Portfolio Optimization Analysis -- Based on Six Stocks from Different Industries in the U.S. Stock Market

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Abstract: As the stock market becomes more and more unstable, investors all over the world pay more attention to risk management. Creating a portfolio is an effective way to make the trade-off between risk and return rate. This paper selects six stocks from the U.S. stock market in different industries, including the real estate industry, high technology, financial services company, pharmaceutical industry, automotive manufacturing industry and gas industry. This paper simulates 1,000,000 different investment portfolios by using the Monte Carlo simulation, then drawing the efficient frontier of the portfolio and figuring out the maximum Sharpe ratio and minimum volatility portfolio. As the results showed, McKesson Corporation accounted for the max proportion in both the maximum Sharpe ratio portfolio and the minimum volatility portfolio, which are 45.7149% and 60.8960% respectively. J.P. Morgan takes up 0.1232%, which is the min proportion in the maximum Sharpe ratio portfolio. However, Amazon accounted for the min proportion in the minimum volatility portfolio, which is 0.4828%. In the last, this paper compared the cumulative return of the three portfolios with the S&P 500 Index, only the return rate of the equal-weight portfolio and minimum volatility portfolio is better than the benchmark. Hence, this result may provide suggestions for picking certain kinds of portfolio in this special period.

Keywords: mean-variance model, equal-weight model, portfolio management

1. Introduction

In the 2020, the global economy experienced some volatility and uncertainty due to the outbreak of COVID-19, the U.S. stock market has performed bad in 2020. In early 2020, the U.S. stock market experienced a brief plunge due to some volatility, including some impacted by political uncertainty and trade tensions. Therefore, investors need to have patience and a long-term perspective to understand the nature of market volatility. it is of great importance to mitigate the risk while improving the return rate by optimizing the portfolio, especially in a market environment of high uncertainty and volatility [1].

Some researchers have shown that technology and Internet stocks perform relatively well under the impact of the epidemic, while energy, financial and aerospace stocks perform poorly by comparing the performance of different sectors of the U.S. stock market in 2020 [2]. Saeed analysis the performance of high-tech, pharmaceutical, energy, financial sectors. Specifically, the high-tech sector has the poorer performance compared to the pharmaceutical and services sectors [3]. Because of the high demand of high-technology and pharmaceutics, the stocks in high-tech and pharmaceutical

in the U.S. stock markets performed well [4]. Cohen and Tang indicate that the COVID-19 affected different sectors in the U.S. differently. Sectors such as technology and health care performed relatively well, while airlines, hotels and travel sectors performed poorly. In addition, market volatility increased significantly, indicating increased investor uncertainty about the market [5]. Therefore, bounds of the researchers focus on the portfolio optimization. Wang examine the relationship of risk and return rate in the U.S. stock market and discusses the construction methods and optimization strategies of equity portfolios [6]. Jiang drawn the conclusion of the strengths and weakness of each portfolio model based on the real performance of the U.S. stock market by using empirical study [7]. It turns out that mean-variance model can achieve higher returns and lower risk by construct numbers of portfolio. Some of the researchers points out that numbers of factors are supposed to be take into consideration when making the choices, including the objective of investment, the style of investment, and risk preference of investors [8-9].

This paper aims to examine how effective asset allocation optimization is during a specific period and offer guidance to individual investors on portfolio management. Meanwhile, this paper focus on how each sector performs under different portfolios. Therefore, this article chooses six different stocks from various industries in the U.S. stock market, which includes the real estate, high-tech, financial services, pharmaceutical, automotive manufacturing, and gas industries. Monte Carlo simulation will be utilized to create portfolios consisting of 6 stocks, and the assets' weights will be derived for the minimum volatility portfolio and the maximum Sharpe ratio portfolio. Subsequently, the cumulative returns of these portfolios will be compared to the equal-weight portfolio and the S&P 500 Indicator.

