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Portfolio Construction and Analysis Based on the Hang Seng Index from 2009 to 2021

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Abstract: Maximizing returns at a low risk is a popular goal for investors. The Hang Seng Index (HSI) is a significant indicator of Hong Kong stocks, which is made up of 66 stocks traded on the Hong Kong stock market. This study aims to analyze and the find optimum portfolio based on the Hang Seng index. For this purpose, Fama experiment are applied firstly to evaluate the systematic risk and the idiosyncratic risk. But for convenience, three stocks from three different companies, Tencent, BYD and the CITIC are chosen to construct the portfolio. Then R is used in this study to analyze the portfolio. The minimum variance portfolio, Sharpe ratio and the tangency portfolio are all calculated by R then. The results show that the weights of the tangency portfolio are -200% for Tencent, 42% for BYD and 258% for CITIC, which may be helpful to make the investment decision.

Keywords: Fama experience, minimum variance portfolio, Sharpe ratio, tangency portfolio.

1. Introduction

Most people are risk-averse, which also implies investors hope to gain higher return at the lowest level of risk. In order to find a good way to analyse the portfolio, Markowitz proposed the modern portfolio theory [1]. Pringle and VanOrden also mentions that Markowitz defined an efficient portfolio based on the mean-variance method as one that delivers least variance for a given expected return and maximum return for a given variance [2]. In the modern portfolio theory, the total risk can be reduced by keeping a diversified portfolio. In this case, this gives the idea of the diversification in order to reduce the portfolio risk. According to Chong, Jin and Phillips [3], In modern portfolio theory, there are two different settings to consider the risk of an asset. For a single asset, the standard deviation is a relevant measurement for risk. For a portfolio that is well-diversified, on the other hand, is more concerned with systematic risk.

Based on the modern portfolio theory, Capital Asset Pricing Model (CAPM) was established by Sharpe and Linterner, respectively, to investigate how a risky asset's expected return correlates with the market risk premium [4]. In part of the CAPM model mentioned by Sharpe, he said that risks can be classified into two types, which are systematic risk and idiosyncratic risk [5-6]. The difference between the two risks is that the idiosyncratic risk could be well diversified by investing in portfolios rather than in individual assets, while the systematic risk cannot. This is part of the diversification. And it is wise for an investor to invest in a portfolio instead of the single stock. That is because the diversification will be good to minimize the risk [7]. A more diversified portfolio can be created by

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reducing the mean-variance portfolio to an evenly weighted one [8]. Zaimovic et al. also argued that the risk can be totally diversified even there are thousands of assets in a portfolio [9].

As Kim et al. mentioned, the modern portfolio theory not only gives us a framework to analyse and select a portfolio based on a mean-variance method, but it also describes the concept of diversifying a portfolio to reduce risk [10]. The diversification used as a method to reduce the portfolio risk. Then, the goal of gaining higher return is still the most popular topic. Sharpe [6] then created the Sharpe ratio to quantify the performance of equities based on the CAPM model. Investors always focus on the positive Sharpe ratio. It would be better to invest in the portfolio which has the higher Sharpe ratio, since this kind of portfolio has the higher return when taking the same risk compared with the portfolios that have lower Sharpe ratio. Then, according to Yang's paper [11], each portfolio has a capital market line that illustrates the link between expected return and volatility. And the capital market line is tangent to efficient frontier. The tangency portfolio is where the capital market line crosses the frontier, which is the only and best option for a risk-averse investors in the theoretical model.

This paper will mainly focus on the portfolio constructed on the Hang Seng index. And the methods used in this paper are all related to the mean-Variance approach and the capital asset pricing model. The Hang Seng index, launched on 24 November 1969, plays a key role in monitoring and recording the changes of the stock market in Hong Kong. Furthermore, the largest and the most liquid Chinese companies are measured by the Hang Seng index. In this case, it would be very helpful and practical for us to study the performance of many Asian companies chosen from the Hang Seng index. And in order to diversify the risk, the method of Fama experiment is used at the beginning, which explains how many stocks we should invest to build the portfolio. According to the Fama experiment, three stocks are chosen to construct the portfolio. And the three stocks are Tencent, BYD and the CITIC.

