Decision-making and Utility Theory

Pai Liu^{1,a,*}

¹Stony Brook University, 100 Nicolls Rd, Stony Brook, NY 11794 USA a.paddyliu2067@gmial.com *corresponding author

Abstract: Decision-making is a process that would affect people's lives from simple daily decisions even to major life decisions. The understanding of judgment is critical for making better decisions as well as improving individual and collective outcomes. This paper would focus on historical decision theory, utility theory, prospective theory, and their applications in real life. The utility theory of the decision-making theory is a crucial part of the decisionmaking theories, as well as the prospect theory in the later paper. According to prospect theory and utility theory, people would make decisions based on the relative importance of the options' predicted values, and biases and heuristics would influence their choices. Both theories have applications in many fields such as economics, psychology, and medicine. They are used to form models and predict consumer behavior or financial decisions. People's lives are strongly associated with different decision makings, from simple to complex. The decisionmaking process is essential to human existence and fully connect to humans. By figuring out the decision-making processes, people can identify the factors that would influence the choices and make adjustments based on them. The application of the theories could be more practical. Thus, it is crucial to analyze or quantify it for more stable and reasonable choicemaking.

Keywords: decision-making, utility theory, prospect theory

1. Introduction

As advanced creatures on Earth, humans have been specially gifted with the ability to think. Decisionmaking plays an irreplaceable role in people's daily life. The decision-making of humans is a significant component in human evolution. This gifted ability enabled humans to survive in different environments.

It is crucial to figure out how people make their decisions and how different situations affect their decision-making. Decision-making involves tons of elements. There are many variables that could affect deciding. As mentioned in Hastie & Dawes [1], for people who lived a hundred or thousand years ago, there are tons of situations that people must think of for their lives. Today, it is still a problem for making the right decision.

For centuries, many people did research on different areas of decision-making theories. On the economic side, as mentioned in Herbert [2], as a science that describes and predicts the behavior of consumers, normative decision-making would be involved. Researchers studied those factors that would influence people's decision making such as preferences. They also lead to a setup for economic frameworks.

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Also, in other areas, such as engineering and management, decision-making is also important. Gregory S. P.'S book Decision Making in Systems Engineering and Management [3] talked about how engineers have to solve problems and think from a more logical perspective. The decision-making is crucial for them to catch the changing circumstance. With the importance of decision-making in people's daily life, it is crucial to quantify this abstract concept.

As well in biology, researchers did research not only on humans but also on animals with the decision-making processes. This had a great impact on evolutionary and psychological perspectives. Some of the studies also explored cellular levels to make the contribution. On the medical side, researchers had their study with the analyzing of patients for their preferences during the medical processes.

This paper will be about decision-making theories, utility theories, and prospect theories. The history and the development of decision-making theory will be first introduced with primary research reasons and the founders of those research which can be traced back to around the 1940s and 1950s. Following the decision-making theories, different areas of application will be introduced for the theory, such as the gambling theory, behavior science, and economic part. Decision-making theories have been developed over the years for individuals and organizations to make rational decisions in complex situations. The utility theory and prospect theory are two of the most prominent theories of decision-making. They have been applied to various fields, such as finance, gambling, and behavioral science, among others. The development of decision-making theories has played a crucial role in the advancement of different areas of study.

2. Decision-making and Judgment

2.1. History and Development of Decision Making

Humans are descended from ancestors who lived in small groups hundreds of thousands of years ago. For those years, they spent most of their time searching for food. If they cannot find resources for eating or drinking, or have a safe shelter for protecting the family, it's a failure of surviving and would be most likely to die. The decision-making plays an irreplaceable role here as a key to survival. The same skills can get you off the planet for a short period of time, but you can certainly use these techniques to develop techniques and tools that can make the planet less resilient if you make the wrong decisions. People have a special ability to choose the right way to achieve their goals. Humans as a dominant specie that spread out the world due to their unique ability for thinking to make the right decisions. The process of thinking can be characterized as the construction of mental images for things that are not immediately present [1].

The first psychological research about decision-making can be traced back to the 1940s and 1950s [4]. At that time, two programs of research began that were motivated by different questions. The first group took notice of the efforts of economists and statisticians to account for and advise people about their decision-making. The other group was motivated by perception.

