Portfolio Optimization for 5 US Stocks

Yi Shi[1,a,*]

[1] *University of California, Davis, Davis, CA, 95616, USA*
a. syshi@ucdavis.edu
*corresponding author

Abstract: Nowadays, the rose of interest in the US and the changes in the value of the dollar case the public attention of investors on stocks. The US company stock was under a great change in this situation. So, the investors are now focusing on finding the portfolio optimization on the investment allocation. This study directly focuses on the selected five stocks of interest company to find the portfolio optimization using the Monte Carlo simulation and com-pare its return with the actual data. By Monte Carol simulation, the maxi-mum Sharpe ratio and the minimum volatility portfolios would be generat-ed, Netflix is regarded as the largest weight in the maximum Sharpe ratio portfolio, while Sunning is regarded as the largest proportion weight in the minimum volatility portfolio. Besides, after evaluating the results, the max-imum Sharpe ratio and minimum volatility portfolio perform better than the standard 1/N portfolio. Using this portfolio optimization, investors can find how to allocate their investment through several stocks to get the maximum returns with relatively low risk.

Keywords: portfolio optimization, maximum sharpe ratio, minimum volatility, US stock, Monte Carlo simulation

1. Introduction

To help address inflation, the US federal decided to raise its federal fund's bench-mark rate by 25 basis points to 0.50% in March 2022, and the Federal Reserve again decided to raise and control the range of federal funds rate between 0.75% and 1% in early May 2022 [1]. This change makes the US GDP growth rate decrease from 5.9% in 2021 to 2.1% in 2022, which shows a decline in economic growth [2]. Owing to the increase in the interest rate, the stock becomes less attractive to the investors since the value of future returns of the stocks seems less attractive under the high-interest rate. The Rise of interest rates would cause a reduction in bond prices, which would also lead to a reduction of bond funds, and shareholders would lose more benefits [3]. Instead, the investors are more willing to invest in bonds and deposits that are more competitive under the increase of interest rate. So, in order to make the stock market more attractive and competitive, Portfolio optimization should be found to attract investors to enter the market.

The model in this paper used is the most efficient and common Mean-variance model that was conceptualized by Harry Markowitz in 1952 [4]. This model focuses on two parameters, portfolio expected returns, and volatility, to measure returns and risks. The expected returns are calculated by the normal returns of the stock price, and the volatility is calculated by the standard deviation. Then, by stimulating thou-sands of allocations, a diagram with these two parameters could be got, and each

allocation would be a single dot on the diagram. The edge of all sets of the dot with a positive slope can be found as the efficient frontier. The portfolio optimization can be chosen by the allocations on the efficient frontier in different ways. The way this paper used in this study is the maximum sharp ratio which is the allocation on the efficient frontier with the highest sharp ratio.

In previous studies about portfolio allocation, there are serval models used by in-vestors. The maximum Sharpe ratio portfolio is a popular model in the financial area. It could be a really good measurement of the performance for compensating money management under multi-parameter situations and frictionless market set-tings [5-6]. In Mete Emin Atmaca's research, the author used the Sharpe ratio model to find the optimized allocation in the electric market [6]. The maximum Sharpe ratio helps her to find the allocation that performs the best by avoiding certain risks. But the different market has their own special situations, and the investor also has different needs. For the target investor in Mete Emin Atmaca's research, investors need the investment decision of balanced risks and returns. So, a maximum Sharpe ratio portfolio may be the best choice for them. But the risk is not the lowest, and it cannot satisfy the investors that need the lowest risk in investment. The minimum volatility portfolio is widely accepted by financial scholars and used in much related research [7]. Also, low volatility investing allows investors to meaningfully mitigate their risk exposure and still participate in the upside. Whereas minimum volatility portfolios will provide a lower risk exposure relative to the parent index and allow investors to beneficially participate in market gains.[8] So, the minimum volatility portfolio may be a better choice for them because it sacrifices some of the returns to ensure the risk are at the lowest level.

The goal of this study is to find the portfolio optimization of a certain set of stocks and analyze the performance and efficiency of this portfolio optimization in order to provide investment suggestions to investors. The portfolio optimization would consist of 5 stocks in the US stock market. The allocation would be stimulat-ed by the Monte Carlo simulation and determine the weight of each stock under the maximum Sharpe ratio method and minimum volatility method. Using portfolio optimization to predict the returns and compare them with the real return to get the efficiency of the allocation.

