CAPM combined these three elements together to refine its practicability. Limitations of CAPM galvanize the problem-addressing procedure in this article. Obvious limitations are its unrealistic assumptions and the complexion of determining beta value. While some alternative methods and theories, like APT, ICAPM, and FFM, emerge with the development of the financial market, investors are obligated to consider the applicability of CAPM before using others as it is the absolute basis of financial modeling. Recent studies have explored several aspects regarding the feasibility of CAPM, scholars are still struggling to choose the best model to match the markets' volatility, despite the fact that it's relatively complicated. Moreover, different interpretations and applications of these models have been discussed and debated over several decades. This article delves more into the selection

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processes of APT and FFM and their respective relationships to CAPM, helping investors make better decisions when involving themselves in different scenarios. Besides, this article concretely examines the analysis of factors in different models and development of these model as well as how these factors affect the financial regression or overall predictability. With the usage of secondary data analysis and literature analysis, this article empowers investors to comparatively precisely choose models based on their needs and give certain insights to corresponding financial markets they encounter.

2 APT and FFM Analysis

CAPM is universally recognized as a derivation of Markowitz's modern portfolio theory, which informs investors on the best way to predict the investment portfolio, and CAPM attributes the return on assets to the risk-free return rate and market return [1]. (Return = Risk-free rate + Beta (Market Return – Risk-free rate). The alternate definition of beta is the covariance of a return and a market return divided by the variance of the market return, and this usually stands for volatility. Thus it can be thought of as the market portfolio return rate. Security Market Line (SML) represents the typical core of CAPM, in other words, the way that CAPM manifests its functionality is SML [2]. SML displays a line that uses the risk-free rate as the intercept, expected return as the vertical axis, beta as the horizontal axis, and market risk premium as the slope. Every asset or portfolio when the market is in equilibrium corresponds to a point on this line, meaning the market price is equal to the expected value. Also, differentiated situations are designated clearly by the line: points above demonstrate undervalued stocks while points below demonstrate overvalued stocks. It is noted that SML may be used with individual assets and specifically only takes into account the systematic risk of an investment as assessed by the beta.

Different from CAPM, the APT model focuses on a more combined effect rather than a singular one, which demonstrates that a linear mix of several factors determines the predicted return on any hazardous asset. The model also says that an asset's riskiness is closely correlated with its sensitivity to certain elements since the components are changed in accordance with the requirements of the intended financial market.

$$R=\alpha+\beta'f+\varepsilon \tag{1}$$

$$E[\varepsilon|f]=0 \tag{2}$$

$$E[\varepsilon\varepsilon]'=\Sigma \tag{3}$$

The above are three equations of asset return proposed by Ross, which are all about the equations of matrices and vectors, and Σ stands for a diagonal square matrix [3]. One thing to note is that the R in the first equation does not involve an expectation mark, meaning unsystematic error cannot be explicated by the fluctuation of asset return. The second and the third equation indicate that unsystematic risk is independent of systematic risk and unsystematic risks will not affect each other respectively [3]. As a multi-factor model, APT does not provide an arbitrary conclusion of what these factors are designated to be, so investors must analytically determine which factors can be taken into account for affecting the asset's returns [4]. Different elements from business cycles, such as changes in interest rates, inflation rates, and oil prices, will have an impact on stock returns [5].

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Furthermore, compared to APT, investors might tend to select CAPM more often, as APT strictly requires investors to quantify multiple factors before using it, though it theoretically holds fewer assumptions.

A specific extension of the Capital Asset Pricing Model is the Fama-French three-factor model that includes two additional factors besides the market factor. Size of the company, which is measured by the market capitalization of the firm, and the value of the company, which is measured by the book-to-market ratio are added to the original model [1]. The formula is shown below, where Bm is the beta of the asset with respect to the market, Bs is the size factor sensitivity of the asset, and Bv is the value factor sensitivity of the asset.

$$Ri=Rf+Bm*RPm+Bs*RPs+Bv*RPv \quad (4)$$

The market risk premium (RPm) is the [expected return on the market - the risk-free rate], and is obtained by the conditions in the financial market and investor expectations. The size risk premium (RPs) is the excess return on small-cap stocks over large-cap stocks, and is intended to capture the premium that investors typically demand for holding small-cap stocks due to their higher risk and lower liquidity [5]. The value risk premium (RPv) is the excess return on value stocks over growth stocks, and is intended to capture the premium that investors typically demand for holding value stocks due to their lower valuations and higher risk. The formula can be easily derived from the basics of CAPM, since it only adds size risk premium and the value risk premium into its original version [5].

