# Comparative Analysis on Benefits of Elective System at Universities Based on Triangle-dynamic-matching System

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*Abstract:* Nowadays, the development of education can improve the level of human capital and significantly promote economic development. As part of the education system, more and more students care about the college admissions, but few of them pay attention to the benefits of a good elective system, which also might influence their whole university life. Taking Chinese elective system as an example, the paper proposes a new elective system at universities with a specific formula: Triangle - dynamic - matching system, which combines the advantages the Auction mechanism, the Wharton Business School Mechanism and the Deferred acceptance algorithm with the true preferences of students, teachers and the school's recommendation dynamically. Through the Systematic Literature Review and Analytical Method, the paper aims to analyse the benefits of a good elective system, its utility, the algorithm and problem in the elective system. This is a new system promising Pareto-efficiency and strategy proof, which might helps to save the school's human, material and financial resources, and greatly enhances the efficiency of students' course selection.

*Keywords:* educational economics, triangle-dynamic-matching system, pareto-efficiency, auction mechanism

## 1. Introduction

According to the statistics of the United Nations Educational, Scientific and Cultural Organization (UNESCO), the total number of the undergraduates in the world was about 90 million in 2022. Educational economics focuses on the analysis of the institutional environment and institutional conditions in terms of educational investment and educational resource allocation, including the college admissions system, course elective system and other issues [1]. To be honest, every students and schools care about the college admissions, but few of them ever spare time to pay attention to the benefits of a good elective system. Most schools spend the most time on their rankings and the teachers pay more attention to the papers and their publication. Both the teachers and the students are not emphasized in some areas, let alone the elective system. It is time for them to stop underestimating the elective system. Through the Systematic Literature Review and Analytical Method, the paper aims to analyse how to maximize the utility of students in course selection in the domestic education course selection system without changing the utility of teachers and achieve Pareto efficiency results. With the contribution of the improved system, the whole matching system will be developed more practically and comprehensively, which might helps to save the school's

human, material and financial resources, and greatly enhances the efficiency of students' course selection.

## 2. Literature Review

Several economists have contributed to education reform. Gale & Shapely proposed the first school-student matching method [2]. Deferred Acceptance Algorithm additionally improves the system: After applying to their preferred institution, students are accepted according to their priorities. After that, rejected students apply to their second-favorite institution, which rejects them based on their updated list and preferences. No seat, no system. Random Serial Dictatorship is another method. This system is ex post Pareto efficient, but not ex ante, because students are grouped randomly and cannot choose classes until their turn [3-5]. Peking University uses auction process, which is efficient. Students receive points and bid for courses and teachers within their budget [6]. This is faster than Deferred Acceptance Algorithm since students already provide points to every teacher. Budish & Cantillon attacked Harvard business school mechanism for being non-incentive compatible and strategy-proof [7]. This study will not describe its complexity and drawbacks. The Wharton Business School Mechanism is dynamic since the budget constraint can vary from -1.6% to 1.6%. Students start with 100 points and bid for their preferred teachers. At the second step, pupils can lower or raise their score within limits [8].

## 3. The Problems of Chinese Elective System

In economics, utility maximization is the ultimate goal of a rational individual. The current course selection system in Chinese universities cannot actually achieve this goal. In fact, the current Chinese elective system is a Random-Top internet speed matching system (RT algorithm). In the current elective system, at first, all the students choose their favorite teacher, and then the elective system will pass someone randomly, the accepted students will be stable unless they withdraw from the courses willingly. After that, all the rejected students only have the right to choose the teachers with vacant seats. Noticeably, which student is chosen in the second part only depends on their internet speed. In other words, if student a is chosen while student b is not, it is totally because a applied for this lesson earlier than b instead of thinking about the teacher's preferences. We can illustrate the system comprehensively with the next example, where we have students<sub>1</sub>,s<sub>2</sub>,s<sub>3</sub> and teacherT<sub>a</sub>,T<sub>b</sub>,T<sub>c</sub>,T<sub>d</sub>. Preferences of students are given in the following table 1.

s <sub>1</sub>	S <sub>2</sub>	s <sub>3</sub>
T <sub>a</sub>	T <sub>a</sub>	T <sub>a</sub>
T <sub>b</sub>	Т <sub>с</sub>	Т <sub>с</sub>
Т <sub>с</sub>	T <sub>b</sub>	T <sub>b</sub>
T <sub>d</sub>	T <sub>d</sub>	T <sub>d</sub>

Table 1: Students' preferences (Source:Author's example).

Since it is a student-proposing system, and if we suppose that teachers are indifferent to every students since they know nothing about them(That is what happened in the real world), there preferences are shown in the next table.

Table 2: Teachers' preferences (Source:Author's example).

$\succeq_{T_a}$	$\succeq_{T_b}$	$\succeq_{T_c}$	T <sub>d</sub>
s <sub>1</sub> ,s <sub>2</sub> ,s <sub>3</sub>			

We can see the result from the first random step, as shown in table 2. If both of them were passed unfortunately, they will be distributed as follows.



Figure 1: Students' distributions.

From figure 1 we can prove that it is not Pareto-efficient. If  $s_1$  withdraws from  $T_c$  to exchange it with  $s_2$ , since both of them would be better, then neither of them can make sure whether  $s_3$  will get this lesson if  $s_3$  has a better internet speed. Actually, after these steps, only  $s_3$  is better with a worse result of  $s_1$ .



Figure 2: Forced students' results.

In summary, the final result of matching between students and teachers cannot strictly improve the utility of students, or even make them worse. Therefore, it is necessary to introduce a new matching system to improve the utility of students and achieve Pareto optimality.