2. Data

This study selects five representative stocks in the US stock market, which is Amazon, Alphabet, Salesforce, Suning, and Netflix (See Table 1). The data is collected from https://finance.yahoo.com with the corresponding ticket 'AMZN,' 'GOOG,' 'CRM,' '002024.SZ,' and 'NFLX' of closing prices from January 1st, 2019, to May 21st, 2019, to get 100 sets of data. Then, Separating them into the first 70 sets as a training set and the last 30 sets as a test set. The training set would be used to calculate the average returns and covariance matrices to get the efficient frontier with the Sharpe ratio. The test data would be used to calculate the real average return that will be used to compare with the predicted return by the portfolio optimization. This paper uses the linear return to calculate the date returns and shown below (See Figure 1 and Table 2).

Table 1: Selected stocks.

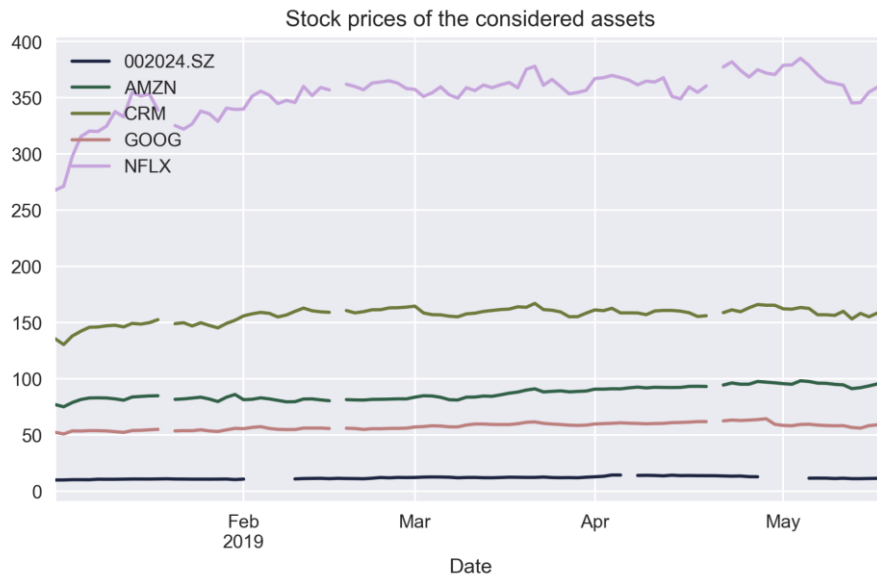| Ticket name	Company
AMZN	Amazon
GOOG	Alphabeta
CRM	Salesforce
002024.SZ	Sunning
NFLX	Netflix

Figure 1: Stock price.

Table 2: Descriptive statistics of the daily return of the five stocks.

	Average returns	Cumulative returns	Minimum	Maximum	Standard deviation
002024.SZ	0.002898	0.240533	0.089139	-0.043199	0.023203
AMZN	0.002417	0.219989	0.050064	-0.053819	0.016943
CRM	0.001552	0.141271	0.057976	-0.042453	0.017396
GOOG	0.001414	0.128697	0.053786	-0.076966	0.016202
NFLX	0.003145	0.286178	0.097234	-0.044907	0.022845

3. Method

3.1. Expected return and Variance

The details for expected return and variance are shown below.

$$\mu_p = x^T \mu \tag{1}$$

Where μ_p indicates the return of the portfolio, μ indicates the return of selected asset a, and x is the weights of assets.

$$\sigma_p^2 = x^T \omega x \tag{2}$$

Where σ_p indicates the standard deviation of returns of the portfolio, and ω indicates the covariance matrix of the selected assets.

3.2. The Monte Carlo Simulation

It is basically a computational technique used to model the probability distribution of various outcomes in a complex system with several parameters.[5] In this study, the expected returns and volatility would be the parameter under the stimulation. In the simulation process, it will generate different weights of each stock to construct different allocations and get the expected returns and volatility of each allocation. Thousands of allocations would construct efficient frontiers.

3.3. Sharpe Ratio

The ratio describes the marginal returns you get as the volatility increases.[6] It measures the performance of the investment under certain risks. In this study, one of the portfolios of the allocation with the maximum Sharpe ratio is an optimized portfolio. The allocation that best balances the returns and risk should be considered optimized.

$$SR = \frac{R_a - R_b}{\sigma_a} \tag{3}$$

where R_b is the zero-risk rate, R_a indicates the expected return and σ_a indicates the standard deviation. The Sharpe ratio is affected by the standard deviation, and Portfolios will have a higher Sharpe ratio as the standard deviation decreases. Therefore, the maximum Sharpe ratio portfolio has better risk-adjustment performance. The minimum volatility portfolio helps investors to minimize the risk better.