Initially, the Fama experiment has been used to examine the reasonable numbers of stocks to construct the portfolio to diversify the idiosyncratic risk well from a portfolio will be detected. Secondly, the empirical rule and the autocorrelation function has been used to test all the chosen data follows the normal distribution and is identical and independent distributed. Thirdly, the investment portfolio of the chosen stocks is constructed both with and without investing in the riskless asset. Fourthly, the minimum variance portfolio is founded by r optimizer. Finally, the maximum Sharpe ratio portfolio is calculated.

2. Methods

2.1. Fama Experiment and Diversification

The Fama experiment is based on the fundamental method of measuring the risk by the standard deviation. And it is also known as the diversification, which is a great risk control strategy by increasing the number of different kinds of stocks in a portfolio. According to Statman's article, a change in the stock weights, variances and the covariances between the stocks will have an influence on the risk of the portfolio [12]. Based on the Fama experience, it is usually noticed that the risk of the portfolio keeps stable when we invest in around 15 to 20 assets in the portfolio. That is the non-diversifiable risk, approximately to the systematic risk.

2.2. Autocorrelation Function (ACF)

Mathematically, Autocorrelation is the correlation between points that are apart by different time delays, which measures the link between a variable's present values and its lagged version of value. And Autocorrelation function (ACF) is a good tool for us to find whether the data is independent and

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identical distributed. That is the pattern of the data What is more, we can compute the autocorrelation by acf function in R.

The autocorrelation function (ACF) at lag k, denoted ρ_k, is defined as $\rho_k = \gamma_k/\gamma_0$ where $\gamma_k = \text{cov}(y_i, y_{i+k})$ for any i. When measuring autocorrelation of returns:

$$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \tag{1}$$

for $k \geq 0$,

$$S_k = \frac{1}{n}\sum_{i=1}^{n-k}(R_i - \bar{R})(R_{i+k} - \bar{R}) = \frac{1}{n}\sum_{i=k+1}^{n}(R_i - \bar{R})(R_{i-k} - \bar{R}) \tag{2}$$

$$r_k = \frac{S_k}{S_0} \tag{3}$$

Where r_k is the autocorrelation function at lag k ; S_k is the autocovariance; R_i is the return at time i; $\overline{R}$ is the mean of time series of return.

2.3. Mean-Variance Approach

The Mean-Variance analysis is one part of the modern portfolio theory (MPT). According to Elton and J., & Gruber's paper, Markowitz is the first person to solve the portfolio problem using the mean-variance approach [13]. The mean-variance analysis is a good mathematical method to analyze a single stock or portfolio. Under the mathematical model, the mean represents the average return of a portfolio while the variance measures the risk, which shows how spread out the returns of a specific security are.

$$\sum_i \omega_i = 1 \tag{4}$$

$$E(r_p) = \sum_i \omega_i r_i \tag{5}$$

Where r_p is the portfolio return; $E(r_p)$ is the expectation of the portfolio return; ω_i is the weights on asset i; r_i is the returns on asset i.

$$\sigma_p^2 = \sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} \tag{6}$$

Where σ_p is the standard deviation of the portfolio; σ_i is the sample standard deviation of the periodic returns on the asset i, and ρ_{ij} is the correlation coefficient on the asset i and j. For a portfolio with three assets:

$$\omega_1 + \omega_2 + \omega_3 = 1 \tag{7}$$

$$E(r_p) = \omega_1 E(r_1) + \omega_2 E(r_2) + \omega_3 E(r_3) \tag{8}$$

$$\sigma_p^2 = \omega_1^2\sigma_1^2 + {\omega_2}^2\sigma_2^2 + {\omega_3}^2\sigma_3^2 + 2\omega_1\omega_2\sigma_1\sigma_2\rho_{12} + 2\omega_1\omega_3\sigma_1\sigma_3\rho_{13} + 2\omega_2\omega_3\sigma_2\sigma_3\rho_{23} \tag{9}$$

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2.4. Efficient Frontier

The efficient frontier shows portfolios with lowest standard deviation for a given level of the expected return. By that means the portfolio on the frontier has the highest returns for the risk assumed. If an investor can perfectly fit a portfolio to its risk based on the CAPM, there would be an efficient frontier which shapes as a curve.