## 4. Triangle-dynamic-matching System

The new Triangle- dynamic- matching system both combines advantages of the Auction mechanism, The Wharton Business School Mechanism and the Deferred acceptance algorithm with different students', teachers' and the school's preference comprehensively and dynamically. The algorithm is as follows.

### 4.1. Research Procedure

First of all, every student will have 100 points, and they can give different lessons with different points based on their preferences (Every point should be given, otherwise they would be separated in proportion and given to different teachers). Noticeably, to solve the problem that it is hard to choose some courses taught by different teachers, the students should give points to different teachers teaching the same course (If they did not give the teacher the point, it means they would prefer not to participate in the course). After that, the school use a formula to calculate their points. Higher score means better preference.

The formula is suggested to be P = A \* T + S. A is the overall grade of this course's prerequisite courses with the maximum of 10 (I suppose it should be one tenth of the weighted average number). In fact, A is not a compulsory but it can show the school's attitude and exclude the students who are

not recommended to participate in this class. For example, the art students who didn't take participate in Maths will get 0 if they want to take Econometrics. However, every problem could be solved with the students' strength. When students put enough points which means they have enough time to learn about this knowledge and overcome difficulties, the design of the system will make sure its massive proportion can ask every class open the door for them. Meanwhile, A will multiple with T, which is decided by the teacher. The teacher can give different scores to different students according to their preference from 0 to 2. If they are indifferent about the students, then maybe they would prefer to bit 1 for everyone. The teachers' decision can not control the whole result, but it is also an improvement to include their preferences.

S is the point given by different students. Since A\*T is not decided by them, so this is the only way to show their preferences. Some people may argue that the seniors should have more priorities, so they should be given more points. To be honest, we will not do this in case manipulation in the new algorithm. We have reasons to believe that some students will make use of this rule to achieve their unfair goals, for example, some of them will choose not to participate in the class unless they get higher scores whey they become seniors. In addition, we do not need to care about the seniors since they have only have few lessons, which means their s would be much higher than the others. Noticeably, the points of S is not allowed to be changed by the students, which promises the fairness. We have to make sure that the points show their true preferences and protect everyone's preferences. The enhanced algorithm protects a triple power balance with student initiative. The formula above will vary A\*T's average from 7 to 10. (If we imitate the numbers in the real world, the average of A is supposed to be from 7 to 9, while T may fluctuate slightly around 1). Each term, students will average 10 courses, giving them 10 points for each teacher in the same course. s may be more weighted because we care more about students' preferences.

The Wharton Business School Mechanism and Deferred-Acceptance algorithm power the system. The algorithm sends applications to the professors with the highest course points after students submit points to different teachers. Then, the algorithm will pass pupils with full points and reject others. After that, the rejected applications will be sent to the students' second preferences, and the algorithm will choose again from the revised list. After repeating, the system stops. Considering that some teachers are extremely popular among students, if the first-round applications are more than twice the initially established seats, it will have a 10% rise only after the teacher accepts it.

### 4.2. Results Analysis

Table 3 is the improved result Before analysing, it needs to set the teachers' new preferences(which already multiples with A) in the next table.

Ta	T <sub>b</sub>	T <sub>c</sub>	T <sub>d</sub>
s <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>	S <sub>1</sub>
S <sub>2</sub>	S <sub>3</sub>	S <sub>1</sub>	S <sub>2</sub>
S <sub>3</sub>	s <sub>1</sub>	S <sub>2</sub>	S <sub>3</sub>

Table 3: Teachers' preferences (Source:Author's example).

From the result, it can appreciatively find that  $s_1$  and  $s_3$  are getting better and  $s_2$  does not get hurt.



Figure 3: Students' choices.

As shown in figure 3, the improved system balances efficiency and equality. Compared to the current system, no student is allowed to show their whole preferences except their favorite, and the elective system does not allow the students to change their lessons which benefits them properly, which always takes a long time to decide and choose their lessons. After the change, the students and teachers have access to rank their preference, and even if they were rejected by their favorite, then they will have access to their second favorite instead of choosing anyone else with vacant seats. With the help of it, the whole elective process would be automatic and quicker. Thanks to the system solves the unfair problem that it only depends on their internet speed and fortune to take class, no one will have to pay extra money to buy their wanted course, which protects the equality.

This new approach is strategy-proof and student-friendly, unlike Harvard business school. As no student knows the others' opinions or can modify points, they must declare their honest preferences. Teachers will not know students' preferences, so they will choose honestly. If we imply they could have participated in their favorite obligatory lesson, it doesn't matter if they truncate. If they were refused, they might have to accept that they won't take this course since they can't keep doing it, but if they tell the truth the next year, it doesn't matter. If this course is optional and students only choose one lesson and reject the others, it suggests they feel better without taking the other teachers' classes. This result is Pareto efficient since it increases the utility of at least one student without affecting other students.

### 5. Conclusion

All in all, the new improved Triangle-dynamic-matching elective system both consider the students' and teachers' preferences and schools' recommendation with their true rankings, efficiency and equality, which combines the advantages of Auction mechanism, The Wharton Business School Mechanism and the Deferred acceptance algorithm. After the online course selection system is put into operation, it is expected to bring considerable benefits and great convenience in management, and will save many unnecessary waste of resources. To be honest, this elective system is only useful for the school with numerous teachers and classes, which means schools with limited resources are excluded. At the same time, due to the lack of time, it is difficult for us to find a real school to experience this system and give us feedback.

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