4. Result

Through the stimulation, by using the Monte Carlo method, this paper gets the scatter plot shown below with the blue edge as the efficient frontier (See Figure 2).
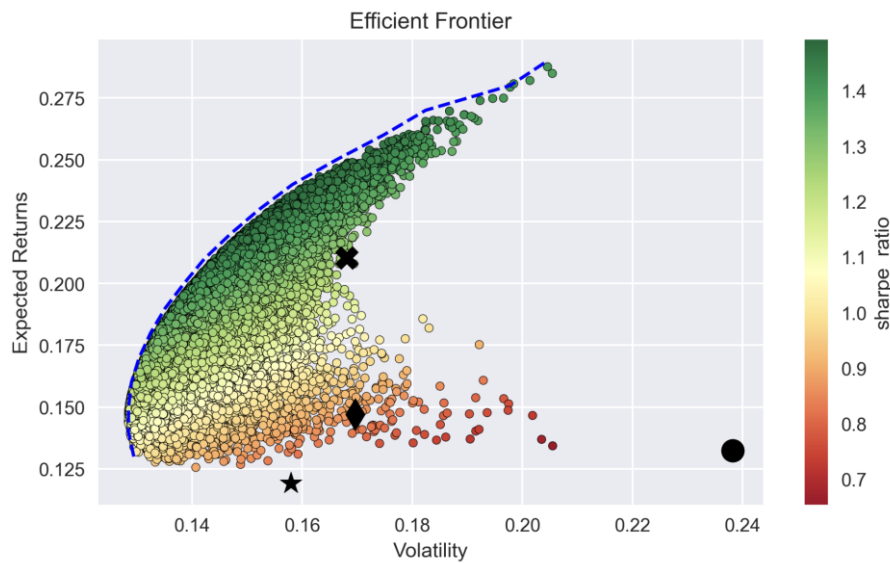

Figure 2: Efficient frontier

Along the efficient frontier, the point that can be found inside the efficient frontier is sub-optimal. In this paper, the point of allocation are chosen with the maximum Sharpe ratio and with minimum volatility (See Figure 3).
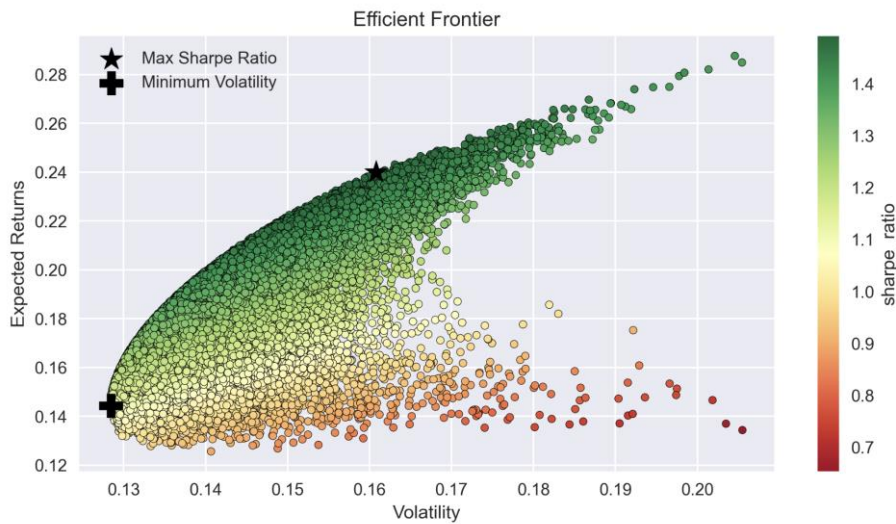
Figure 3: Max sharpe ratio and minimum volatility allocation

In Figure 3, the point of maximum Sharpe ratio and the point with minimum volatility are labeled, and they are the target portfolio.

For the maximum Sharpe ratio portfolio, the stock weight is AMZN: 15.02%, GOOG: 39.21% CRM: 1.87% 002024.SZ: 0.56% NFLX: 43.34%. The Sharpe ratio of the maximum Sharpe ratio portfolio is 841.43%. Google stock takes most part of the weight at nearly 40%, and salesforce and sunning stock only take a tiny weight. For the minimum volatility allocation, the stock weight is AMZN: 22.84%, GOOG: 16.51% CRM: 25.23% 002024.SZ: 35.39% NFLX: 0.03%. The volatility of the minimum volatility portfolio is 72.43%. The Suning stock takes almost 35% of the weight as the biggest part, and Netflix stock takes only 0.03% owing to its high volatility (See Table 3 and Table 4).

Table 3: Stock weights of portfolio.