After closer examination about the purposes of Rolls and Ross, the fundamental objective was not to reinforce the importance of “Multi-factor”, but refining the assumptions proposed by Sharpe in CAPM [3]. As the CAPM was established under a strong macroeconomic world and such assumptions are impossible to be satisfied holistically, which lead to its limitations. Ross states that the corresponding statistical stipulation (equation 1 in APT) can proceed to approximately obtain a basic beta pricing equation from “no arbitrage condition” [3]. We intuitively can notice that diversification enables us to overlook the specialized risk (ε), and investors are only exposed to systematic risks (f), thus investors who hold relevant assets should be compensated. The procedure explained above will ultimately leads to a relationship between beta and expected return, and pricing factor can be seen as a statistical factor, and that logistic determines the basis of co-movement of financial assets. Apart from this, Sharpe and Ross both proposed the requirement of market equilibrium, the major defect that occurred in their researches are the ambiguity of pricing factors.

Fama-French Model, in contrast, provides a more concrete but fixed factorized formula for investors to choose, and the reason behind this should also take “time” factor into account. At the age of 80s, universal recognition of CAPM can be separated as beta (relevant) and alpha (irrelevant) [6]; at the age of 90s, the market development started behaving rampantly for different fields and more factors were added; after 2000, the emergence of style factor and strategy factor make investors realize that the original alpha was majorly composed of beta, but in a way that opposes the tradition.

From the graph below, which was made by Liu, we can see that from 1962 to 2012, the number of factors increased exponentially. Based on reasonable estimation, the number of factors will be more than 500 in ten years [7].

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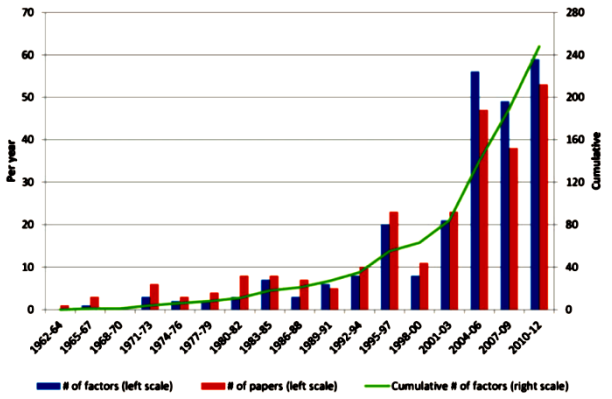

Figure 1: Cross section of expected returns.

Model selection from these three models can be regarded as "the factors' selection", indicating that the combinations of factors might generate certain "chemistry". That is also the question of 1+1=2 or 1+1<2 or 1+1>2.

3 Comparison Analysis

From the reasoning above, it is clear to conclude that FFM is a multi-factor model, but APT tends to be more "statistical" other than "financial" as it only becomes a financial form (factor pricing model) under certain circumstances. Here we can derive the functionality of FFM from the comparison between APT and CAPM [8].

In actuality, cross-sectional pricing is the basis of pricing model. Chen proposed an experiment regarding the validity of APT based on cross-sectional pricing perspective with the data gathered from S&P 500 index for different time periods [9].

We can generate a null hypothesis around the APT equation from Chen's analysis: $ri=\lambda 0+\lambda 1\beta i1+\ldots+\lambda k\beta ik+\varepsilon$, and CAPM equation: $ri=\lambda 0+\lambda 1\beta I+\eta i$. That is, null is $\lambda 1=\lambda 2=\lambda 3=\lambda 4=\lambda 5=0$(the expected return of all assets are the same and equal to $\lambda 0$). Certainly, the alternative hypothesis would be at least one of the $\lambda$is is not equal to zero which indicate difference across groups are discernable and the validity of APT. Despite the comparative low power of the test when all five factors are included, the F statistic is significant at the level of $\alpha = 0.1$ for every period [10]. And we are confident to reject the null hypothesis at the level of 0.1 and conclude that the expected return of different assets are not constant all the time, and that means APT is a reasonable model for explaining cross-sectional variation in asset returns.

Table 1: Comparing CAPM and APT in the Chinese stock market.

