The 6th International Conference on Economic Management and Green Development (ICEMGD 2022)
DOI: 10.54254/2754-1169/4/20221024

# Application Research of Portfolio Related Theory--Based on Hong Kong Stock Data

**Jialong Li**[1,a,*]

[1]*Xi'an Jiaotong University, Shanxi Province, China*
*a. e1010457@u.nus.edu*
**corresponding author*

***Abstract:*** With the development of the stock market, people have carried out in-depth and comprehensive research on the portfolio theory. This paper aims to verify and analyze the practical application of the relevant theories. By collecting the stock price information of Hong Kong stocks as the target, this paper verifies and discusses basic theories such as tangent portfolio theory, portfolio diversification effect, Markowitz portfolio theory and Sharpe ratio. In the minimum variance portfolio and the tangent portfolio, this paper obtained the weight of each asset under the conditions of assets and without risk-free assets. Finally, the differences in the weights of investment targets in the portfolio are analyzed and discussed. The results show that, first, in the minimum variance portfolio, when considering risk-free assets, only investing in risk-free assets is considered, and when risk-free assets are not considered, assets with lower variance are given higher weights. Second, in a tangent portfolio, regardless of whether risk-free assets are considered, assets with higher Sharpe ratios in the portfolio are given higher weights. This can give some guidance and suggestions for future investment behavior.

***Keywords:*** Sharpe Ratio, Minimum Variance Combination, Tangent Combination.

## 1. Introduction

With the continuous development of the investment market, various products emerge in an endless stream [1]. How to choose products and determine the investment ratio is a problem that all investors need to think about. Some investors avoid risks in a timely manner through reasonable allocation of their own assets and achieve a relative balance between risk and return. However, more people invest haphazardly and end up with huge losses. Therefore, for investors, they need to consider the risk level and return rate of each asset under the condition of existing funds to achieve a relative balance between the risk and return rate of the investment portfolio.

After considering the risk level and rate of return of the investment portfolio, the academic community has carried out in-depth research on the basic method of selecting the investment portfolio. In the past few years, Markowitz established the portfolio theory. And this theory gives the return of the portfolio is the weighted average of the returns of individual assets, but the risk of the portfolio is not different from the return. Actually, Markowitz believe that the risk can be smaller than that of each asset in the portfolio which means that the portfolio can reduce the unsystematic risk. On the other hand, the Sharpe ratio is established to evaluate different portfolios and then select the optimal

DOI: 10.54254/2754-1169/4/20221024

portfolio. This paper aims to use data to study minimum variance portfolios, portfolio diversification effects, and Sharpe ratio and tangent portfolios.

Portfolio theory was first proposed by Markowitz in his 1952 article, which studied the basic methods for investors to select assets and determine the weight of corresponding assets. In this theory, the portfolio corresponding to the maximum expected return or the portfolio corresponding to the minimum risk is called the efficient portfolio, and the set of efficient portfolios is the efficient frontier. Therefore, the key to Markowitz's portfolio theory is to determine the efficient frontier for a given investment object [2].

Before discussing and analyzing the application research of portfolio theory, this paper first sorts out and sorts out the research on portfolio related theory in academic circles in the past few years. On the basis of Markowitz's portfolio theory, William Sharp and others proposed CAPM in 1964 [3]. Fisher Black and Robert Litterman in 1992 combined the Markowitz portfolio model, CAPM, Bayesian method, reverse optimization and other models and methods, in order to overcome the Markowitz model in the investment portfolio is highly concentrated in a certain type of Asset and input sensitivity issues [4]. Idzorek combined the insights from some articles related to the BL model, gave a step-by-step explanation of the model, and conducted an empirical analysis of the model with eight types of assets such as U.S. Treasury bonds. The authors also propose a new method for suppressing excessive bias caused by investor views, increasing the usability of the model [5]. Bertsima constructed his model by inverse optimization. In his model, he considered investor information and market dynamics, making it possible to significantly expand the application scope of the BL model. The authors also introduce and study the corresponding portfolios of two new BL estimators, MV-IO and RMV-IO [6]. The domestic research on the portfolio model of securities started relatively late, but in recent years, with the rapid development of my country's securities market, more and more attention and research have been paid to the portfolio model. Many scholars have used different methods to optimize the BL model and used the optimized BL model to conduct empirical research on stocks in China's A-share market. Ma introduced liquidity risk measurement constraints on the basis of the BL model, proposed a new optimal asset allocation model and method, and improved the portfolio selection model based solely on historical returns and historical volatility. The author uses the stock portfolio data of A-share market fund companies to conduct empirical analysis [7].