2.5. Sharpe Ratio

The Sharpe ratio is one of the indexes derived from CAPM, which investors use to determine an investment's excess return in relation to its risk. The Sharpe ratio is calculated as bellowed:

$$Sharpe\ ratio = \frac{r_p - r_f}{\sigma_p} \tag{10}$$

Where r_p is the expected return of the portfolio; r_f is the risk-free rate; σ_p is the standard deviation of the portfolio.

3. Results

3.1. Fama Experiment

The original data set in this paper is extracted from the Hang Seng index, which represents the monthly stock price of 66 stocks from December 31, 2009, to December 31, 2021. Since there are some missing periods in the whole dataset, all the stocks that cannot spread all the period are deleted. As a result, currently there are only 50 stocks in the whole dataset instead. Before we can construct the portfolio to invest in the market, fama experiment is used to measure the risks. Based on figure 1, we could know the scatter plot of the standard deviation of the Fama experiment. As we can see, the standard deviation is going to keep stable at about 10 stocks, which means that we need about 10 stocks to eliminate the idiosyncratic risk. And the systematic risk is approximately 0.05 here.

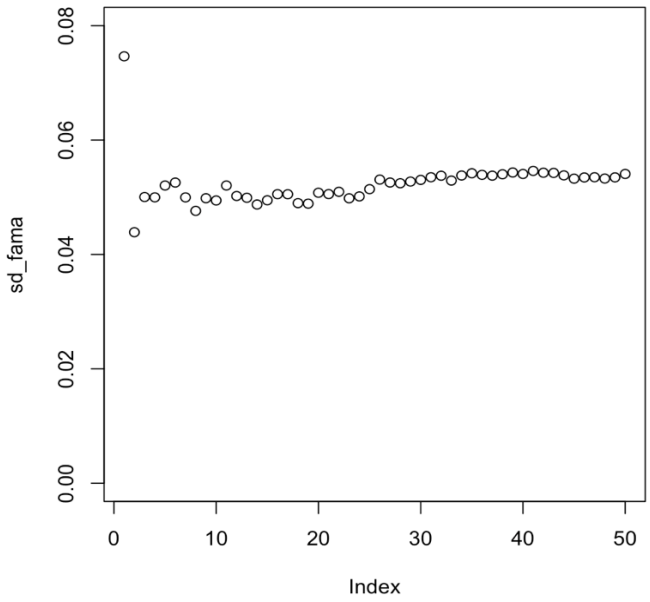

Figure 1: Scatter plot of Fama experiment.

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After we replicated the Fama experience in R, we can also get the bar plots shown in the figure 2. To describe it, the light grey represents the idiosyncratic risk and the dark grey bar added in the diagram 3 represents the systematic risk.

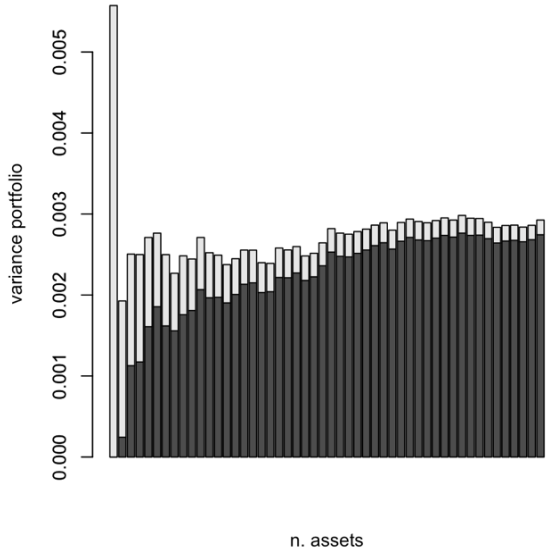

Figure 2: Fama experiment: bar plots.