Around 1944, von Neumann and Morgenstern published the book Theory of Games and Economic Behavior. This is one of the influential articles in this area. In 1954, Ward Edwards published a review of the field in Psychological Bulletin which had a lot of impacts as well. Until today, people are still having the research in different fields of decision makings.

The decision-making theory also have applications on different areas. In the book written by Kate Sweeny, Megan L. Robbins, Lee M. Cohen [5], talked about the importance of decision-making in medical area that those research utilizes features of people's psychological processes in systematic ways. It is important to observe the patients or to find out what they want in this area. Sholom M. W. [6] showed a great example of how people use decision making to formulate the CASNET model in

medical area for analyzing decision making for patients. Beside this, there are many research and experiments were made with the medical decision making. In Medical Decision Making in Situations That Offer Multiple Alternatives by Donald A. R. [7], they had the research on additional options would make the patients increase the difficulty of choosing medication. Neeraj K. A. and Colleen A. M. [8] tried to find out what would the patient choose in different situations in Patient Preferences for Medical Decision Making: Who Really Wants to Participate?

Also, in economic area, John B. D. [9] mentioned in The Handbook of Economic Methodology that it is important to find out the situations and factors that would address people's decision. Economic decision-making is based on the principle of identifying and choosing the best course of action in order to maximize profit. Numerous difficult decision-making circumstances involve the application of preferences, both by individuals and by organizations [10]. Douglas et. al. [11] also involves with the decision making in the article Deep Rationality: The Evolutionary Economics of Decision Making. Rational decision was introduced in the article and the importance of a rationality framework in economic area. Martin J. B. [12] also talked about the development of economic evaluation of health technology and the impact on decision-making. Those decision making sometimes are called as the economic decisions. Based on this, Harlé, K. M., Allen, J. J. B., & Sanfey, A. G. [14] had research to figure out the impact of mood disorders or depression on the economic decision making.

In biology area, the study of decision making would benefit the study of both humans and animals. Based on David P. R. & R. Bruce Hull [15], it mentions that the success of conservation biologists' knowledge should be measured by how well it can affect conservation decisions, as well as the knowledge of all those who generate, review, and apply conservation research [15], and the conservation knowledge are based on the decision making. A great example is provided in H. P. Possingham [16] of the application of decision-making theory with the nature conservation. Nevertheless, David J. S. [17] had research with his fellows about how the decision making related to the group size with fish in Consensus Decision Making by Fish. Theodore J. P., and Peter S. S. [18] even had research on the decision-making strategies on cellular level.

2.2. Theory of Decision Making

According to David et al., there are three general approaches to a decision: The descriptive approach focuses more on individual behavior, which can be used to characterize or predict behavior, and how and why people think and act the way they do. The normative approach focuses on how individuals would think and conduct if they were free from cognitive worries or concerns, which is how people would behave in a perfect world. The prescriptive approach is linked to each person's uniqueness, their varied psyches and emotions, capacities, and requirements, and the necessity for various approaches or designs [19].

Rationality is a core variable of decision making which often varies whether a decision is good or not. A reasonable choice can be defined as meeting the following four criteria: 1. The decision-maker's current assets serve as the foundation. In addition to money, assets can also be things like good health, intelligence, connections with others, and feelings. 2. It is founded on pickable outcomes. 3. If these outcomes are unsure, assess their likelihood using the fundamental principles of probability theory. 4. Within these probabilities and the value or pleasure connected with each decision outcome, it is a choice that may be modified [1].

Von Neumann and Morgenstern's work Theory of Games and Economic Behavior [20] had a significant impact on people of their interest in utility theory. There were many researchers tried to draw out the consequences of maximizing the expected utility based on the initial formulation. Additionally, there is a remark that the fundamental formulation might be incorrect. Despite this, they did

not advocate doing away with the four criteria for rationality; rather, they suggested that rational decision-makers might make decisions in accordance with some rational principle other than maximizing expected utility. These early studies emphasized the normative issue of how decision-makers ought to make their choices. Traditional economists are certain that the maximization of anticipated utility principle accurately captures what occurs when they examine the collective behavior of numerous individual decision-makers in broad economic environments.

A study done by Huang S. et al [21] showed that older adults, compared to younger adults, made more deontological decisions in DI dilemmas but did not differ from younger adults in UI dilemmas. Older adults are less likely to support instrumental injury or sacrifice maximal collective utility, especially if that support is intuitively compelling.