Hong Kong is one of the global financial centers. At the same time, in recent years, the overall market of Hong Kong stocks has not been good, but the volume of Hong Kong stocks is relatively large. As of July 10, 2022, there are 4,650 listed companies in Hong Kong stocks, and the valuation of hundreds of listed companies exceeds 100 billion. Since the establishment of the Hong Kong stock market, the Hang Seng Index has been on an upward trend all the year round. However, the Hong Kong stock market is currently volatile and is in a downward stage, so it has high research value and thus, in this paper, we focus on the portfolio optimization in the Hong Kong stock market.

## 2. Data

The data source of this article is the wind database. In this paper, we select three representative stock assets in the Hong Kong stock market: China-Construction-Bank-Corp (CCBC), China-Merchants-Bank-Co-Ltd (CMBCL) and Bank-of-China-Ltd (BCL). And we get monthly stock price data of three stocks from 2009/12/1 to 2021/12/1. These three companies are more representative than general companies, especially China-Merchants-Bank-Co-Ltd. The range and variance of this company are at a high level, so after being included in the investment portfolio, it can better reflect the problem. The basic situation of stock price data is shown in Table 1:

DOI: 10.54254/2754-1169/4/20221024

Table 1: Descriptive analysis of monthly share prices of three stocks.

| | 'CCBC' | 'CMBCL' | 'BCL' |
|---|---|---|---|
| Mean | 6.156 | 26.171 | 3.489 |
| Variance | 0.567 | 196.897 | 0.313 |
| Max | 9.020 | 71.750 | 5.330 |
| Min | 4.550 | 11.710 | 2.400 |

From the above table, it can be observed that the stock prices of CCBC and BCL are less volatile, with lower variance and range. The share price of CMBCL fluctuates wildly with a variance of 196.897.

## 3. Methods

The basic methods involved in this paper mainly include calculation of monthly and annual return and standard deviation (SD) of a single stock and the portfolio, calculation of Sharpe ratio calculate. This article considers short and long portfolios.

$$r_{ij} = \frac{P_{ij} - P_{i-1,j}}{P_{i-1,j}} \quad (1)$$

Where, $r_{ij}$ is the monthly rate of return of the $i$-th month and the $j$-th stock, and $P_{i,j}$ is the $i$-th month and the $j$-th stock price.

$$\sigma_j = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(r_{ij} - \overline{r_{ij}}\right)^2} \quad (2)$$

$$\overline{r_{ij}} = \frac{1}{N}\sum_{i=1}^{N} r_{ij} \quad (3)$$

Where, $\sigma_j$ is the SD of the monthly return of the $j$-th stock, and $\overline{r_{ij}}$ is the average monthly return of the $i$-th month and the $j$-th stock. Annual rate of return and its SD

$$r_j' = 12 \times \overline{r_{ij}} \quad (4)$$

$$\sigma_j' = \sqrt{12} \times \overline{\sigma_{ij}} \quad (5)$$

Where, $r_j$ is the annual rate of return of stock $j$, $\overline{r_{ij}}$ is the average monthly rate of return of stock $j$, $\sigma_j'$ is the SD of the annual rate of return of stock $j$, and $\sigma_j$ is the SD of the average monthly rate of return of stock $j$.

The investment portfolio established in this paper mainly considers 3 stock assets and 1 risk-free asset, then the monthly return and SD of the investment portfolio $R_{ij}$ are:

$$R_{ij} = \omega_{i1} \times r_{i1} + \omega_{i2} \times r_{i2} + \omega_{i3} \times r_{i3} + \omega_{i4} \times r_f \quad (6)$$

Where, $R_{ij}$ is monthly rate of return of the $i$-th and $j$-th investment portfolios, $\omega_{i1}, \omega_{i2}, \omega_{i3}$ are the weight of the three stock assets, $\omega_{i4}$ is the weight of the risk-free assets in the $i$-th investment portfolio, $r_{i1}, r_{i2}, r_{i3}$ is the annual rate of return of stock assets, and $r_f$ is the annual investment rate of return of risk-free assets.

