

# *An Assessment of a Real Course Allocation System*

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**Abstract:** This research mainly focuses on the course allocation problem in schools. In different schools, the matching mechanism is usually different. So far, people have not reached a consensus on an optimized mechanism for course assignments. In this essay, some important concepts for judging the quality of a matching mechanism will be introduced, and some instances will be quoted to support the assessment of the course allocation system in HuaFu International (HFI). In HFI, the course allocation procedures have some controversial parts that show the deficiency of the reasonability and validity of the mechanism. In HFI, courses accept applications based on the student's priority, the student's preference ranking for the course, and the grade of student. There are three important characteristics that cannot be satisfied in the system because the current mechanism partially focuses on the preference of one side of the matching objects, which leads to a large amount of welfare loss. In order to maintain the benefit of both students and teachers, this research suggests an available method to improve the mechanism.

**Keywords:** matching mechanism, course allocation, supply and demand

## **1. Introduction**

In school, courses usually have limited seats, but the demands of numerous students need to be met. The scarcity leads to a system of course allocation that decides the matching between students and courses. Since the course allocation problem is relevant to the market design, specifically relating to the application of market design techniques for the practical issue, the course allocation system can be viewed as a model of market design. The supply of seats and the demand from students form a market of courses. In the market, the main purpose of the course allocation mechanism is to improve the welfare of both teachers and students. The key to solving the real matching problem of the course allocation is to build up a direct matching mechanism that separates agents into two sides and turns the preference profile into the matching result. The quality of a matching system is related to some characteristics. The characteristics can be examined by some mathematical methods and logical demonstration. In the course allocation system, there are some mechanisms balancing the supply of courses and the demand of students, and controlling the result of allocation. The rationality and validity of the mechanism affect the welfare and fairness of the result in the system. If some serious errors exist, the students may get unsatisfied with their application results. So, for the purpose of exploring the possible problems of course allocation system, the example of a real course allocation system in a senior high school is chosen as the research object. In this senior high school, there is a complete allocation system that includes the process from course selection intention survey to

teachers' confirmation. The research will investigate whether the main purpose has been fulfilled by the course allocation system in HFI.

To examine the effectiveness of the allocation mechanism, the real items of the course registration requirements in the student handbook will be introduced, and the circumstance will be used as the foundation to form a simulative course application situation to test the quality. By using the mechanism, the survey of student's preference for the courses will be output into a result, which can be analyzed by applying certain formulas of market design. In this research, there is investigation and discussion mainly about the general welfare, the stability, the strategy-proof, and the individual rationality of the course allocation system. The specific research topic is formed with the four research questions, whether the mechanism has Pareto-efficiency, whether the mechanism is stable, whether the mechanism is strategy-proof, and whether the mechanism has individual rationality. According to the research about course allocation problems, in the literature, there have been some practical optimization-based mechanisms, but a common agreement for one specific mechanism has not been formed [1]. In order to contribute to solving the course allocation problem, this research will try to find an available mechanism to improve the current mechanism in senior high school, based on the analysis of the current mechanism.

## 2. Literature Review

The four characteristics have a significant influence on the rationality and validity of an assignment mechanism. First, the stability of a matching determines whether the matching is achievable. An unstable matching cannot ideally cover the demand of all participants, which may cause the matching to be vulnerable [2]. Participants will not follow the unstable matching result where they can personally join with another mutually acceptable object to form a better matching pair. If they find the matching unreasonable and unprofitable for themselves, and meanwhile the possibility of matching with a preferred agent exists, people will respectively go ahead to pair themselves with their own preferred object, disarranging the former matching result. Secondly, Pareto-efficiency is a powerful exhibition of the common benefit. To be specific, a Pareto-efficient matching system can provide more welfare to the participants. A Pareto-efficient matching, which is also called Pareto-optimal matching, is the matching that cannot be Pareto improved by another matching, which means, there will be no such a matching that participants are at least the same benefited and at least one participant can be better off [3]. According to the literature, stability is not complementary to welfare, so being stable is not identical to being optimal. Pareto-efficiency is a more direct exhibition of the level of aggregate welfare. By reaching a Pareto-efficient matching, the result can be controlled in a balance between stability and welfare [4]. Besides, strategyproofness is crucial in many matching systems, since it can help maintain the robustness of the forecast of a matching system, avoiding unexpected and unpredictable strategical behaviors. When people choose to cheat to get a beneficial consequence, the prediction of the output of a mechanism will be inaccurate, which greatly obstructs the conduction of the assignment [5]. Individual rationality is one concept within the definition of stability, which indicates that no one should be matched with an object that is unacceptable. In a rational mechanism, it is required that the supply side will not make a revenue that is lower than the cost and the demand side will not spend more than their attainable utility. If not, then people are not making reasonable decisions to make a benefit for themselves, which means the mechanism does not have individual rationality [6]. To ensure individual rationality, the matching system usually should offer some ex-post measures for participants to adjust their matching.

