The 6th International Conference on Economic Management and Green Development (ICEMGD 2022)
DOI: 10.54254/2754-1169/4/20221079

# Cattle's Optimal Diet

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***Abstract:*** Linear programming is an important mathematical technique, which is widely used in military operations, economic analysis, management, and engineering technology to find the optimal choice. Moreover, it is useful when trying to find the best outcome that involves several resource constraints, like animal diet. In this paper, we will become cattle ranchers, trying to figure out the best combination of feeds for the three groups of cattle. We also considered the cattle's environmental impact, methane emission, into the linear program. The goal is to minimize the cost while balancing the cattle's nutritional needs and lowering methane emissions. The Simplex Loop used to solve linear programs is also explained, however, we used MATLAB to arrive at the solution.

***Keywords:*** cattle diet problem, linear optimization, methane emission, Simplex Method.

## 1. Introduction

Linear programming is also called linear optimization. It is extremely useful when we need to minimize costs in managing a business, or maximize daily nutrients taken in managing everyday diet, linear optimization will help us to find the optimal solution. Specifically, we will be looking at cattle diet and the overall goal is to determine the optimal diet for beef cattle.

Eating beef had a long history throughout human's development. In 1534, the first longhorn cattle were introduced to America by Spanish Explorer. Since then, cattle gradually played a more and more important role in Americans' daily diet. To boost meat production, initial nutrition studies, which were called 'Nutritional Science', began in the 1920s [1]. It not only studied cattle, but also studied sheep, swine, horses, and turkeys [2]. Besides cattle production, methane emitted by cattle is also accounted for. Ocko (2019) suggests that methane has more than 80 times the warming power of carbon dioxide, which means methane emission is also a major cause of global warming [3],[4]. Therefore, reducing methane emission during cattle production is important.

Our paper will be divided into parts: introduction, problem, setup and data, LP (linear program) formation, methodology, solution, analysis, conclusion, and finally citation for our references.

## 2. Problem

Our problem simulates the cattle ranchers in Cameron, Texas who are trying to figure out the best diet for their cattle. The cattle are of the same kind—Angus. The goal is to make the most money, in other words, the goal is to minimize the cost to maximize the profit. All data researched are based on

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the prices in Cameron city [5]. Since the price of feeds varies from one seller to another, we took the average price. We are pretending this is reality as if we are the cattle ranchers. There are three factors we focus on: weight of the cattle and nutrition requirements of cattle, and methane emission. First, the weight of the cattle is directly proportional to the income, so we want the cattle to be heavier. Then, to reduce food waste and spending, we want to feed the cattle so that they have the right amount of nutrition to fulfill their daily nutrition requirements. In this case, we spend less money, but the cattle can still grow heavier. Again, the objective is to maximize the profit. Thus, any factor that will affect the profit is part of the constraint. Due to climate change, the government fines those who emit methane above the methane threshold. Therefore, we want the cattle to produce the least amount of methane to minimize the penalty.

Moreover, the amount of methane emitted is determined by the total gross energy of the cattle's diet. As cattle digest the feeds, chemical reactions break down the food, methane is produced, then emitted to the environment through belching [6].

## 3. Setup and Data

For this ranch, there are 11 types of feeds to choose from. Although lots more choices on feeds are available and cattle need a variety of nutrients, we will only focus on the 6 most important nutrition requirements. Data are shown below:

Table 1: shows the amount of each nutrient in the 11 types of feeds. Each row contains the nutrition facts for the feed and each column corresponds with the nutrient. All nutrition information is the average across resources [7-15]. *Note: salt and water are special. Salt maintains mineral balance at the right level and keeps cattle healthy*.

| | Protein (g/kg) | Fiber (g/kg) | Potassium (g/kg) | Calcium (g/kg ) | Sodium (g/kg ) | Energy (MJ/kg) |
|---|---|---|---|---|---|---|
| Alfalfa Hay | 182 | 289 | 24.6 | 16.8 | 0.3 | 18.2 |
| Orchard Grass Hay | 130 | 314 | 3 | 1.7 | 0.02 | 8.38 |
| Corn | 30 | 70 | 10 | 0.3 | 0.8 | 14.5 |
| Oats | 110 | 139 | 4.9 | 1.1 | 0.1 | 19.5 |
| Barley | 110 | 281 | 14 | 4.9 | 0.9 | 17.3 |
| Wheat straw | 42 | 415 | 11.2 | 4.8 | 0.1 | 18.5 |
| Rice | 103 | 310 | 16 | 2.9 | 0.01 | 16.2 |
| gamba Grass | 76 | 367 | 16.4 | 4.1 | 0.09 | 18.1 |
| Beet | 78 | 81 | 15.2 | 2.5 | 0.78 | 16.9 |
| Salt (6g) | 0 | 0 | 0.000498 | 0.00138 | 387.5 | 0 |
| Water (240g) | 0 | 0 | 0 | 0 | 0.04 | 0 |