	Max Sharpe Ratio	Min Volatility
Amazon	15.02%	22.84%
Google	39.21%	16.51%
Salesforce	1.87%	25.23%
Suning	0.56%	35.39%
Netflix	43.34%	0.03%

Table 4: Return, volatility, and Sharpe ratio of portfolio.

	Return	Volatility	Sharpe Ratio
Max Sharpe Ratio	23.99%	16.08%	149.20%
Min Volatility	14.43%	12.84%	112.38%

The Maximum Sharpe ratio portfolio has the highest Sharpe ratio of 149.20% that, results in a high return of 23.99%. Also, the Minimum Volatility portfolio has the lowest volatility of 12.84%, which results in a relatively low but good real return of 14.43%. Both are the optimized portfolio in training data with their specific weight of stocks. To assess the effectiveness of a proposed asset allocation strategy put forth by financial research in the investment area, the 1/N rule should be considered as a

primary criterion, to minimize duplication in evaluation [9-10]. So, the 1/N portfolio is a really good standard model to test the efficiency and performance of other optimized portfolios. To test the efficiency, the Maximum Sharpe ratio portfolio and Minimum Volatility portfolio should be tested by the testing data to show the results and compare the results with the 1/N portfolio (See Table 5). The training data is the first 70 days of 100 days period, and the testing data is the remaining 30 days.

Table 5. Performance of the test set.

	Real Return	Volatility	Sharpe ratio
Max Sharpe Ratio	4.24%	6.72%	63.10%
Min Volatility	1.49%	6.39%	23.30%
1/N	-1.41%	8.21%	13.47%

So, for the testing data, the results show that the real return in the testing period of the 1/N portfolio is only -1.41%, the real return for the Maximum Sharpe ratio portfolio is 4.24%, and the real return for the Minimum Volatility portfolio is 1.49%. Both real returns of the Maximum Sharpe ratio portfolio and the Minimum Volatility portfolio are higher than 1/N, showing that they are optimized. The Maximum Sharpe ratio portfolio has the highest 63.10% with 6.39% volatility which results in a relatively excellent performance in real return of 4.24%. Also, the Minimum Volatility portfolio has the lowest volatility of 6.39%, which results in a relatively low but good real return of 1.49% because the low volatility leads to the low risk of the investments.

As a result, the testing data shows that compared to the 1/N portfolio, both the Maximum Sharpe ratio portfolio and the Minimum Volatility portfolio are efficient since the results of the testing data still show the benefits of each portfolio. The testing data results with the weight of the Maximum Sharpe ratio portfolio still have the highest Sharpe ratio to ensure the risk management performance is at a good standard. The testing data results with the weight of the Maximum Sharpe ratio portfolio also has the lowest volatility to avoid the risk of investment at the highest level. Both of them get a higher real return compared to the 1/N portfolio, showing that the special aspects of the portfolio do help them get good results. So, they can be applied to real investment problem and helps the investor to make an investment decision based on their needs.

5.   Conclusion

This paper investigated the optimized portfolios: minimum volatility and maximum Sharpe ratio portfolios composed of 5 selected US stocks. Based on the data from yahoo finance, the data sets are created and separated into two sets, training sets and testing sets. The cumulative return of the stocks can be calculated over a specific period of each data set. Then, thousands of random portfolios were simulated by the Monte Carlo simulation to generate the plot of the portfolio with the parameter of expected returns and volatility. The data was visualized by plotting the portfolio return against the volatility, and the efficient frontier can be pointed out. The asset weights in the two optimal portfolios were known. Together with the 1/N portfolio, where all assets weigh equally, the cumulative portfolio returns were accessible using the test set data. Then compared with returns of the 1/N portfolio, where all assets weigh equally, the efficiency and performance could be visualized.

The results show that the maximum Sharpe ratio and minimum volatility portfolios are efficient and perform well based on the testing data set. The Maximum Sharpe ratio portfolio has a balanced combination of returns and risk. The minimum volatility portfolio satisfies tiny returns to get safe risks. Both of them are efficient since their performance in returns and risk is better than that in the 1/N portfolio.

This paper's results show the usefulness and applicability of the maximum Sharpe ratio and minimum volatility portfolios in the US stock market. Also, owing to the different needs of different investors and different market situations, the maximum Sharpe ratio and minimum volatility portfolios can provide two suggestions for investment.

Still, owing to the limited data set in this paper, more than the results are needed to apply to all kinds of markets worldwide. The results of this paper can only show how efficient, and performance is compared with the 1/N portfolio but need to quantify the efficiency and difference. In the future, it can be achieved by exploring the data sets and making advanced calculations to quantify the difference.

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