2.3. Criticism of Decision-making Theory

As Hastie & Dawes mentioned [1], the theory did not include any specific settings for the definition of satisfaction which means that satisfaction could be different among people. For example, a gambler may choose to take the risk for a greater win, but an economist may choose a safer way or not take the risk. In the same situation, another gambler may choose the safer way to not take the risk, or the economist may take the risk for a greater prize.

Because humans lack a clear understanding of what they want, they frequently predict future events incorrectly, sometimes failing to maximize the value of their past experiences [22].

3. Utility Theory and Prospect Theory

3.1. Utility Theory

3.1.1. History of Utility Theory

The utility theory is based on decision-making. While people are forced to make a decision, they will have their own preferences.

As a blur definition, a lot of people did research and add features to the theory. In Fishburn P. C. [23], it mentions final preliminary section summarizes the linear utility theory of John von Neumann and Oskar Morgenstern was first appeared in 1944, which was often used in subjective expected utility theories.

Extraneous scaling probabilities and the use of probability lotteries in subjective expected utility. Theories by Anscom be and Aumann (1963), Pratt, Raiffa and Schlaifer (1964, 1965), Fishburn (1967, 1969) and others [24].

Luce and Krantz's (1971) conditional acts on nonnull events, and a theory of conditional subjective expected utility.

Jeffrey's (1965a, 1978) and Balker's (1967) mono-set theories, with utilities and subjective probabilities defined on the same entities. Domotor's (1978) finite version"

In the book written by Fishburn, P.C. [24], the history was mentioned. For early periods, the utility was seen as a measurable psychological magnitude with the notion $u_1(x_1) + u_2(x_2) + \ldots + u_n(x_n)$. Edgeworth, Fisher, Pareto and Slutsky [23] thought that utility represents the agent's preference.

3.1.2. Expected Utility

There is a distinction between the two senses of the term utility. In Kahneman & Tversky [22], the concept "want ability" was mentioned: "it is inferred from choices and used to explain choices. In contrast, experienced utility refers to the hedonic experience associated with an outcome"

The expected utility theory also has great implications in other areas. In the The Handbook of Economic Methodology written by John B. Davis, D. Wade Hands, and Uskali Maki [9] the expected utility theory has always been used in normative way within the economic field that it helped economic methodology from general movements in history to philosophical themes.

3.1.3. Applications of Utility Theory

The best way to comprehend this expected utility criterion is to look at examples of straightforward gambling situations. Since gambling scenarios are well-known and well-defined, we will extensively depend on them (as have most scholars in this field) to demonstrate fundamental concepts, though we'll also make an effort to give a variety of non-financial, real-world instances. Consider, for example, mentioned in Hastie & Dawes [1], a choice between two gambles: (1) With a probability .20 win \$45, otherwise nothing. (2) With a probability .25 to win \$30, otherwise nothing. The expected value of each is equal to the probability of winning multiplied by the amount to be won. Thus, the expected value of gamble (1) is \$9, while that of gamble (2) is \$7.50. People need not, however, prefer gambling (1) simply because its expected value is higher. Depending upon their circumstances, they may find \$30 to have more than four-fifths the utility of \$45, in which case they would—according to the theory—choose gamble (2). For example, an individual may be out of money at the end of a week and simply desire to have enough money to eat until the following Monday. In that situation, the individual may find the difference in utility between \$30 and \$45 to be negligible compared with the difference between a one-fourth and a one-fifth chance of receiving any money at all. Such a preference is represented in the von Neumann and Morgenstern theory by the conclusion that .25 times that individual's utility for \$30 is greater than .20 times that individual's utility for \$45.

3.1.4. Criticism of Utility Theory

There are also some shortcomings for the theories according to Fishburn P. C. [23]:

For what Balker (1967) and Jeffrey (1978) did for the theory, there is some difficulty for sorting out the decisional aspects with their mono-set format. Pratt, Raiffa and Schlaifer (1965)'s finding was fantastic, but it objected the direct use of extraneous scaling probabilities. Fishburn (1969, 1972) had a simpler structure but there a few less-intuitive axioms and some potential problems with the comparisons. The theory of Luce and Krantz (1971) is appealing in its conditional approach and has reasonably straightforward axioms, but there are some interpretational difficulties which comes from the combinations and restrictions of conditional acts. And those would make them avoid extraneous probabilities.