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Table 2: shows the cost for each feed in $ per kg. All costs are specific to the price in Cameron, Texas which is where the ranch is located.

| Type of Feed | Cost ($/kg) |
|---|---|
| Alfalfa Hay | 0.205 |
| Orchard Grass Hay | 0.3 |
| Corn | 0.222 |
| Oats | 0.454 |
| Barley | 0.224 |
| Wheat straw | 0.13 |
| Rice | 2.003 |
| Gamba Grass | 0.95 |
| Beet | 4.89 |
| Salt | 8.99 |
| Water | 0.02 per gallon |

However, it would be unrealistic if all the cattle are the same size. The size of cattle is determined by their age and different size require different amounts of nutrients. Thus, we need to create the optimal diet for adult cattle, baby cattle (veal), and growing cattle. Growing cattle is between baby cattle and adult cattle, older than baby cattle, but younger than adult cattle. Daily nutrition requirements of cattle by size are shown in **Table 3**.

Table 3: shows the amount of each nutrient the three groups of cattle need *daily* [16-20]. Units are consistent with data in table 1.

| | Net Energy for Maintenance (Mj) | Dry matter intake (kg/day) | Protein (g) | Water (gal) | Crude Fiber (%) | Potassium (g) | Calcium (g) | Sodium (g) |
|---|---|---|---|---|---|---|---|---|
| Adult Male Cattle (1000kg) | 78.8684 | 14 | 671.317 | 10.8 gal | 1269.8 | 21.768 | 30.838 | 25 |
| Baby cattle (60kg) | 37.82 | 7 | 201.4 | 21 gal | 932.91 | 10.23 | 201.4 | 10 |
| Growing cattle (450kg) | 51.21 | 12 | 306 | 9 gal | 12.5 | 19.817 | 29.204 | 23 |

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In addition, baby cattle are vulnerable and need more care. Other costs like paying the vet for giving birth also need to be considered into the linear program. Data are shown in **Table 4**:

Table 4: shows the extra expenses for taking care of one baby cattle [21]. Note: transportation fee also applies to adult and growing cattle.

| | |
|---|---|
| Vet | $25 |
| Breeding | $40 |
| Marketing | $25 |
| Transportation (applies to all cattle) | $15 |

Since baby cattle (veal) has a limited market meaning the demand is not very high, we decided to lower the selling price for the veal to reduce the business competition between ranches.

## 4. Forming the Linear Program

The objective is to maximize the total income subtracted by the costs, which is the profit. We want to find x, the amount of each type of feed. Obviously, there are constraints. First, sign constraints. The quantity of feeds (x) must be positive, as well as the estimated methane emitted by cattle ($\widehat{M}$) and the change in weight of cattle ($\Delta W$). Next, the total amount of nutrients provided by the feeds (Ax) must meet the nutrition requirements ($\hat{b}$). Cattle is different than a human. They do not know if they are full or hungry. They just eat what's given to them. To not waste food and money, we need to set a limit, which is the reasonable amount of dry matter intake per day (D). Full LP is shown below, see Table 5 for symbol representation:

$$\begin{aligned} \text{Objective:} \quad \max \quad & p \cdot \Delta W - 0.015W - c^T \cdot x - f \cdot \widehat{M} \\ \text{s.t.} \quad & x \geq 0 \\ & \widehat{M} \geq 0 \\ & \Delta W \geq 0 \\ & A \cdot x \geq \hat{b} \\ & e^T \cdot x = D \\ & \widehat{M} \geq \left(\frac{m}{mec}\right) \cdot C^T \cdot x - M_0 \\ & \Delta W = 13.91(C^T \cdot x - E_m) \cdot W^{-0.6837} \end{aligned}$$

We can change the linear program to general LP form by changing the objective to a minimization problem. To simplify the linear program, we plugged the last constraint into the $\Delta W$ in the objective. The new linear program is:

$$\begin{aligned} \text{Min} \quad & (c - 13.91pW^{-0.6837}C + 0.015W)^T x + f\widehat{M} \\ \text{s.t.} \quad & x \geq 0 \\ & \widehat{M} \geq 0 \\ & Ax \geq \hat{b} \\ & \left(\frac{m}{mec}\right) \cdot C^T \cdot x - \widehat{M} \leq M_0 \\ & e^T \cdot x = D \end{aligned}$$

DOI: 10.54254/2754-1169/4/20221079

Table 5: shows what each symbol represents in the linear program.