2. Data

This paper use Yahoo Finance (https://www.yahoo.com) to select six companies that are from different industries to make portfolio. The ticker of the six stocks are MCK, TSLA, EQIX, XOM, AMZN, JPM. The adjusted closing price data of the six stocks are from January 1st, 2020, to December 31st, 2020. The data was divided into two parts, which are training part from January 1st, 2020, to August 31st, 2020, and testing part from August 31st, 2020, to December 31st 2020. The reason to choose the adjusted closing price data is to figure out the whether the portfolio is effective under the influence of COVID-19. What's more, choosing stocks from different industries can describe the market comprehensively and avoid the bad influence on the whole returns due to the recessionary of specific one industry.

The 147 pieces of data were collected. Linear returns were used to calculate the return of the adjusted closing price. The basic description of the returns are the following Table 1.

Table 1: Descriptive statistics of the daily return.

	Max	Min	Mean	Std Dev	Cumulative Return
MCK	0.1500	-0.0852	0.0011	0.0344	0.1659
TSLA	0.1989	-0.1858	0.0103	0.0617	1.5048
EQIX	0.1160	-0.1266	0.0025	0.0307	0.3687
XOM	0.1269	-0.1222	-0.0018	0.0379	-0.2650
AMZN	0.0793	-0.0792	0.0039	0.0257	0.5757
JPM	0.1801	-0.1496	-0.0007	0.0413	-0.1090

As the data has shown, when it comes to the average returns, TSLA is the highest one and XOM is the lowest one. However, TSLA has the highest standard deviation. The AMZN has the lowest standard deviation which makes it steadier. Moreover, the max return and the min return were all appear at TSLA. The following Fig. 1 represents the cumulative returns of the six stocks.
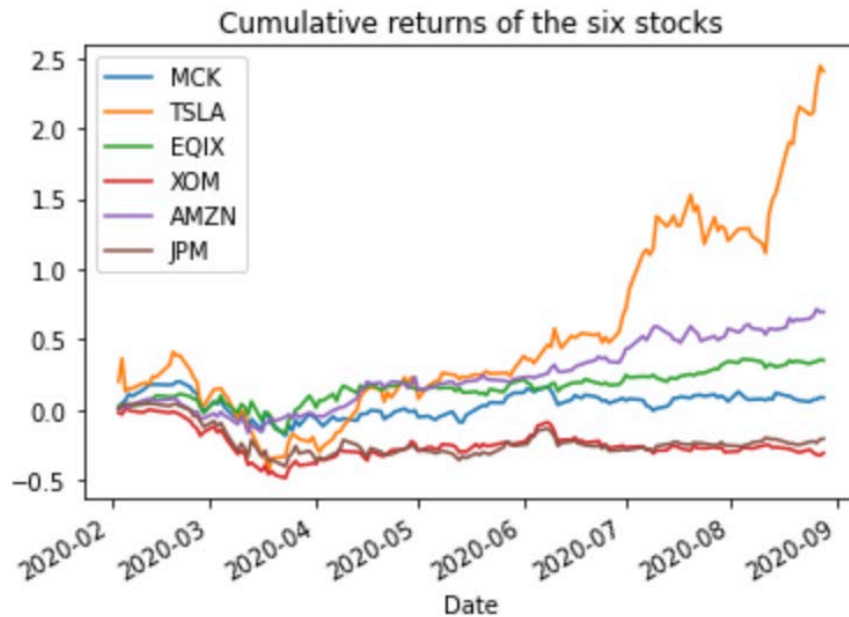

Figure 1: Cumulative returns of the six stocks.

According to the diagram, TSLA remains a higher cumulative return among those six stocks. XOM and JPM have a lower cumulative return on the contrary.

3.   Methods

3.1. Linear Returns

Linear returns is particularly suitable for making short-time trading decision [10]. Furthermore, linear rate of return has a wider range of applications and can be applied to different markets without additional conversions and adjustments [11]. By using linear returns instead of logarithmic return, the price changes of six stocks are explained intuitively. This paper focus on the portfolio during only one year, as a result, the linear returns explain the changes in a both clear and simply way.

