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Application and Evaluation of Valuation Methods of Volatility Derivatives

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Abstract: With the integration of the world and the development of financial markets, investors have more investment options and are accordingly exposed to greater and more complex risks. To better adapt to such changes as well as cope with risks, financial derivatives have been created. Volatility derivatives are a more sophisticated class of financial derivatives whose value is based on the volatility of a certain asset. Through literature analysis, comparative analysis and case studies, this paper focuses on the definition of volatility derivatives, the role they play for investors, and examines whether volatility derivatives are correctly priced by studying their pricing methods. The paper finds that volatility derivatives provide investors with efficient tools to speculate and hedge. In terms of pricing, despite the availability of metric values such as B-S models and stochastic volatility models, their pricing is subject to various limitations that need to be further explored.

Keywords: volatility, volatility derivative, pricing, hedging

1. Introduction

As financial markets developed and expanded, the impact of volatility on investments was recognised and as a result, volatility derivatives, financial derivatives that use the volatility of an asset as the underlying asset, were created to address risk [1]. Due to the relatively short history of volatility derivatives and the relatively more complex valuation of this class of derivatives, past articles have typically focused on other types of derivatives, while volatility derivatives have been less studied. In this context, this paper systematically examines the concept, role and valuation of volatility derivatives.

In the next section, the paper provides an introduction to the meaning of volatility and its derivatives to give the reader a comprehensive understanding of the topic and introduces the concept of Volatility Index (VIX). In Section 3, the role of volatility derivatives in hedging and speculation is presented with specific examples and the differences between the two are compared. In Section 4, the B-S model, Stochastic Volatility Models such as Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) and Heston model and stochastic volatility model are presented. In order to analyse and compare the different valuation methods, the assumptions or characteristics of each model are also detailed. Building on previous research findings, this paper provides academics with a more systematic and comprehensive introduction to the role of volatility derivatives and a comparison of pricing models, and one will gain a deeper understanding of such derivatives, as well as a clearer appreciation of the strengths and weaknesses of different models.

2. Definitions of Volatility Derivatives

2.1. Volatility

Volatility is the standard deviation of the fluctuation in the value of an asset around its mean over a period of time and it usually represents the uncertainty of an asset's return. The value of an asset may go up or down, but the volatility must be greater than zero. The time standard for measuring volatility varies in different situations. If used in pricing options, it is usually measured over a period of one year, and if used to measure risk, it is usually limited to one day. The formula for calculating volatility over a given period of time is as follows, with the volatility for n trading days $\mathrm{T_n}$ being the volatility for one day $\mathrm{T_1}$ multiplied by the square root of the number of days, n, as shown in equation(1):

$$T_n = \sqrt{n}T_1 \tag{1}$$

An important international measure of volatility is the Volatility Index(VIX), an index created by the Chicago Board Options Exchange, which in 1993 selected eight stocks from the S&P100 to form the underlying of the VIX, and the range was later expanded to include the S&P500, making the index compilation more accurate. The Volatility Index is also known as the Panic Index, referring to the fact that this index reflects market sentiment, which also provides a good reference for predicting market movements and reducing portfolio risk.

After the world financial crisis in 2008 and the subsequent European debt crisis, the VIX has come to be seen as an important indicator of the global economy. Major financial markets around the world took this very seriously and created their own volatility indices for their local financial markets after the CBOE . To date, the world's major options markets such as the US, Europe and Asia have all developed their own volatility indices. Apart from the CBOE, the European Futures Exchange has the most extensive volatility index.

2.2. Volatility Derivatives

Financial derivatives are derivative financial instruments that rely on the price of the underlying asset to calculate its value. The growth of financial markets has required more efficient and targeted financial products thus volatility derivatives have been created. The value of such derivatives is based on the magnitude of volatility rather than the price of a traditional financial asset. This extends the role of volatility from its original role in the pricing equation to an asset class of its own.

As a special financial instrument, volatility derivatives give market participants the opportunity to trade at the volatility of a particular asset [2]. One of the most common examples is the variance swap or volatility swap on a volatility index such as the VIX. The innovative nature of such derivatives has created new investment options for investors, while their complexity and specificity have also created difficulties in pricing and valuing volatility derivatives.

3. Application of Volatility Derivatives

3.1. Hedging

3.1.1. Definition

In the financial markets, hedging is an operation carried out by investors to reduce the risk of their portfolio. Hedging allows investors to hedge potential risks while maintaining a certain level of profitability. Specifically, when an investor buys a certain asset, he can sell futures of the same value on the futures market, a process also known as long hedge, the opposite of which is known as short

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hedge. However, the long hedger may make a loss in the futures market. Conversely, when the price of the underlying asset falls, the long hedger is able to sell its holdings at a loss but make a profit in the futures market [3]. It is important to note that while hedging reduces risk, it also inevitably reduces the return on the portfolio. Therefore, when deciding to hedge, investors should give careful consideration to the size of the hedging position and the underlying hedge.

