Application of Two-sided Matching Theory in Financing of Small and Medium-sized Enterprises

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Abstract: The nation's small and medium-sized businesses (also known as SMEs) constitute an essential component of the economy as a whole; yet, many companies are right now struggling to obtain adequate funding. On the one hand, increasing the amount of capital invested in venture firms by venture capitalists can considerably promote the development of venture enterprises. On the other hand, venture capitalists look for businesses in which they may invest, with the goal of achieving high levels of success and gaining substantial profits. Therefore, in order to make a reasonable and stable match between venture capitalists and SMEs according to their respective preference lists, this paper makes use of the Gale-Shapley algorithm, which is part of the two-sided matching theory. This helps improve cooperation efficiency and maximizes the benefits for both parties. In addition to that, based on the algorithm, this article offers some policy recommendations for the venture capital market in China.

Keywords: SME, venture capitalist, two-sided matching, market design

1. Introduction

SMEs are crucial to China's economy. SME flexibility, adaptability, and a wide variety of operations improve the market economy, advance technology, and boost international trade [1]. Despite their substantial contribution, SMEs have limited financial resources. SME finance is constrained in China due to the inefficient financing infrastructure, lacking related laws, and the high risk of SMEs and their immature business system [2]. Due to risk aversion, banks are wary of lending to SMEs when credit demand is high. Hence, SME financing is a pressing issue because traditional techniques are difficult to suit SME needs and protect banks. Venture money is needed because SME development is high-risk, and high reward.

Venture capital (VC) is risk money for high-return investments. China's macroeconomic performance has been good amid global economic recovery, with new industries growing, economic restructuring accelerating, and more people joining the VC market. Consequently, the rising VC market can assist Businesses to overcome financial challenges.

Several research has examined venture capitalist-venture enterprise matching. This paper will focus on SMEs and the Gale-Shapley algorithm to match SMEs and venture investors. The appropriate matching of venture capitalists and SMEs can help both sides obtain satisfactory results,

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which improves venture capital activity success and is crucial to the long-term cooperation and development of venture capitalists and venture firms.

2. The Gale-Shapley Algorithm in Two-sided Matching Theory

The two-sided matching theory focuses on the difficulty of matching the preferences of two entirely non-overlapping subjects. Matching between public schools and students, personnel and job matching in the human resource management field, marriage matching, and complex task-matching services in the manufacturing field are representative examples of the widespread application of research on matching models and solution algorithms in various fields.

The market for venture capital is a classic two-sided market (Figure 1). In the real investment process, the preferences of the venture investor and the SME will determine which parties are selected to collaborate. The venture capitalist will make the best choice based on the strength of the applicant companies and other criteria, while the SME must consider the actual situation of the venture capitalists when deciding on financing. In such a bilateral market, the two-sided matching model can accommodate both parties' requirements.

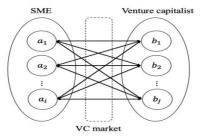


Figure 1: Two-sided matching decision-making problem between SMEs and venture capitalists [3].

The Gale-Shapley algorithm used in this paper is a classical algorithm in the two-sided matching theory, a market mechanism devised by Gale and Shapley to find a stable match. The objects a_i on one side of the market, i=1,2,...,m, send an offer to objects b_j , j=1,2,...,n, on the other side according to their preferences. Each b_j compares the offers received, keeps the best ones, and rejects the others. The a_i whose invitation is rejected continues to send new invitations to other b_j until no a_i wants to send another invitation. At this point, each b_j finally accepts its own reserved offer. A key aspect of the algorithm is that the consensual invitations are not accepted immediately, but are only temporarily reserved from rejection, i.e., the "Deferred Acceptance (DA) algorithm ". In this paper, the algorithm is mainly used for one-to-many matching [4].

3. The Application of Two-sided Matching Theory in the Venture Capital Market

In the two-sided matching of the VC market, the SME is a_i , i = 1,2,...,m, the venture capitalist is b_i , j = 1,2,...,n. Each b_i can invest q_i .