Returns Boards	Mean True in Returns	Average Mean Forecast In Returns		Average Mean Difference	
		CAPM	APT	CAPM	APT
SME Board	0.003940025	0.00201	-0.000399305	0.00209855	-0.037802138
SME Board	0.006313103	0.003295	0.00239149	-0.0030181	-0.003921613

Although the APT model explains the variation well in some modern market, some evidence from SME and ChiNext board in Chinese stock market distends the conclusion we drew above (SME board stands for Small and Medium Enterprise board of the Shenzhen Stock Exchange and ChiNext stands for largest and most liquid A-share stocks of Shenzhen Stock Exchange). For the samples from the

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SME Board, the deviation based on the CAPM is 0.00209855, which is lower than the deviation based on the APT model, which is 0.0378021375. This comparison shows that the CAPM is more accurate at forecasting for the samples used by the SME Board [6]. That means, APT is not suitable in every situation in this type of stock. The deviation based on the CAPM is 0.0030181 for the samples from the ChiNext Board, and the deviation based on the APT model is 0.0039216125. As a result, the APT model's deviation is higher than the CAPM model's deviation [6]. From this comparison, we cannot say that the APT model is superior to the CAPM in all the cases. It is noted that absolute values are utilized here since the accuracy is determined by the "deviation" from the true in returns. However, the results may vary for different scenarios. Although APT is more suitable for regression analysis because of its large acceptance of regressors beta, some problems like over-fitting might also tend to balance between flexibility and accuracy of prediction. This would potentially lead to confusion in future financial analysis, and this requires further investigation.

3.1 Comparison Between APT and CAPM

When the asset has a non-linear risk-return relationship: The CAPM assumes a linear relationship between risk and return, but in reality, the relationship may not be linear. In these cases, the APT may be more appropriate, as it allows for the consideration of multiple factors and does not assume a linear relationship between risk and return [8].

When the asset is part of a concentrated portfolio: The CAPM assumes that investors hold a well-diversified portfolio, but in reality, many investors hold concentrated portfolios. In these cases, the APT may be more appropriate, as it allows for the consideration of multiple factors and may be better suited for evaluating the risk and return of a concentrated portfolio.

3.2 Comparison Between CAPM and FFM

Suppose the investors are considering investing in a small-cap company that operates in a highly cyclical industry. The company's financial performance is affected by the overall market conditions, the performance of the industry, and the company's size (measured by the market capitalization).

In this case, the Fama-French three-factor model may be a more appropriate model to use, as it allows for the consideration of multiple factors that may affect the required return on the investment. The specific factors and risk premiums could be determined based on the characteristics of the small-cap company, such as the market risk, the size risk, and the industry risk. However, if the investment goes to a large-cap company that operates in a mature industry with stable cash flows, CAPM may be a more appropriate model to use, as the company's financial performance is primarily driven by the market risk. The required return on the investment can be estimated by using the company's beta (a measure of the volatility of the company's stock relative to the market) and the expected market risk premium.

4 Conclusion

In conclusion, there are three factors that investors should be mindful of when deciding which model to use between the Capital Asset Pricing Model (CAPM), the Arbitrage Pricing Theory (APT), and the Fama-French three-factor model. The first issue that investors should consider is the data availability. The CAPM and the Fama-French three-factor model require historical data on the returns of the asset and the relevant benchmark or factor portfolio. If investors do not have sufficient data, they may not be able to use these models. The APT is more flexible and does not require as much data, but it may still be helpful to have some data to validate the model. The investment horizon should also be considered. If investors have long-term investment horizons, they may want to consider using a model that accounts for multiple risk factors, such as the APT or the Fama-French

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three-factor model. These models may be more appropriate for evaluating the long-term risk and return of an asset. Finally, investment goals are diverse and should also contribute to deciding which model to choose. If investors are evaluating an asset with specific risk factors that are not captured by the market, or if they have a concentrated portfolio, they may want to consider using a model that allows for the consideration of multiple factors, such as the APT or the Fama-French three-factor model.

From this article, investors might sense that CAPM is the basis of the subsequent model and its alternatives help investors to find several ways to examine the market. Moreover, investors should keep in mind that APT and FFM are both used to evaluate the relationship between a security's returns, and the one notable difference is that APT capture a set of macroeconomic variables while FFM captures characteristics of the security. To quantify the difference between these models, we will need more investigations in the future, and market volatility somehow confines a deeper interpretation of these models in this article.

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