3.2. Data Analysis

In the part of Fama experiment, we can know that the idiosyncratic risk of 3 stocks is low. And in case of R is running slowly, 3 stocks are chosen to analyse. i.e., Tencent Holding ltd, BYD Co. Ltd and CITIC.Ltd. The three companies are all in the different area. Tencent is one of the most successful internet and technology firms in the world. Tencent not only takes advantage in many fields like social media, video games, but it also offers a range of services such as cloud computing, FinTech. And BYD Company Ltd. is also one of China's largest private firms, which mainly focuses on the car manufactory area. It has grown to become a major manufacturer of automobiles, especially the electric and hybrid cars. For the CITIC limited, it is also one of China's largest enterprises of the Hang Seng Index. It has great influence on the area of finance, technological manufacturing, sophisticated materials, new consumption, and urbanisation. Overall, the three companies are all big and influential Chinese companies that have great development potential, and they are all risky assets. Hence, the three stocks are selected for the further study. Then, the index of the three chosen stocks is transferred to monthly returns for further investigation. And some basic information is shown in table 1 as follows.

Table 1: Descriptive data.

	Tencent	BYD	CITIC
Mean	-0.0142	0.0028	0.0110
Sd	0.0866	0.1578	0.0926
Median	-0.0207	-0.0001	0.0052
Max	0.2610	0.5836	0.4129
Min	-0.1771	-0.3657	-0.2221

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According to table 1, it is clearly that “Tencent” has the lowest average return of -0.01424437, while the “CITIC” has the highest monthly return of approximately 1.1%. Furthermore, we can observe that the “BYD” has the highest risk, then it is “CITIC” and “Tencent” has the lowest risk. When it comes to the median, “BYD” has the highest median of 0.1578367 while the “Tencent” has the lowest and negative median of -0.02070. What is more, we could find that the highest monthly return of each stock is above 25%, which are much more than their mean. So as the minimum returns. Then, the following figures (Figure 3) show the time series of the three stocks. As the figure of Tencent shows the Tencent has the highest monthly return of more than 20% just before 2012. However, then it reaches the bottom in 2012. The red line shows the time series of BYD, while the green line stands for the CITIC. According to these figures, the monthly returns of the three companies are fluctuated in last ten years. In general, it is very hard for us to predict the stock price based on the past data. In other words, the data are independent and identical distributed.

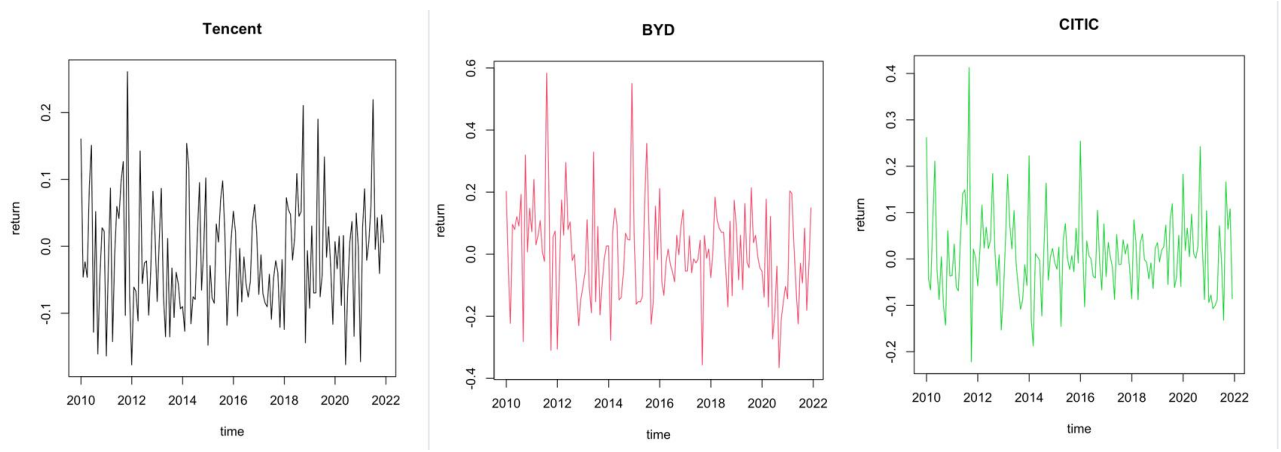

Figure 3: Time series of three stocks.