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

In the formula, r_i represents the return of one asset in period i while r_{i+1} represents the return of one asset in the next period of i. The method is to compare the change range of this period with the previous period.

3.2. Monte Carlo Simulation

The application of Monte Carlo method in financial risk management can handle complex problems and take various uncertainties, flexibility and scalability into considerations [12]. When using Monte Carlo method, python will randomly and automatically select various probability distributions.

3.3. Mean-Variance

Mean-variance model is created by H. M. Markowitz, which is used to make the tradeoff between volatility and returns rate [13]. To make the best choices for the different people who have different styles in investment, using mean-variance model is optimal to find out all the possible portfolios of six stocks. After then, return and matched risk are used to form the efficient frontier. The following formulas are used in mean-variance model.

$$Return\ Rate\ Vector\colon R = (r_1, r_2, \ldots, r_i)^T \tag{2}$$

$$Weight\ Vector\colon W = (w_1, w_2, \ldots, w_i)^T \tag{3}$$

$$\sum_i w_i = 1 \tag{4}$$

Returns and weight for each stock, the main elements in mean-variance model, constitute the portfolio return together. The r_i stands for the returns of six stocks and w_i is the weight of each asset.

$$Expected\ return\colon E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{5}$$

$$Variance\colon \sigma_P^2 = Var(r_p) = \sum_{i=1}^{n} w_i^2 Var(r_i) + \sum_{i \neq j} w_i w_j Cov(r_i, r_j) \tag{6}$$

$E(r_P)$ is the expected return of the portfolio while $Var(r_p)$ represents the risks of portfolio.

$$Sharpe\ Ratio = \frac{E(R_P) - R_f}{\sigma_P} \tag{7}$$

Sharpe ratio can indicate the relationship between portfolio's risk and return. R_f is the annualized risk-free rate.

3.4. Equal-weight Method

Compared to the other portfolio method, equal-weight method is a basic way to set the weight of each asset. The use of an equal-weight model reduces the risk and increases the return of the portfolio and is easier to implement and maintain than a market-weighted model [14].

$$w_i = w_j = \frac{1}{N} \tag{8}$$

N stands for the number of assets that have been chosen.

4. Results

According to the prices data of those six stocks from January 1st, 2020, to December 31st, 2020, 1,000,000 portfolio weights by Monte Carlo Method are used to draw the efficient frontiers. The detailed figure is shown as the following Fig. 2.
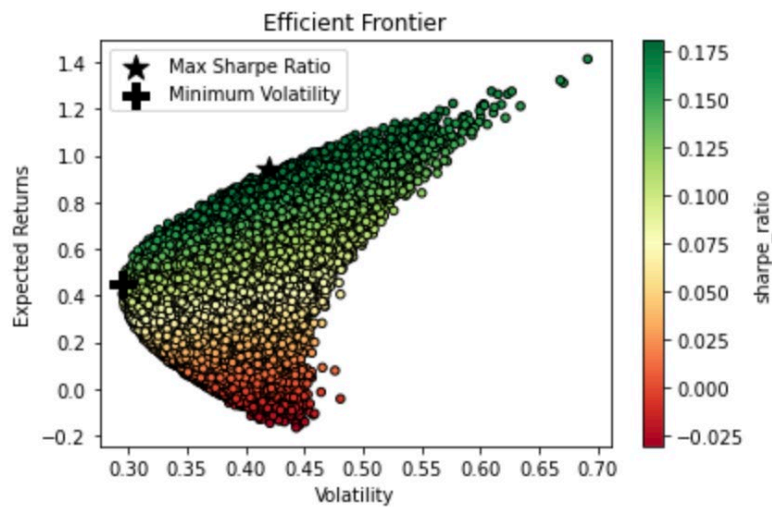

Figure 2: Efficient Frontier of six stocks.