The SD of the monthly return on the portfolio and the monthly return is:

$$Var(R_j) = \omega^T \Omega \omega \quad (7)$$

DOI: 10.54254/2754-1169/4/20221024

$$E_{jR} = \omega_i E(R_j) \tag{8}$$

where $\omega$ is the weight vector, $\Omega$ is the variance-covariance matrix, and $R_j$ is the return of the $i$-th portfolio.

$$E'_{jR} = 12 \times E_{jR} \tag{9}$$

$$Var(R_j)' = 12 \times Var(R_j) \tag{10}$$

where $E'_{jR}$ is the annual expected rate of return of the $j$th portfolio, $E_{jR}$ is the expected monthly rate of return of the $j$-th portfolio, $Var(R_j)'$ is the SD of the annual rate of return of the $j$-th portfolio, and $Var(R_j)$ is the monthly rate of the $j$-th portfolio SD of expected return.

$$SH_j = \frac{E(R_{ij}) - R_f}{\delta'_j} \tag{11}$$

Where $SH_j$ is the Sharpe ratio of the $j$-th portfolio, $E'_{jR}$ is the annual expected return of the $j$-th portfolio, $R_f$ is the annualized risk-free rate, and $Var(R_j)'$ is the SD of the $j$-th portfolio's annual return [8].

## 4. Results

Based on the data of three stocks, this paper replaces the risk-free interest rate with the short-term treasury bond interest rate of 2.84% and uses R software to study the minimum variance portfolio, the diversification effect of the portfolio, and the Sharpe ratio and the tangent portfolio.

In real life, there are risk-free assets such as short-term treasury bonds, which are also favored by investors in order to balance the risk of investment portfolios. However, risk-free assets have a greater impact on the research on portfolio-related theories. Therefore, it is necessary to the investment portfolio is discussed in two cases.

### 4.1. Minimum Variance Portfolio

Minimum variance is a commonly constructed portfolio with only the consideration of risk rather than return [9]. Without considering the risk-free assets, only considering the portfolio composed of three stock assets, the variance level and expected return corresponding to the portfolio under different weight levels are studied, as shown in Figure1:

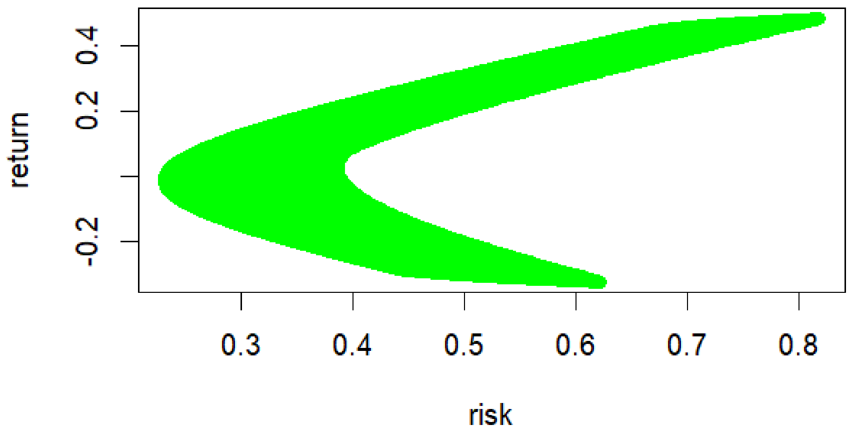


Figure 1: Portfolio without risk-free assets.

The composition of the minimum variance portfolio under this condition is shown in Table 2.

DOI: 10.54254/2754-1169/4/20221024

Table 2: The minimum variance portfolio without considering the risk-free asset.

| | 'CCBC' | 'CMBCL' | 'BCL' |
|---|---|---|---|
| Weight | 0.676 | -0.127 | 0.452 |
| Min_variance | 0.231 | | |

It can be observed that in this portfolio, BCL and CCBC with smaller variance are given higher weights, and CMBCL with larger variance is given a lower weight. After considering risk-free assets, the variance levels and expected returns corresponding to investment portfolios with different weight levels are studied, as shown in Figure 2:

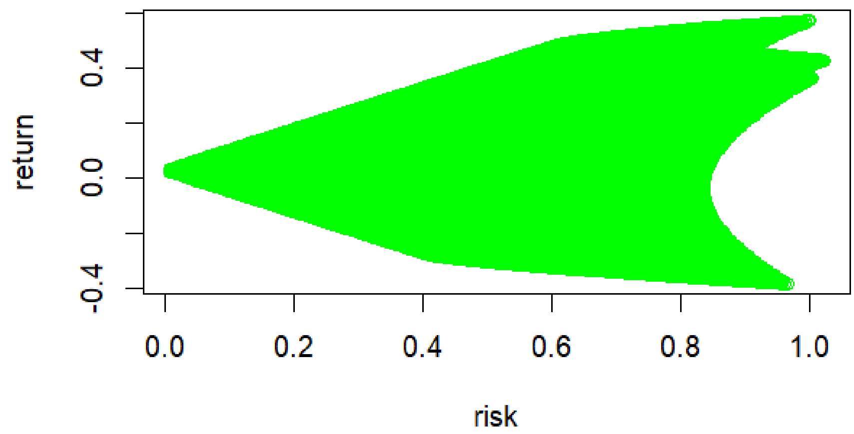


Figure 2: Portfolio Considering Risk-Free Assets.

The weight distribution of each asset in the minimum variance portfolio under this condition is shown in Table 3

Table 3: The weights of each asset in the minimum variance portfolio considering risk-free assets.

| | 'CCBC' | 'CMBCL' | 'BCL' | 'Rf' |
|---|---|---|---|---|
| Weight | 0 | 0 | 0 | 1 |
| Min_variance | 0 | | | |

It can be observed that in this portfolio, in order to minimize the risk, that is, minimize the variance of the portfolio, all assets other than risk-free assets are assigned a weight of 0. With and without risk-free assets, it can be observed that as investments are diversified, the variance of portfolios relative to individual assets decreases significantly, with the minimum variance portfolio in both cases much lower. The variance corresponding to a single asset.

## 4.2. Sharpe Ratio and Tangent Portfolio

A tangent portfolio is the line between the point corresponding to the portfolio that invests only in risk-free assets and the point corresponding to the portfolio with the largest Sharpe ratio [10]. When risk-free assets are not considered, the tangent portfolio are plotted as shown in Figure 3:

DOI: 10.54254/2754-1169/4/20221024

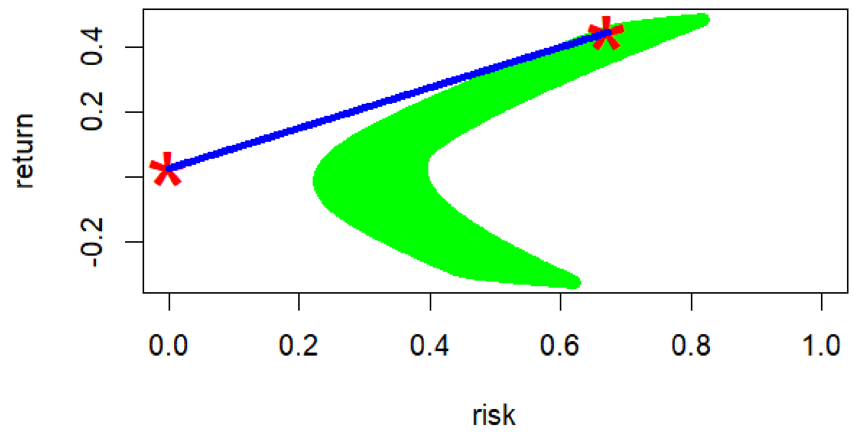


Figure 3: The tangent portfolio (without considering risk-free assets).

At this point, the weights of each asset are: -0.027, 3.000, and -1.973, and the Sharpe ratio for this portfolio is: 0.623. It can be observed that in this portfolio, although the variance of CMBCL is larger, the average return of this asset is significantly higher than the other two assets, so it is given the highest weight.

Table 4: The weights of each asset in the max Sharpe Ratio portfolio without considering risk-free assets.

| | 'CCBC' | 'CMBCL' | 'BCL' |
|---|---|---|---|
| Weight | -0.027 | 3.000 | -1.973 |
| Sharpe Ratio | 0.623 | | |