Much former literature on course allocation has analyzed different assignment mechanisms in schools all over the world and has advocated several kinds of theoretically improved mechanisms. In research about the analysis of the course allocation at Harvard Business School (HBS), the authors introduce the course allocation mechanism at HBS and verify the lack of strategyproofness of the

system and the deficiency of Pareto-efficiency. At HBS, students input their course preferences on the Internet and then the preferences are put into several rounds. In the first round, the choosing order is random and then reverses in the next round. In this matching, students can easily conduct strategic behavior by over-reporting the more popular courses that they want and under-reporting the less popular courses that they want because they can make use of the priority of earlier rounds to increase their probability to enter the more popular courses they prefer. The non-strategyproofness mainly affects welfare in two ways. First, the strategic behavior may quickly consume the vacant seats of the popular courses, which makes the matching result unfair for those whose favorite courses are popular for most students. Second, the strategic behavior is conducted based on the popularity of the courses, instead of students' own preference, so the under-reported courses, which are less popular, will possibly be missed due to the maximum number of courses to enroll in has been exhausted. So when students all use the equilibrium strategic behavior, many of them will receive a less beneficial matching result [7].

Another research investigates the course allocation mechanism at the Ross School of Business, University of Michigan (UMBS). UMBS has upgraded its course allocation mechanism from a preference-ranking mechanism to a bidding mechanism. In the bidding mechanism, the bids can not only show students' preference for the courses but also can represent the possibility for them to register for one course. The coexistence of the two functions of the bids brings on a deviation of the matching result from the actual market outcome. The field investigation implemented at UMBS reveals that there is a large efficiency loss [8].

In earlier years, the literature about the course registration system can also be found. In 1993, Graves, Schrage, and Sankaran have done research on the system at the University of Chicago Graduate School of Business (GSB) [9]. In GSB, students use their bidding points to bid for their preferred courses. As they enter the university, they get 8,000 initial points and can get more points when they finish each course. In each season, the student may start a course registration procedure, which involves one part called the registration bidding system (RBS) and one following part called drop/add/swap (DAS). In RBS, students will report four desired courses in a form, with the expenditure of bidding points for each course, and list the four courses from 1 to 4, representing their personal preference for the four courses. Then in the procedure of course acceptance, at most one of the bids will be accepted, determined by three factors: first, the bidding points will be considered from higher to lower, and the student who spends more points will precede the student who spends fewer points; second, when the amount of bidding points is the same, the serial number of the preference on the form will be considered, for example, bid 1 has higher priority than bid 2; third, in the same condition, the earlier bidding has higher priority than later bidding. If in one round of course acceptance, no bidding is accepted, students must enter the DAS system. In DAS, students can change their course selection by dropping a course to get the corresponding bidding points of the course in the round, adding a course by paying the number of bidding points that is higher than the price of the course in the round and swapping a selected course to another course by adding and paying for a substitute course [9]. In the GSB course allocation system, the RBS effectively prevents the congestion of course allocation by impelling students to put the most important course in the first bidding place. Individual rationality can be guaranteed since students have the chance to change their courses in four rounds of DAS.

### 3. Research Methods and Discussion

This research mainly focuses on the course allocation system in Huafu International (HFI). HFI adopts a mechanism similar to the preference-ranking mechanism for course allocation by using the course registration survey. Y10 and Y11 students need to finish the survey to choose one English course, which is necessary for everyone and is therefore excluded from the discussion, and three other

courses. Most of the courses have a basic requirement for the basic course. Courses are not equally popular, and those popular courses may usually offer more seats. Some courses have priority for Y10 or Y11 students because of the difference in course difficulty. Some teacher has their own extra requirements, and those requirements will not be considered in the project. To reveal their preference for the courses, they will fill out the course application survey that lists the course preference from ranking 1 (highest preference) to 10 (lowest preference). So, ideally, in order to register for more preferred courses, students will fill out the survey according to their actual preference for courses. After all, students have turned in their surveys, teachers will use four procedures to choose students. First, they will start at the students who put their course in ranking 1, and then continue to rank 2, then ranking 3, until there is no vacant seat or all applications have been accepted. Second, teachers will accept the students with priority at the current stage. Third, teachers will accept students who pass the required score at the current stage, then go to the next stage. Fourth, if all students in the second and third procedures are accepted and there is still an available seat, they will go back to stage 1 to compare the grade of students who put the course in ranking 1, and accept students sequentially from higher grades to lower grades. Teachers' preference towards students is based on their grades, which means teachers prefer students with higher grades in the basic course. After the mechanism completing, at the beginning of the semester, students can propose to change some courses. If they can satisfy the requirements and there is enough seat, they can join the class. To make sure that all students have enrolled in enough classes, the school prepares some courses with abundant seats for students.