3.2. Prospect Theory

3.2.1. Prospect Theory and Cumulative Prospect Theory

Prospect theory assumes that individuals' deviations from benchmarks rather than net worth levels, and their identification with that benchmark is a key variable. They value losses over comparable gains and are generally gain averse and accept risk over the loss.

Prospect theory distinguishes between two stages in the selection process: classification and evaluation. During the framing phase, decision-makers construct representations of decision-related behaviors, contingencies, and outcomes. During the evaluation phase, decision-makers assess the value of each potential customer and make corresponding choices.

In Kahneman & Tversky [25], it talked about that for simple prospects with monetary outcomes and probabilities with complex options, the prospect theory was developed. It distinguishes two stages for the selection process that the first is editing stage and the second one is evaluation stage. As mentioned in Kahneman & Tversky [25], during the editing phase, the initial examination of the available options would be carried out, which would often cause a more straightforward portrayal of those options. For the next stage, the updated options are appraised and the one with higher value would be chosen. For accommodating the effects in the prospect theory equations, it is crucial to assume the values are related to changes compares to the final states, as well as the decision weights are not consistent with the stated probabilities. There are chances that the deviations from the theory would lead to unacceptable consequences. Usually, the preference would be corrected when people noticed that preference are not fit to the situation. But for many cases, the decision-maker do not have the chance to correct the preference. Since prospect theory has been proposed as a choice model, the inconsistency between offers and choices means that the measurement of decision values and weights should be based on choices between specific prospects, rather than on offers or other production tasks.

There are three tenets of prospect theory:

(i) Expectation: $U(x_1, p_1; ...; x_n, p_n) = p_1 u(x_1) + \dots + p_n u(x_n)$

Which U represents the overall utility of a prospect. $(x_1, p_1; ...; x_n, p_n)$ as the prospect that contracts outcome xi with the probability p_i , and $p_1 + p_2 + \cdots + p_n = 1$

(ii) Asset Integration: $(x_1, p_1; ...; x_n, p_n)$ when $U(w + x_1, p_1; ...; w + x_n, p_n) > u(w)$

(iii) Risk Aversion: u is concave (u'' < 0).

A person considered as risk averse if he prefers certain prospect (x) to any risky prospect with expected value x

When people were asked to choose between two situations:

A: 500 with probability 0.30; 400 with probability 0.69; and 0 with probability 0.01

B: 400 with certainty

18% percent of the people chose A and 82% percent people chose B

With the formular from the expected utility theory:

u(400) > 0.3u(500) + .69u(400) or it can be shown as .31u(400) > .33u(500)

Cumulative prospect theory describes a "framing pattern" of risk-seeking and risk aversion in the same individual. A person exhibits "risk aversion" when they prefer the expected value of the bet to the bet itself. For example, most people would prefer \$50 to a risky bet with a 50% chance of winning \$100 and nothing. When a person prefers a bet to its expected value, the person is described as a "risk seeker". In the "quadruple model," a typical participant exhibited risk-seeking for binary bets with a low probability of winning and risk aversion for winning with medium to high probability. For players with strictly non-positive consequences, the pattern is reversed. This inversion is called the reflection effect. Finally, cumulative prospect theory describes risk aversion in mixed betting as a tendency to favor safe returns over mixed bets with an expected value equal to or greater.

3.2.2. Applications of Prospect Theory

Here is an example: If there are two kinds of lotteries: (1) with 2 percent to win \$1000 but 98 percent to win nothing; (2) with 1 percent to win %1000 but5 percent to win \$500. In prospect theory, people would cancel out the 1 percent to win \$1000 but consider about the other 1 percent to win \$1000 or 5 percent to win \$500.

Also, in Tversky & Kahneman [26], it shows that: If S is a finite set of natural state variables, then its subsets are known as events. The decision-maker doesn't know that exactly one state exists, therefore it is presumed that it does. Let X be a collection of consequences or outcomes. We limit the subject at hand to financial results in the interest of simplicity. We consider the neutral outcome of X to be represented by the number 0, and we interpret all other X components as gains or losses, represented by positive or negative numbers, respectively. The calculations are also showed as following.