3.1. Establishing the Matching Evaluation Index Systems of Both Sides

This study refines the respective preferences and creates a matching evaluation index system for mutual evaluation between venture capitalists and SMEs in order to conform to the actual investment environment and increase the matching rate. Let S_k , k=1,2,...,8 be the evaluation index of the venture capitalist to the SME and V_x , x=1,2,...,5 be the evaluation index of the SME to the venture capitalist. The index systems of both sides are showed in Table 1 and Table 2.

Table 1: Matching evaluation index system of venture capitalists to SMEs [5].

Indicator	Description
Payback period (S ₁)	The time elapsed between the start-up of the SME's
	anticipated investment project and the date when the
	cumulative total reaches the total investment
Annual Return on Investment (S ₂)	The ratio of annual profit to total investment of the SME
Technology level (S ₃)	The novelty and advancement of the technology (or product)
Capability of risk-averse (S ₄)	The ability of SME's to hedge technology (or product) development risks and marketing risks
Accessibility of the market (S_5)	Technical barriers and trade barriers in the target market
Entrepreneurship (S ₆)	SME entrepreneur's sense of responsibility, innovation,
	leadership and organization
Investment Environment (S ₇)	The economic development level and investment
	environment of the surrounding area where the SME is
	located
Tax Benefits (S ₈)	Preferential policies provided by the government

Table 2: Matching evaluation index system of SMEs to venture capitalists [5].

Indicator	Description
Investment amount (V ₁)	Amount of investment that venture capitalist can offer to each
	SME
Investment Strength (V ₂)	The total existing capital size of venture capitalist, the number
	of companies it has invested in and the size of its investments
Investment Success Rate (V ₃)	Success rate of SMEs in which investors have invested
Prestige (V ₄)	Credit evaluation of the venture capitalist investing in other
	SMEs
Entrepreneurship (V ₅)	Venture capitalist 's sense of responsibility, innovation,
	leadership and organization

Both the venture capitalist and the SME assign weights to each indicator in their own evaluation system, with the venture capitalist and SME's weight being w. Lastly, for each indicator, the opposing side is ranked and substituted into the weight calculation method (1) & (2) to determine the respective ultimate preference.

For SME a_i , the final score α_{ij} is obtained by a_i after being ranked by venture capitalist b_j in its evaluation system is

$$\alpha_{ij} = \sum_{k=1}^{8} w_j(S_k) r_{ij}(S_k), \sum_{k=1}^{8} w_j(S_k) = 1$$
 (1)

where $r_{ij}(S_k)$ represents a_i 's ranking in each of the evaluation index S_k of venture capitalist b_j . The smaller the α_{ij} , the more popular a_i is with venture capitalist b_j .

For venture capitalist b_j , the final score β_{ji} is obtained by b_j after being ranked by SME a_i , in its evaluation system is

$$\beta_{ji} = \sum_{x=1}^{5} w_i(V_x) r_{ji}(V_x), \sum_{x=1}^{5} w_i(V_x) = 1$$
 (2)

where $r_{ji}(V_x)$ represents b_j 's ranking in each of the evaluation index V_x of SME a_i . The smaller the β_{ji} , the more popular b_j is with SME a_i .

3.2. Matching Process

Step 1: After each side ranks the other side according to their respective evaluation index system to form a preference list, SME a_i applies to its favorite venture capitalist. For each venture capitalist b_j , up to q_j applicants who have the highest j – priority is tentatively assigned to venture capitalist b_j . The remaining applicants are rejected.

Step k, $k \ge 2$: Each SME rejected from a venture capitalist at step k-1 applies to its next favorite b_j . For each venture capitalist b_j , up to q_j students who have the highest j-priority among the new applicants and those tentatively on hold from an earlier step, are tentatively assigned to b_j . The remaining applicants are rejected [6].