According to the efficient frontier, the results of the mean-variance model form a sector area. The upper part of the efficient frontier is called effective set. Meanwhile, to clearly show the special points on the frontier. The max sharpe ratio point is marked out by star mark and the minimum volatility point is marked out by plus mark. To make portfolio in a comprehensive way, the weight of equal-weight method is also calculated. The following Table 2 is the weights of each stock in max sharpe ratio point, minimum volatility point and equal-weight method. Besides, Table 3 shows some characteristics of the portfolio.

Table 2: Asset weights under two criteria.

	MCK	TSLA	EQIX	XOM	AMZN	JPM
Max Sharpe Ratio	45.7149%	13.3321%	0.6872%	1.6735%	38.4690%	0.1232%
Minimum Volatility	60.8960%	19.4306%	2.0467%	10.9055%	0.4828%	6.2383%
Equal weight	16.6667%	16.6667%	16.6667%	16.6667%	16.6667%	16.6667%

Table 3: Portfolio characteristics of the two portfolios.

	Sharp Ratio	Volatility	Returns
Max Sharpe Ratio	224.2856%	41.9919%	94.1818%
Minimum Volatility	153.1349%	29.5361%	45.2301%
Equal weight	108.9922%	36.1481%	39.3987%

As is shown in the Table 2, the results of the max sharpe ratio and minimum volatility are different. The reason behind this phenomenon is that max sharpe ratio focuses on the maximum of returns of portfolio. However, the minimum volatility focuses on the minimum risk of portfolio. Under both max sharpe ratio and minimum volatility, the weight of MCK is the highest, which is 45.71% and 60.90% respectively. The lowest weight under max sharpe ratio is 0.12%, which is JPM's weight. AMZN has the lowest weight under minimum volatility, only 0.48%. Compared with these two portfolios, the weights of all assets are quite different.

After getting the weights of each asset in different portfolio, data from August 31st, 2020, to December 31st, 2020, is used to test the performance of each portfolio. This paper use the weight of each portfolio and the real return of each asset to calculate the return of the portfolio during the four months. Since this paper pays attention to the performance of portfolio during certain period, only the last day's return among the for months is compared to the S&P 500 Index, the market level of US stock market. If the return of the optimal portfolio in the four months is greater than the S&P 500 Index, this portfolio will be considered effective. The following Figure 3, Figure 4 and Figure 5 demonstrate the daily return of different portfolios in the four months in details.
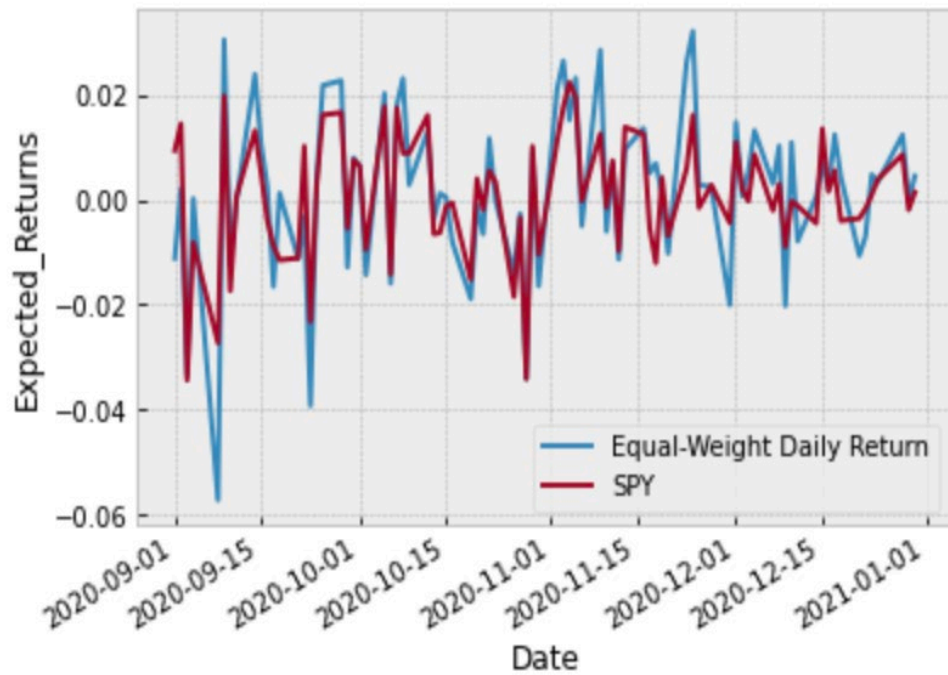
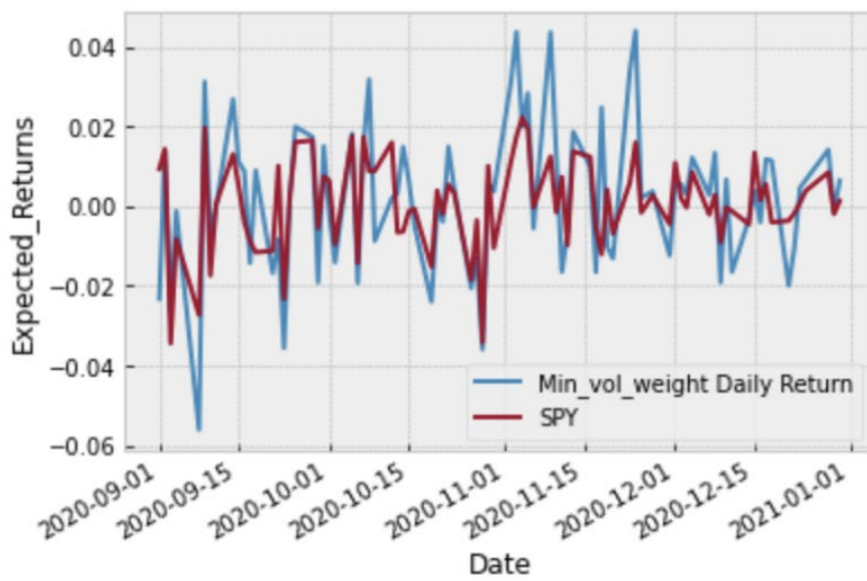Figure 3: Comparison of Equal-Weight portfolio and the market level