To test the four characteristics of the matching mechanism, an artificial model, which is composed of five students (a, b, c, d, and e) and five courses (Calculus, Physics, Economics, Biology, and History), is introduced as an example. Each course has a score requirement for the basic course in the former year. Economics has the priority for Y10 students, and both a and b are Y10 students. Calculus, Physics, and Economics offer 3 seats, and the other two offer 4 seats. Supposing that students are reporting their preferences honestly, each student finishes the survey and needs to enroll in three courses. Then, two basic preference profiles are set in Table 1 and Table 2. By using the four procedures, a matching result can be output, as follows:  $\mu(a) = \{E, C, P\}$ ,  $\mu(b) = \{H, E, C\}$ ,  $\mu(c) = \{H, B\}$ ,  $\mu(d) = \{E, P, C\}$ ,  $\mu(e) = \{P, H, B\}$ ,  $\mu(C) = \{a, b, d\}$ ,  $\mu(P) = \{e, a, d\}$ ,  $\mu(E) = \{b, a, d\}$ ,  $\mu(B) = \{e, c\}$ , and  $\mu(H) = \{e, b, c\}$ .

Table 1: Preference profile of students in the artificial model.

a(10)	b(10)	c	d	e
E	H	H	E	P
C	E	E	P	H
P	C	B	C	E
H	P	C	H	C
B	B	P	B	B

Table 2: Preference profile of teachers in the artificial model.

C	P	E <sub>10</sub>	B	H
e	e	b	e	e
a	c	a	c	a
b	a	e	d	d
d	b	d	b	b
c	d	c	a	c

The matching is mainly decided by three ingredients, which are the ranking of courses set by students, the priority of students, and the grades of students. The preference ranking in the survey determines students' possibility of registering in each course. The priority and the grade of student can determine the teacher's final choices of students. However, the determinant of their matching result may be not identical with their preference. Student' preference for courses is personal, and if they all avoid strategic behaviors, it is the same as the course preference in the survey. Teacher's preference for students is the grade, however, in the process to recruit students, the teachers will primarily consider the course preference ranking of students, which means the mechanism will mainly focus on the preference of students. So, obviously, the matching system is not Pareto-efficient for teachers. If teacher's preference of grade is more dominant in the mechanism, they may be better off. For students, it is also not Pareto-efficient, because they may face the risk of fail to enter some less popular courses when conducting strategic behavior. The precondition is that the mechanism is not strategy-proof. Since the order for their application to be taken into consideration is not relevant to their true preference for the course, but is only determined by the report of their course preference, they may prefer to conduct costly and risky manipulation, which is achieved by placing the more popular courses in higher ranking and placing the less popular courses in lower ranking. For example, in the simulative situation, if student e change the ranking 2 for Economics and the ranking 3 for History, he can earn more benefit since he can enter both Economics and History, and he has a higher preference for Economics than Biology. The strategy will help increase the opportunity to enter those courses with fierce application competition. Nevertheless, this may also cause loss when student fails to enroll in some courses because they mistakenly estimate the popularity of some courses and put them in a excessively low ranking.

According to the matching system, the individual rationality, one of the sub-concepts of stability, can be guaranteed because students can freely apply to change one course to another achievable course. With regard to pairwise blocking, due to unequal consideration to the preference of two sides, some better matching cannot be achieved, so there is not a robustness to pairwise blocking. When the situation, in which a student and a teacher mutually prefer each other but they have not been combined in a pair because their current matching is formed according to students' reported preference ranking, exists, the mechanism is not stable. This may generate the alteration of the matching result and finally cause the matching result to deviate from the predicted result [10].

#### 4. Conclusion

The course allocation problem is a worth discussing market design problem. In school, an efficient assignment mechanism can highly improve the benefit for both students and school, enhancing the learning experience and maintaining the quality of teaching. This research mainly focus on the important characteristics of a matching mechanism, including the Pareto-efficiency, the stability, the strategyproofness. The stability is composed with individual rationality and robustness of pairwise blocking. In the analysis of course allocation system of HFI, it is found that the mechanism in HFI is short of those characteristics, which indicates the disadvantages of the mechanism. This may damage the fairness and utility of the course allocation system. In order to protect the course allocation system, one feasible improvement is to use course-proposing deferred acceptance (DA) algorithm. The course-proposing DA algorithm can relieve the partiality for the students, and therefore balance the benefit of the two aides. In the result given by a DA algorithm, every applicant can at least have the same benefit as in another stable situation. With a DA algorithm, the matching mechanism will be stable and Pareto-efficient, which ensures the matching result to be what teachers want, and improves the common welfare.

Besides, some problems exist in the process of the research. The research can be more cogent by correcting the problems. First, the amount of samples is not enough. The artificial preference profiles

are too simplified, which makes the accidental errors possible to exist. The conclusion may be just derived by chance, with the deficiency of scientificity, accuracy and cogency. To improve this, a more complex circumstance can be introduced, with more detailed items in the course registration handbook and more objects set in the artificial condition. Second, the relevant supporting materials are not enough. The research is mainly about the senior high school, but the cited literature has only covered the course allocation system in the university, which can only provide the reference for the investigation, but not evidence. It still needs a lot of time to find more applicable and helpful resources.

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