3.3. Test Normality and i.i.d

Before constructing the portfolio, we should test whether the assets we chosen follow the normal distribution and independent. Firstly, we can draw the histogram of each company and then add the probability density function on it. In the figure 4. We could see that the pdf of each stock is perfectly match the corresponding histogram. What is more, it is almost mound-shaped just as the normal distribution. Therefore, we can prove that the stock returns are normally distributed. For a double check, we can also use the empirical rule.

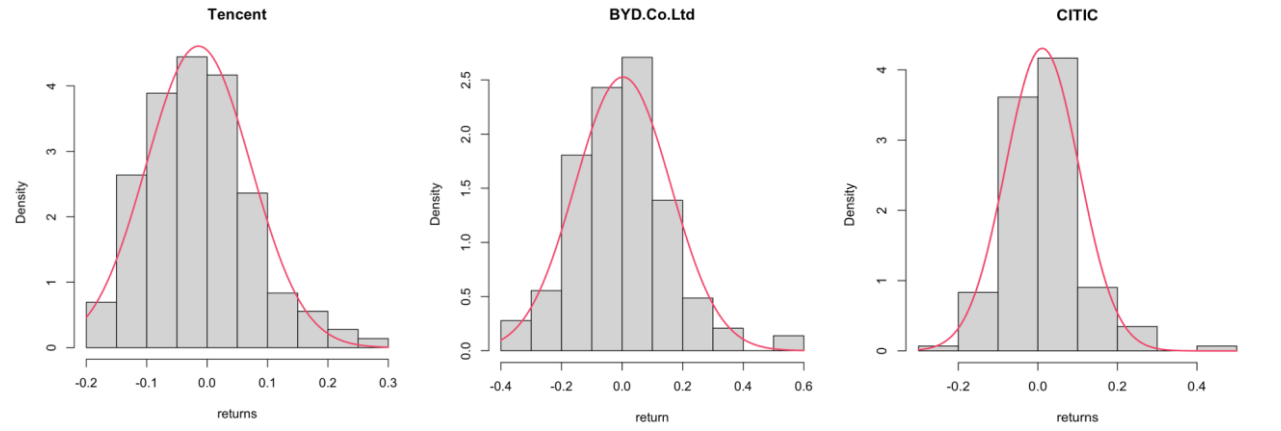

Figure 4: Normality fit.

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Using the autocorrelation function(acf) in R, we could check that the chosen samples are independent and identical distributed. Based on the below figures (Figure 5), the correlations between the current and lagged values are low. Consequently, the samples from Tencent, BYD and CITIC are independent and identical distributed. After these steps, we can then construct portfolios based on the three risky assets.

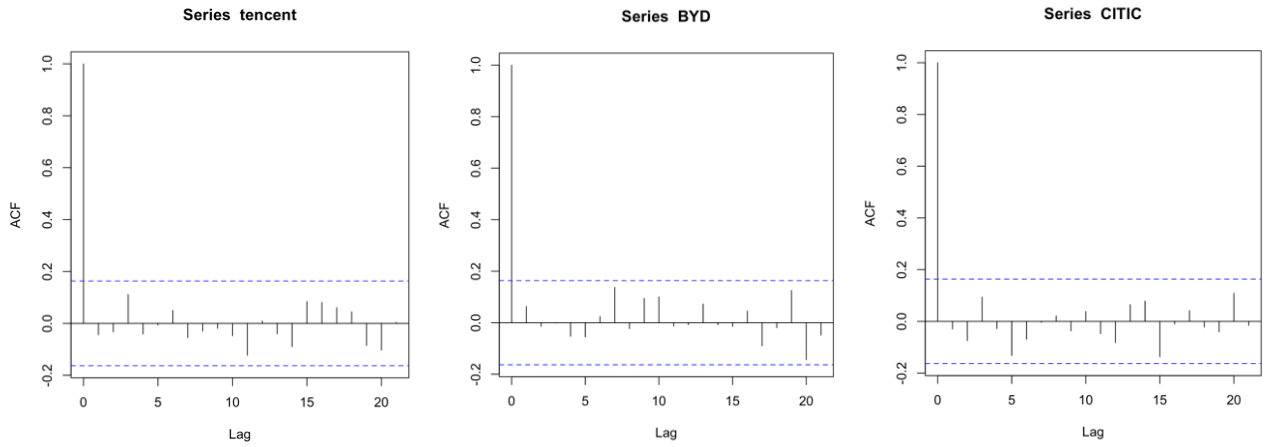

Figure 5: Acf test.