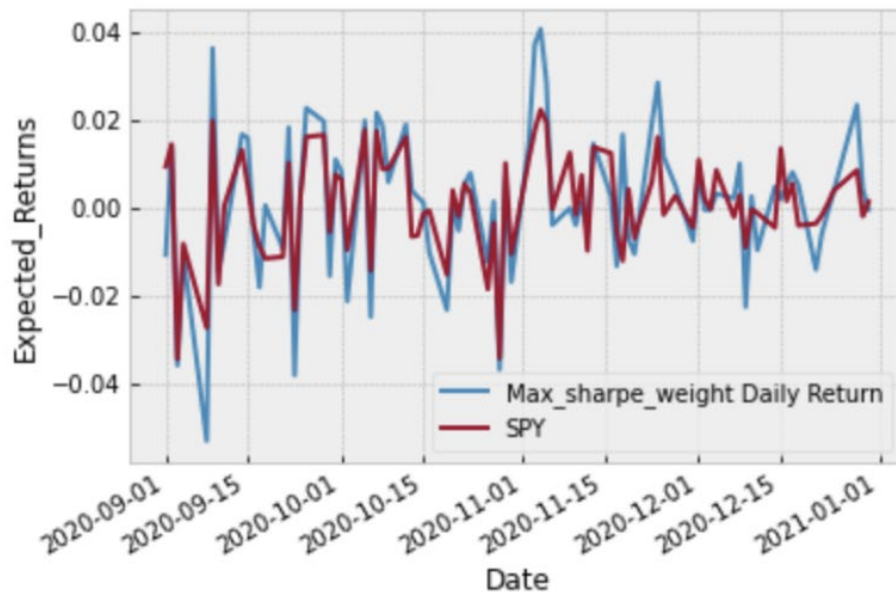Figure 4: Comparison of Minimum Volatility portfolio and the market level