An uncertain prospect f is a function from S into X that assigns to each state $s \in S$ a consequence f(s) = x in X. To define the cumulative functional, we arrange the outcomes of each

prospect in increasing order. A prospect *f* is then represented as a sequence of pairs (x_i, A_i) , which yields x_i if A_i occurs, where $x_i > x_j$ if and only if i > j, and (A_i) is a partition of S. We use positive subscripts to denote positive outcomes, negative subscripts to denote negative outcomes and the zero subscripts to index the neutral outcome. A prospect is called strictly positive or positive, respectively, if its outcomes are all positive or nonnegative. Strictly negative and negative prospects are defined similarly; all other prospects are called mixed. The positive part of *f* denoted f^+ , is obtained by letting $f^+(s) = f(s)$ if f(s) > 0, and $f^+(s) = 0$ if f(s) < 0. The negative part of *f*, denoted f^- , is defined similarly.

As in expected utility theory, we assign to each prospect of a number V(f) such that f is preferred to or indifferent to g if and only if $V(f) \ge V(g)$. The following representation is defined in terms of the concept of capacity, a nonadditive set function that generalizes the standard notion of probability. A capacity Wis a function that assigns to each $A \subset S$ a number W(A) satisfying W((b) = 0, W(S) = 1, and W(A) \ge W(B) whenever $A \supset$ B. Cumulative prospect theory asserts that there exist a strictly increasing value function v:X--+ Re, satisfying v(x0) = v(0) = 0, and capacities W^+ and W^- , such that for $f = (x_i, A_i)$, -m <- i < n,

$$V(f) = V(f^{+}) + V(f^{-})$$
$$V(f^{+}) = \sum_{i=1}^{n} \pi_{i}^{+} v(x_{i}), \quad V(f^{-}) = \sum_{i=-m}^{n} \pi_{i}^{-} v(x_{i})$$

Also, the decision weights are defined by:

$$\pi_n^+ = W^+(A_n), \pi_{-m}^- = W^-(A_{-m})$$

$$\pi_i^+ = W^+(A_i \cup \dots \cup A_n) - W^+(A_{i+1} \cup \dots \cup A_n), 0 \le i \le n-1$$

$$\pi_i^- = W^-(A_{-m} \cup \dots \cup A_i) - W^-(A_{-m} \cup \dots \cup A_{i-1}), 1 - m \le i \le 0$$

3.2.3. Criticism of Prospect Theory

According to Tversky & Kahneman [26], The tradition of assuming the rationality of economic agents is replaced by prospect theory, which is put up as a descriptive theory rather than a normative one. The idealized assumption of rationality in economic theory is sometimes defended on the basis of two premises: the belief that only rational behavior can survive in competitive information and the concern that any approach that forgoes rationality will be chaotic and unsolvable. Both justifications are questionable. First, evidence suggests that someone can spend their entire life in a competitive setting without learning to avoid framing effects or use linear decision weights consistently. Second, and more importantly, the data suggest that although they are not always logical in the strict meaning of the word, human choices are orderly.

4. Discussion

The decision-making process is the basis of people's survival, which plays an irreplaceable role in human for their choice making. Different kinds of factors would bring up various effects on the final choice-making, which also makes it crucial to figure out those factors that would influence decision-making. Research about decision-making has been through hundreds of years with different experts and people are still researching about it to find the process behind it. Many researchers tried multiple ways to try to find a concluding theory or formula that would explain the decision-making process as a whole, which including the utility theory and prospect theory.

As mentioned in the paper, decision-making plays an irreplaceable role in people's daily life. Utility theory and perspective theory are two such examples. Utility theory suggests that people make decisions based on maximizing their expected utility or satisfaction, whereas prospect theory suggests that people make decisions based on how they perceive potential outcomes, and the likelihood of those outcomes occurring. Those formulas mentioned in the utility theory and prospect theory part are not always applicable. There are chances that unpredictable variables come up in the real life.

Since these theories are all based on humans, those quantified data will be affected easily by many factors, such as emotions. Future studies can focus on elements such as emotions that can affect the choice making for people. Emotion is one of the factors influencing the decision-making process, and future research could focus on exploring the role of emotion in decision-making. Since these theories are based on human behavior, quantitative data are susceptible to various factors such as emotions. People may have different thoughts with different emotions instead of their rational thinking.

As Christopher K. Hsee and Reid Hastie mentioned in their article, it is hard to say if people can choose what is best for them. Many choices were made based on happiness besides those survival problems. However, what really is "happiness" to people, also other emotions going to affect people's choice-making process. Those would be some crucial focuses for future research. By gaining a deeper understanding of the factors that influence decision-making, researchers can provide insights that help individuals make better decisions in their daily lives.

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