3.4. Portfolio Construction and Mean-Variance Analysis

First, the portfolio of the three chosen assets is constructed without risk-free asset. The figure 6 shows all the opportunity sets excluded shorting. In this case, the range of weights that can invest in the market is from 0 to 1. According to this figure, there is no doubt that the Tencent has both the lowest mean and the lowest risk. And we can know that the risk of the BYD is the largest. Therefore, based on this mean-standard plot, the CITIC would be the best choose of highest average monthly return of around 1% at the level of risk just a little more the Tencent.

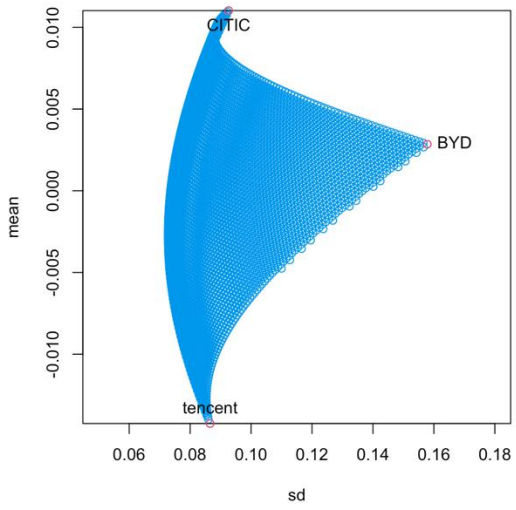

Figure 6: Portfolio constructed by three risky assets without short.

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3.5. Minimum Variance Portfolio

Table 2: Minimum variance portfolio.

	Tencent	BYD	CITIC	Minimum variance portfolio
Mean	-0.0142	0.0028	0.0110	-0.0030
Sd	0.0866	0.1578	0.0926	0.0728

Table 3: weights of the portfolio with lowest variance.

Minimum variance portfolio	Tencent	BYD	CITIC
Weights	0.55	-0.02	0.47

Table 2 compares the minimum variance portfolio's mean and standard deviation to three single stocks. Conclusively, the average return of the portfolio with the lowest risk is around -0.3%, which is lower than the return of BYD and CITIC. However, the minimum variance portfolio has the lowest risk, which is 0.0728. And the table 3 shows the proportions of the portfolio that has the minimum variance, accounts for approximately 55%, -2%, 47%, respectively. That means we should buy 55% Tencent, 47% CITIC stocks, but short the BYD by 2%.

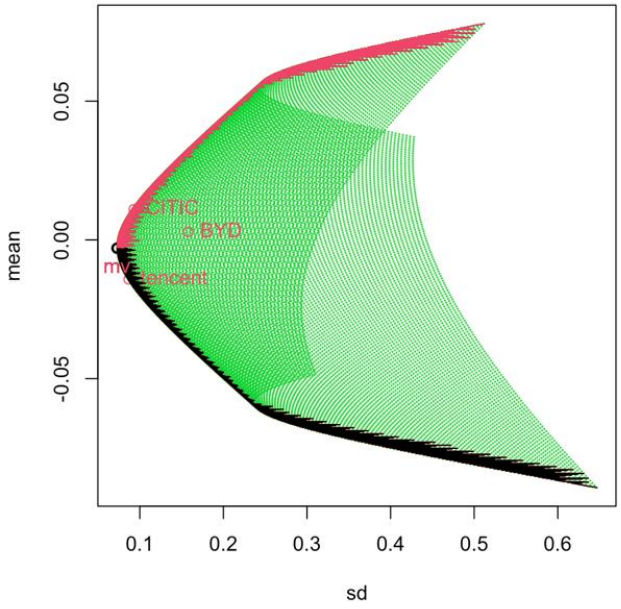

Figure 7: Minimum variance portfolio and the efficiency frontier.