Once no company's application is rejected, the solicitation process concludes. Remember that all DA allocations are temporary till the final phase. Owing to the limited number of small and medium-sized enterprises (SMEs) and venture investors, this process must end after a limited number of phases. All remaining SMBs on the pre-recorded list then enter into venture capital agreements. Under such a two-sided matching model, the Gale-Shapley algorithm can achieve the stable pairing of SMEs.

4. Practical Application of the Model

Supposing that SMEs are $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$, and the venture capitalists are $B = \{b_1, b_2, b_3\}$, and $q_1 = 2, q_2 = 1, q_3 = 2$.

The results of assigning weights w_j and w_i to their own evaluation index system by both sides are in Table 3 and Table 4.

$w_j(S_k)$	S_1	S_2	S_3	S_4	S ₅	S ₆	S ₇	S ₈
b_1	0.19	0.20	0.1	0.25	0.06	0.05	0.067	0.096
b_2	0.12	0.10	0.13	0.125	0.113	0.148	0.125	0.15
b ₃	0.075	0.125	0.20	0.125	0.14	0.10	0.11	0.125

Table 3: Weight of each index set by venture capitalists.

Table 4: Weight of each index set by SMEs.

$w_i(V_x)$	V_1	V_2	V_3	V_4	V_5
a_1	0.2	0.35	0.23	0.1	0.12
a_2	0.15	0.3	0.2	0.2	0.15
a_3	0.1	0.4	0.3	0.15	0.05
a_4	0.3	0.2	0.25	0.15	0.1
a_5	0.33	0.2	0.12	0.2	0.15
a_6	0.2	0.2	0.1	0.3	0.2

After both sides ranked the other side according to each index, the final scores were calculated by substituting (1) & (2), and then the scores were ranked from lowest to highest; the lower the score, the higher the ranking, which indicates that the SME or venture capitalist is more popular; the results are presented in Tables 5 and 6.

Table 5: The preference lists of venture capitalists.

Rank for a _i	a ₁	a ₂	a_3	a ₄	a ₅	$\overline{a_6}$
b_1	1	2	3	6	4	5
b_2	3	1	6	4	5	2
b_3	5	3	1	2	4	6

Table 6: The preference lists of SMEs.

Rank for b _j	b_1	b_2	b_3
a_1	1	3	2
a_2	2	3	1
a_3	1	3	2
a_4	2	1	3
a_5	3	2	1
a ₆	3	1	2

Based on Table 6, all SMEs will submit their financing applications to their respective most preferred venture capitalists.

$$a_1, a_3 \to b_1$$

 $a_4, a_6 \to b_2$
 $a_2, a_5 \to b_3$

For b_1 and b_3 , because $q_1 = 2$, $q_3 = 2$, they will be accepting all applications for the time being. However, in Table 5, for b_2 , $q_2 = 1$, and $a_6 >_{b_2} a_4$, so it will only accept a_6 and reject a_4 .

In the second round of application, rejected a_4 submits financing application to second favorite venture capitalist b_1 . For b_1 , $a_1 \succ_{b_1} a_3 \succ_{b_1} a_4$, and $q_1 = 2$. So, it will also reject a_4 .

In the third round of application, rejected a_4 submits financing application to the least favorite venture capitalist b_3 . For b_3 , $a_4 >_{b_3} a_2 >_{b_3} a_5$, and $q_3 = 2$. Then b_3 will reject a_5 and remain a_2 and a_4 .

In the fourth round of application, rejected a_5 submits financing application to its second favorite venture capitalist b_2 . For b_2 , $a_6 >_{b_2} a_5$, and $q_2 = 1$. Then b_2 will reject a_5 and keep a_6 .

In the fifth round of application, rejected a_5 submits financing application to its least favorite venture capitalist b_1 . For b_1 , $a_1 \succ_{b_1} a_3 \succ_{b_1} a_5$, and $q_1 = 2$. So, it will also reject a_5 .