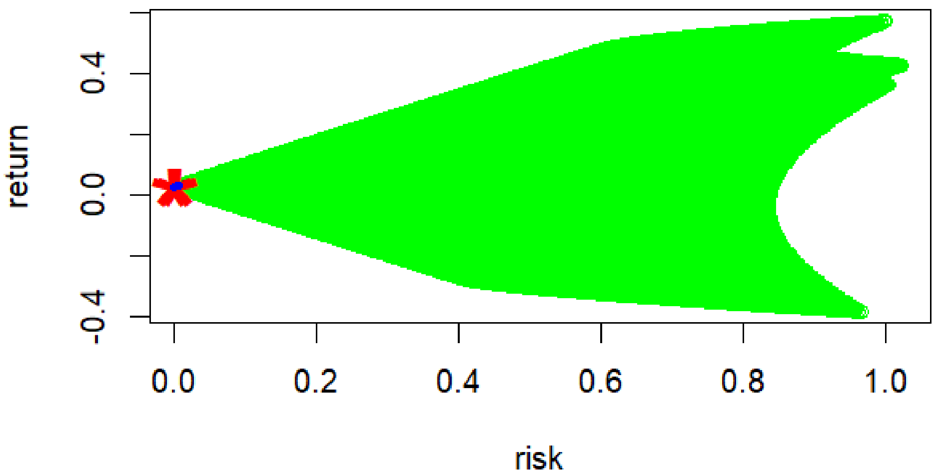


Figure 4: The tangent portfolio (considering risk-free assets).

In Figure 4, it can be observed that a risk-free asset has a higher return but a lower variance relative to a risky asset, so the proportion of this asset in the tangent portfolio is higher.

DOI: 10.54254/2754-1169/4/20221024

Table 5: The max Sharpe Ratio portfolio without considering the risk-free asset.

| | 'CCBC' | 'CMBCL' | 'BCL' | 'Rf' |
|---|---|---|---|---|
| Weight | 0.020 | 0.020 | -0.020 | 0.980 |
| Min_variance | 1.612 | | | |

## 5. Conclusion

After analysis, with the diversification of investment, portfolio will get different expected return and the SD of the portfolio can be different. When considering and ignoring risk-free assets risk-free assets, the expected return of the investment portfolio changes relative to a single asset. It is unpredictable, but the risk of the investment portfolio will be gradually spread out. At the same time, the Sharpe ratio is a common indicator for evaluating funds. Through analysis, it can be obtained that for a given situation of multiple risky assets and one risk-free asset, there is always an investment portfolio, so that the Sharpe ratio corresponding to this portfolio of risk-free assets and risky assets ratio is maximum. Among the minimum variance portfolios, this paper considers two situations. Without adding risk-free assets, the weight of each asset in the minimum variance portfolio is obtained, among which China-Merchants-Bank-Co-Ltd has the lowest weight, because the corresponding annual return of the stock fluctuates very far, so is given a lower weight. After adding risk-free assets, since the risk of risk-free assets is 0, and the variance of other stock assets is greater than 0, in order to achieve the variance of the portfolio, only investment in risk-free assets is considered when investing. In the tangent portfolio and the portfolio with the largest Sharpe ratio, this paper also considers two situations, including considering risk-free assets and not considering risk-free assets. When the risk-free assets are not considered, since there is no portfolio with a variance of 0, it can be clearly observed that the tangent portfolio, that is, the connection between the portfolio with the largest Sharpe ratio and the corresponding point of the risk-free asset, where China-Merchants-Bank-Co-Ltd is given the highest weight. When considering risk-free assets, risk-free assets have the highest weight in the portfolio. The results in this paper may benefit certain investors when constructing financial portfolios.

## References


[1] *Elton, E.: Modern portfolio theory and investment analysis, (2007).*

[2] *Harry M.: Portfolio Selection.The Journal of Finance, 7 (1), 77-91(1952).*

[3] *Sharpe W F.: Capital Asset Prices: A Theory of Market Equilibrium. Journal of Finance, 425-442(1964).*

[4] *Black F, Litterman R.: Global portfolio optimization. Financial Analysts Journal, (9/10) :28-43(1992).*

[5] *Idzorek T M.: A step-by-step guide to the Black-Litterman model: Incorporating user-specified confidence level. Duke University, working paper, (2004).*

[6] *Bertsimas D, Gupta V, Paschalidis IC.: Inverse Optimization: A New Perspective on the Black-Litterman Model. Operation research, (2014).*

[7] *Ma J.: An asset allocation model with liquidity risk measurement constraints based on the Black-litterman model. Zhejiang University, (2005).*

[8] *Ledoit O, Wolf M.: Robust Performance Hypothesis Testing with the Sharpe Ratio. IEW - Working Papers, (2008)*

[9] *Zhou X Y, Yin G.: Continuous-time mean-variance portfolio selection with regime switching// Decision and Control, (2002)*

[10] *Kandel S., Stambaugh R F.: On correlations and inferences about mean-variance efficiency. Journal of Financial Economics, 18(1), 61-90(1987).*