We could also use the R studio to represent the minimum variance portfolio. In this situation, the short selling of stocks is allowed by setting the weights ranging from -2 to 3. As the figure 7 shows, the green area represents all the possible portfolios. If we keep changing the weights, it can spread over more area. And both the red and the black line are the efficient frontier, which shows the portfolios that have the maximum return at the fixed level of risk. And the red line shows the returns of the portfolios on the frontier that are higher than the return of the minimum variance portfolio. What is more, we could clearly observe that the point of the minimum variance portfolio lies on the far left of the frontier. And its standard deviation is 0.0728 related to the table 2 above.

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3.6. Sharpe Ratio and Tangency Portfolio

Using the formula 10 above, we can calculate the Sharpe ratios. In the situation of not investing in the riskless asset, the risk-free rate is 0. The figure 8 shows all the Sharpe ratio on the efficient frontier. As is apparent, the value of the Sharpe ratio firstly goes up, then it decreases after reaching the maximum Sharpe ratio.

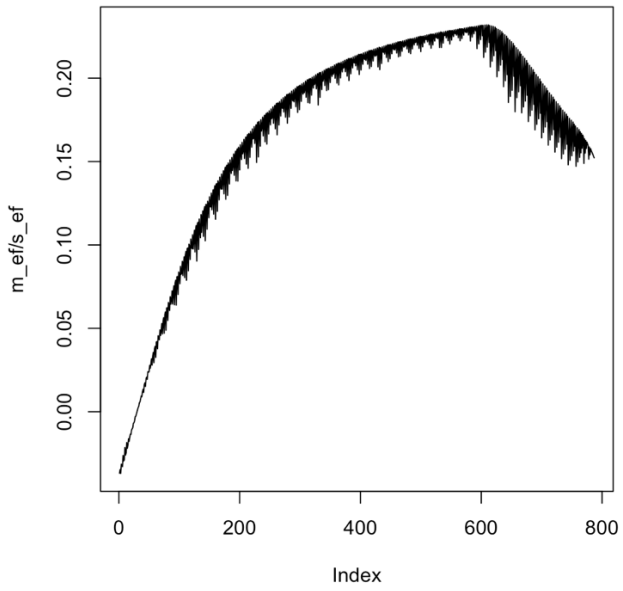

Figure 8: Scatter plot of Sharpe ratios.

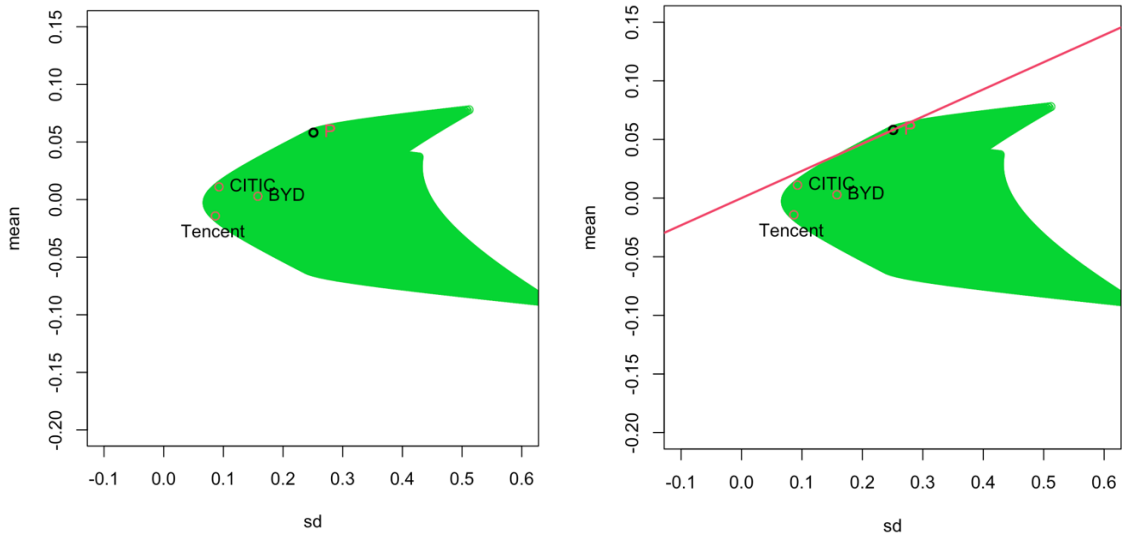

Figure 9: Maximum Sharpe ratio and the Tangency portfolio.