| symbol | Representation |
|---|---|
| p | constant for the sell price of the cattle ($) |
| $\Delta W$ | weight increase of cattle (daily) |
| c | cost ($) |
| $c^T$ | feed cost matrix |
| x | vector for the amount of each type of feed (kg) |
| $f \cdot \widehat{M}$ | financial penalty when methane emission is over the limit |
| $\widehat{M}$ | methane emitted by cattle in kg that is above the limit (daily) |
| A | a M x N matrix for the amount of nutrients each feed has |
| $\hat{b}$ | cattle's nutritional requirement (daily) |
| $e^T$ | a vector that contains only ones |
| D | total allowed (reasonable) dry matter intake amount (daily) |
| m | constant for Methane conversion |
| mec | constant for methane energy |
| C | vector of energy concentration in each feed |
| $M_0$ | methane emission threshold |
| $E_m$ | amount of energy of maintenance a cattle need (daily) |

Table 6: shows the value of the constant used in the linear program [22-24].

| Constant | Value |
|---|---|
| Price for 1 kg of adult cattle (p) | $6.04 |
| Price for 1 kg of veal (p) | $20[1] |
| Price for 1 kg of growing cattle | $6 |
| Penalty for methane emitted above the limit (f) | 15% of selling price[2] |
| Constant for methane conversion (m) | 0.03 |
| Constant for methane energy (mec) | 55.65 |
| Threshold for methane emission ($M_0$) | 0.125 |

[1]*The actual selling price for veal is $25.7/kg. We decided to lower it to $20 (veal) has a limited market meaning the demand is not very high, we decided to lower the selling price for the veal to reduce the business competition between ranches.*
[2]*The penalty for emitting too much methane varies. In this case, we decided the fine is 15% of the cattle's selling price which for adult cattle is $0.9, baby cattle is $3, and growing cattle is also $0.9.*

However, to use the simplex loop, the linear program has to be in canonical form. There is the primal canonical form and dual canonical form. We are using primal for this diet problem which takes the form of:

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & x \geq 0 \\ & Ax = b \end{aligned}$$

There are three criterias:

1. The objective function must be a minimization problem

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2. Sign constraints, variables have to be non-negative
3. Equality constraints, Ax has to equal b

To change the linear program to canonical form, we used slack variables v and μ where

$$v \stackrel{\text{def}}{=} Ax\text{-}\hat{b} \geq 0$$
$$\mu \stackrel{\text{def}}{=} M_0\text{-}[\left(\frac{m \cdot k}{mec}\right) \cdot C^T \cdot x - \widehat{M}]$$

LP in canonical form:

$$\begin{aligned} \min \quad & \hat{c}^T \hat{x} \\ \text{s.t.} \quad & \hat{x} \geq 0 \\ & \widehat{A}\hat{x} = \hat{b} \end{aligned}$$

$$\widehat{A} = \begin{pmatrix} A & 0 & -I & 0 \\ e^T & 0 & 0^T & 0 \\ \frac{m}{mec} C^T & -1 & 0^T & 1 \end{pmatrix}, \hat{x} = \begin{pmatrix} x \\ \widehat{M} \\ v \\ \mu \end{pmatrix}, \hat{c} = \begin{pmatrix} c - 13.91pW^{-0.6837}C + 0.015W \\ f \\ 0 \\ 0 \end{pmatrix}, \bar{b} = \begin{pmatrix} \hat{b} \\ D \\ M_0 \end{pmatrix}$$

## 5. Methodology

Remember that inequality in vector sense means element-wise comparison. Therefore, sets of linear inequalities formed a convex polygon in the n-dimensional Euclidean space. This result, on the other hand, also leads to the conclusion that the optimal solution to the standard LP problem, could NOT exist inside the open polyhedron, which is the closed polyhedron minus its boundary. Furthermore, if an optimal solution exists for the LP problem, then it must sit on a vertex of this polyhedron. Assuming non-degeneracy of the LP problem, we could use simplex method as a rigid algorithm to solve it. Using the Simplex Loop, we start by finding a special solution of the constraints' equations (which are derived from restraint polynomials) called basic feasible solution (BFS) which could be geometrically interpreted as one vertex of this polyhedron in the n-dimensional Euclidean space.

Starting from the BFS, an iteration process, through which, step by step, we swap out one variable of the chosen basis, and swap in one variable out of the chosen basis, till we have all the coefficients for the non-basis variables in the objective function are negative, while satisfying the constraint that all variables must be non-negative. This constraint is equivalent to the step at which we have all $c_j$-$z_j \geq 0$, for all j not in mathbb{B}. This way, we know the optimal value of the objective function must be the non-zero constant term, by setting all non-basis variables as zero.

On the other hand, it is possible that some LP problems are ill-posed (degenerated LP problems) and do not have basic feasible solutions (BFS). For example

$$\begin{aligned} \text{Min} \quad & x \\ \text{s.t.} \quad & y \geq 0 \end{aligned}$$

However. if we can form the bounded polyhedron by given inequalities in canonical form, we can always expect one or infinite optimal solutions for the LP problem by traversing vertices of this polyhedron using the Simplex Method, with n being the number of vertices.