Figure 5: Comparison of Maximum Sharpe Ratio portfolio and the market level

According to the figures, the return of Minimum Volatility portfolio and Equal-Weight portfolio in the last day is clearly higher than the return of S&P 500 Index. On the contrary, the last day's return of Maximum Sharpe Ratio portfolio is slightly lower than the return of S&P 500 Index. The cumulative return for the Minimum Volatility portfolio, Equal-Weight portfolio and Maximum Sharpe Ratio portfolio are 19.44%, 13.28% and 11.30% respectively. Minimum Volatility portfolio's cumulative return is more than twice the return of the market level, which is 7.69%. As a result, the performance of Minimum Volatility portfolio is not only better than that of the market level, but better than that of Maximum Sharpe Ratio portfolio and Equal-Weight portfolio.

5.   Conclusion

By comparing the result of different portfolios, this paper figured out that the Minimum Volatility portfolio performs excellent unexpectedly. It is possibly caused by that the minimum variance strategy efficiently manages risks and benefits the investors through reducing the volatility in their expected return. This risk-reducing strategy helps the investors survive in the unstable market accidently.

Although the maximum Sharpe ratio aims to maximum the return of the portfolios, it finally turns out that the cumulative return ratio of the maximum Sharpe ratio portfolio did not perform better than the benchmark index. Hence, when investing in the real life, bunches of factors such as policy, regulation and the operation of corporation are supposed to be taken into consideration instead of using portfolio model solely.

This paper only focuses on the influence of each stock's return rate to portfolio weight without considering the effect of business cycle. In fact, in the second half of 2020, the U.S stock market performances much better than before. In the future, if this factor can be taken into considerations, the portfolio with higher return and lower volatility. Furthermore, due to the volatility of stock market, the time span should be smaller, which can better verify the fitness of each portfolio model, especially the maximum Sharpe ratio portfolio in the U.S. stock market.

References

[1] *Harvard Business Review.: Looking Back on 2020: Lessons for the Future. Harvard Business Review 98(12), 12-13 (2020).*

[2] *London Stock Exchange.: 2020 Review and 2021 Outlook. London Stock Exchange 16(1), 8-10 (2021).*

[3] *Saeed, A.: US stock market sector performance during COVID-19. Journal of Asset Management 22(1), 1-12 (2021).*

[4] *Kang, W., Kim, S., Kim, S.: COVID-19 and stock market: The case of US sectors. Sustainability 12(22), 9369 (2020).*

[5] *Cohen, L., Tang, Y.: Industry-specific shocks and the relative performance of US sectors during the COVID-19 pandemic. Applied Economics Letters 28(18), 1499-1504 (2020).*

[6] *Wang, J.: Risk and Return Analysis of US Stock Investment Portfolio. Financial Economics Review 4, (2016).*

[7] *Jiang, X.: An Empirical Study on the Construction of US Stock Investment Portfolio. Economic Research Journal 7 (2018).*

[8] *Sun, Y.: Research on the Construction of US Stock Investment Portfolio Based on Mean-Variance Model. Statistics and Decision 19(4) (2019).*

[9] *Qiu, W.: Portfolio Optimization Model and Its Application in the US Stock Market. Economic and Management Research 9(4) (2019).*

[10] *Röthig, A., Chiarella, C.: Linear or logarithmic returns? Evidence from the US and the German stock markets. Physica A: Statistical Mechanics and its Applications 344(1-3), 576-582 (2004).*

[11] *Schürhoff, M., Großmann, A.: Linear versus logarithmic returns in the quantitative analysis of financial markets. Journal of Economic Dynamics and Control 35(9), 1568-1581 (2011).*

[12] *Haugh, M., Iyengar, G.: A review of Monte Carlo simulation techniques in financial risk management. Communications in Statistics - Simulation and Computation 33(1), 1-28 (2004).*

[13] *Markowitz, H.: Portfolio selection. The Journal of Finance 7(1), 77-91 (1952).*

[14] *Arnott, R. D., Beck, C., Kalesnik, V., West, J.: The case for equal-weighting equities. Financial Analysts Journal 73(1), 17-26 (2017).*