According to the above two figures (Figure 9), this proves the statement of Engels (2004). It is obvious that the portfolio that has the largest Sharpe ratio intersects the tangent line which starts from the origin. And that is the reason why it is called the tangency portfolio. Therefore, the optimal portfolio is the point p that intersects the red tangent line.

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Table 4: Sharpe ratios.

Portfolio	Tangency portfolio	Tencent	BYD	CITIC
Sharpe ratio	0.2319	-0.1646	0.0180	0.1190
Weights		-2.00	0.42	2.58

As the table 4 indicates, the Sharpe ratio of the Tencent, BYD and CITIC are -0.1646, 0.0180, 0.1190, respectively. And the Sharpe ratio of the tangency portfolio is 0.2319, which is also the point of the largest Sharpe ratio portfolio. The weights of the tangency portfolio are also shown in this table. To describe it, the percentage of each stock being invested in this portfolio should be -200% for the Tencent, 42% for the BYD and 258% for the CITIC. So, we should short the Tencent and long the BYD and CITIC to achieve the goal of optimal portfolio.

3.7. Consider the Risk-free Asset

The figure 10 shows the case of investing in one risk-free asset and three risky assets. For convenience, 2% is chosen to be the annual risk-free rate. As we can see, clearly the minimum variance portfolio is now changed to only invest in the riskless asset like the government bond since the variance of the riskless asset is 0. And the tangent line now is through the risk-free rate instead of the origin, the efficient frontier is now a straight line.

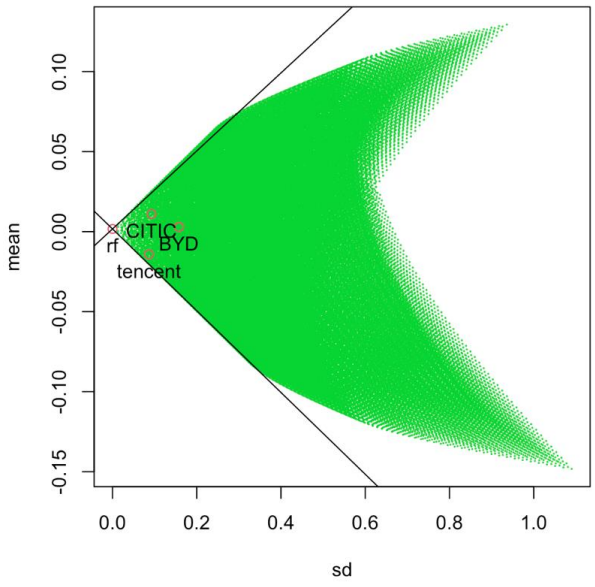

Figure 10: One risk-free asset with three risky assets.

4. Conclusion

Currently, the mean-variance analysis is still a popular and basic method to measure a portfolio. The study aims to find the tangency portfolio constructed by three different stocks based on the Hang Seng index from December 2009 to December 2021. Furthermore, the three chosen companies are all influential companies in China, which focus on the main area of the internet technology, automobile manufactory and Finance etc. In this paper, the Fama experiment is used to analyze the portfolio risks. Then, the method of the acf test is used to analysis the feasibility of the chosen stocks.

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Then, the mean-variance approach is used to do the portfolio analysis. The minimum variance portfolio and the portfolio with maximum Sharpe ratio are both founded by using R. The results show the tangency portfolio is the optimum portfolio reached by shorting Tencent and buying BYD and CITIC. The study may be benefit to the potential investors when making investment decisions. The tangency portfolio seems to be the best choice of investing in three risky assets since the Sharpe ratio is highest. However, the investment decision may differ due to investor's different ability to take the risks. If an investor is extremely risk averse, the minimum variance portfolio would be a good choice. What is more, the result is only based on the past data, which cannot be used to predict since there are many unknown variables in the future.

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