Since there are many variables and constraints present for our cattle diet linear program, we used MATLAB to solve this linear program. Thus, we simplified the LP to the following:

$$\begin{aligned} \min \quad & \hat{c}^T \hat{x} \\ \text{s.t.} \quad & \hat{x} \geq 0 \\ & A_1 \hat{x} \geq b_1 \\ & e^T \cdot x = D \end{aligned}$$

$$A_1 = \begin{pmatrix} A \\ -\frac{m}{mec} C^T \end{pmatrix}, \hat{x} = \begin{pmatrix} x \\ -\widehat{M} \end{pmatrix}, \hat{c} = \begin{pmatrix} c - 13.91pW^{-0.6837}C + 1.75W \\ f \end{pmatrix}, b_1 = \begin{pmatrix} \hat{b} \\ -M_0 \end{pmatrix}$$

The objective stays the same, as well as the sign constraint on x. Instead of $\hat{A}\hat{x}=\hat{b}$, we took off the slack variables and changed the equality to inequality. We also separated out $e^T \cdot x = D$ so it is a constraint on its own. Since the inequality has to be greater than or equal to ($\geq$), we multiplied both sides of the methane emission constraint by -1 and flipped the sign. To solve this LP, we just plugged the values into MATLAB using the code:

x = linprog(f,A,b,Aeq,beq,lb,ub) where f corresponds with $\hat{c}^T$ in the LP above; A corresponds with $A_1$; b corresponds with $b_1$; $e^T$ corresponds with Aeq; and D corresponds with beq [25].

## 6. Solution

| For baby cattle: | For adult cattle: |
|---|---|
| $x_1 = 0.5916$ | $x_1 = 0$ |
| $x_2 = 0$ | $x_2 = 2.2556$ |
| $x_3 = 0$ | $x_3 = 0$ |
| $x_4 = 6.3847$ | $x_4 = 8.0383$ |
| $x_5 = 0$ | $x_5 = 3.7062$ |
| $x_6 = 0$ | $x_6 = 0$ |
| $x_7 = 0$ | $x_7 = 0$ |
| $x_8 = 0$ | $x_8 = 0$ |
| $x_9 = 0$ | $x_9 = 0$ |
| $x_{10} = 0.009$ | $x_{10} = 0.024$ |
| $x_{11} = 79.4936$ | $x_{11} = 40.8824$ |
| $x_{12} = 0$ | $x_{12} = 0$ |

There is no optimal solution for growing cattle. The reason is that growing cattle have about the same selling price as adult cattle, but they are not as heavy as adult cattle. This also makes sense in real life, if they are the same price, one would want to raise the growing cattle to adult cattle to sell more meat.

## 7. Analysis

According to the solution to the linear program, the optimal diet for adult cattle is 2.2556 kilograms of orchard grass hay, 8.0383 kilograms of oats, 3.7062 kilograms of barley, 0.024 kilograms of salt, and 40.8824 kilograms of water. For baby cattle, the best diet is 0.5916 kilograms of alfalfa hay, 6.3847 kilograms of oats, 0.009 kilograms of salt, and 79.4936 kilograms of water. Since baby cattle won't be able to eat solid food, we will grind the fodders with water to turn them into solid food. This way, methane emissions will decrease which helps us to achieve our goal of minimizing the cost.

## 8. Conclusion

For this research, a revised cattle diet problem was formulated by taking into consideration methane emissions. The problem was presented as a LP problem and solved using the Simplex Method. The result provides optimal rationing of 12 cattle feeds for maximal profit, considering 15% emission fine taken on the sale price. Additionally, sensitivity analysis indicates that the emission penalties played a pivoting role in the cattle business. Furthermore, this model could provide optimal solutions for different farm animals and different nutrition requirements by simply refactoring a few parameters of the given model.

There are several places with our simplified model that could be improved for future studies. First, cattle weight gain could be formed as a dynamic programming problem by introducing a discrete-time weight function. Secondly, the same weight-growth model was applied to all age groups.

The optimal solution we obtained from our LP problem indicates that we should only provide two to three types of food out of 12 total. Even though economically this is the optimal solution for maximal profit, ecologically, over-simplified food sources could lead to malnutrition, in which case supplements must be provided outside the calculated food rationing for cattle's health. In fact, many crucial nutrients were not included in our linear program. The parameters we obtained might be inaccurate or the model is over-simplified, and some factors were overlooked. On the other hand, we successfully found the optimal diet for cattle while reducing methane emissions. Cattle diet optimization problem relates to our daily lives. Our final solution is significant because we tried to simulate real life as much as possible with real life data and constraints like business competition. Ranchers could use our result or use our linear program when making decisions and this paper could raise awareness for controlling methane emission.

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