At this time, except for a_5 , all other SMEs have found suitable venture capitalists, and each venture capitalist has no vacant seat for a_5 , so the final result is as follows:

$$a_1, a_3 \rightarrow b_1$$

 $a_6 \rightarrow b_2$
 $a_2, a_4 \rightarrow b_3$

According to the results, both parties match according to their preference lists, and the matching procedure optimizes utility. An agent or pair does not obstruct the final match, therefore it is a stable and Pareto-efficient match, resolving the problem of SME financing challenges. Simultaneously, it is strategy-proof for SMEs, protects the fairness of the investment market, raises awareness of the legitimacy of SMEs' operations, and enhances their reputation. SME-optimal stable matching respects the right of invested firms to choose and has, to some extent, increased their excitement for R&D and output, so fostering economic growth.

5. The Applicability of the Stable Matching Theory in the SME Financing Problem and Recommendations on the Venture Capital Market

5.1. Establishing a Strict Evaluation Index System

Due to the enormous number of small and medium-sized firms in China and their distribution across numerous industries, their financing requirements are bound to alter, thus they frequently employ multidimensional and hierarchical classification [7]. This indicates that the evaluation indicators of both parties are robust enough for the matching theory to be fully utilised. There are disparities in investor expectations of SMEs, from the perspective of investors. Diverse venture capitalists evaluate SME prospects according on their risk tolerance, investment horizon, and rate of return. From the perspective of small and medium-sized enterprises (SMEs), since they are in a critical period of development, the choice of venture capitalist is crucial, as it can influence the enterprise's future development to some extent [8].

5.2. Promoting Industry-finance Cooperation

The main causes of the financing difficulties of small and medium-sized companies in our country are the unbalanced allocation and low efficiency of the financial market, as well as the excessive concentration of loan funds in key enterprises, which impedes the development of small and medium-sized businesses. In recent years, however, China's Ministry of Industry and Information Technology has consistently directed financial institutions to enhance their support for small and medium-sized businesses.

Thus, the application of matching theory is the main method for resolving the structural mismatch of the financial market, enhancing the efficiency of capital allocation, and fostering the growth of inclusive finance. The matching of the financial market facilitates the financing arrangement between capital-demanding businesses and capital-supplying financial institutions [9]. Using the theory of stable matching, the benefits and characteristics of financial institutions and small and medium-sized businesses are thoroughly analysed and then integrated with the requirements to achieve the best allocation of capital supply and demand.

5.3. Policy Recommendations on the Venture Capital Market

The government should continue to improve the multi-level inclusive financial system, continuously enrich capital supply subjects, vigorously develop the financing market, improve the multi-level financing system between state-owned capital and social capital, and optimise capital matching efficiency.

To overcome the issue of information asymmetries between venture funders and businesses, a centralised matching agency should be established. Establish online matching institutions, create enterprise preference databases, enhance the matching efficiency of capital supply and demand, and prevent the non-standard development of private lending.

6. Conclusion

The fast growth of small and medium-sized enterprises (SMEs) necessitates a substantial amount of financial assistance; but, due to high business risks and poor rules and regulations, they continue to encounter several financing challenges. This paper employs the Gale-Shapley algorithm from the stable matching theory to solve the financing issues of SMEs and conducts one-to-many matching between venture capitalists and SMEs in order to make a rational choice of financing methods, prevent the capital rupture of enterprises, and ensure the health and stability of enterprises. Not only

does the algorithm account for the diverse needs of both parties in the VC market, but it also achieves the ideal match. To promote the future development of SMEs, it is recommended that the government not only raise investment and reform rules and regulations but also encourage the construction of matching platforms and construct multi-level financial markets. Nevertheless, this work does not apply the model to the actual matching on the investment market; there is insufficient data support. In order to increase the model's viability, it is important to implement the algorithm in future studies and then make more improvements.

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