3.1.2. Role of Volatility Derivatives in Hedging Strategies

Volatility tends to increase during market crashes, which known as Black leverage [4]. In the past, investors have often used delta-hedged hedges to hedge volatility as there are no products that hedge against volatility. However, this hedging method is often dependent on the price of the underlying asset and does not hedge volatility alone.

Volatility derivatives provide traders with a new hedging tool. As it can be hedged against volatility in isolation, it has gained wide popularity among investors in risk management. For example, variance swaps and volatility swaps have become very popular. Szado found that by using VIX derivative positions, investment risk of S&P 500 index can be hedged and this protection is significant, especially during the economic recession [5-6].

3.2. Speculation

3.2.1. Definition

Speculation is a type of financial behaviour that relies on the judgement of the investor. For example, if a speculator decides that the price of a financial asset will fall, he will sell it short for a profit. This includes many things such as real estate and financial instruments.

Unlike hedging, which is a risk-controlled, guaranteed return strategy, speculation does not guarantee any return or even principal and carries a high degree of risk. Of course, if the speculator is correct, the returns received will be much greater. It is because of the huge potential returns that there are a large number of speculators in the market. Another significant difference between the two is that hedgers need to match hedging positions carefully, whereas speculators tend to rely on a more unidirectional judgement.

3.2.2. Role of Volatility Derivatives in Speculation

Derivatives often require only a portion of the investor's margin to be traded, i.e. the leverage effect is significant. Investors need only a relatively small amount of capital to enter into a derivative contract, greatly increasing the potential returns. The advent of volatility derivatives allows speculators to make investment decisions and capture returns based on their judgement of the future movement of volatility. For example, if an investor believes that current volatility is at an abnormal level and that volatility will rise or fall in the future, he can buy or sell volatility derivatives short in order to make a profit. The political and economic volatility now temporarily caused by debt problems or election expectations is a good example of how investors can forecast the market in order to speculate on volatility derivatives.

It is worth noting that volatility derivatives are more complex than ordinary financial assets or other entry derivatives and speculating on them carries significant risks and speculators need to have a clear understanding of future market dynamics.

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4. Evaluation of Valuation Methods for Volatility Derivatives

4.1. Black-Scholes Model

As a complex class of financial derivatives, the pricing of volatility derivatives has received a lot of attention [7]. The Black-Scholes model(B-S model) is a widely used model in derivatives pricing problems, with the equation(2)-(4).

$$C = S * N(D_1) - e^{-r*T} * L * N(D_2) \tag{2}$$

$$D_1 = \frac{ln\frac{S}{L}+(r+0.5*\sigma^2*T)}{\sigma*\sqrt{T}} \tag{3}$$

$$D_2 = D_1 - \sigma * \sqrt{T} \tag{4}$$

Where C represents the price at the start of the option; L represents the option delivery price; S represents the price of the underlying financial asset; T is the life of the option and r represents the constant risk-free rate. The σ in the formula is Implied volatility (IV). It is derived from the B-S model based on other factors.

However, though the model itself is straightforward, it is important to note that the Black-Scholes model relies on several assumptions, which become its drawbacks. Firstly, it assumes that the prices of financial assets follow a log-normal distribution. Secondly, it assumes that the risk-free rate and the financial asset return variables remain constant throughout the option's lifespan. While in the real financial markets, the volatility is fluctuating and has some characteristics such as volatility clustering and volatility smile. Thirdly, the model assumes a frictionless market, acquiring there are no taxes or transaction costs, which is also conflict with the real world. Lastly, it assumes that the option is a European option, meaning it cannot be exercised until it expires. Due to these assumptions, the Black-Scholes model may not accurately capture the dynamics of volatility and is more suitable for European-style options.

4.2. Stochastic Volatility Model

The stochastic volatility model relaxes the assumption of volatility and allows for variation in volatility. These models, such as the Heston model and GARCH, assume that volatility depends on different patterns of change.

The Heston model, introduced by Heston in 1993, assumes that asset prices follow a diffusion process. The Heston model takes into account the correlation between volatility and the price return of the underlying asset, with the correlation parameter being particularly important as it reflects the skewness of price movements and largely characterises the spikes and thick tails of price returns.

Similarly, the GARCH model assumes that volatility changes over time, under the influence of its past volatility. In this model, the impact of positive and negative shocks on the conditional variance is symmetrical, so it does not provide a good measure of the specificity of the impact of changes in different directions on volatility and the asymmetry of fluctuations in the conditional variance of returns [8].

A significant difference between the two models is the use of different volatilities. The GARCH model uses historical volatility to estimate future volatility levels, whereas the Heston model relies on current options data and the market's implied estimates of future volatility.

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4.3. Merton Jump Diffusion Model (MJD).

Equity returns are not always smooth and continuous and are susceptible to major fluctuations and exhibit jumps, as is often the case with changes in volatility. In financial markets, dramatic changes occur from time to time and this is what underpins the jump model, especially in times of crisis when we can see rapid changes in prices.

Assuming that there is a jump process, many different models exist which use a sudden discrete change process. 1976 saw the first suggestion by John Carrington Cox and Stephen Ross that stock prices follow a jump process [9]. Later, Robert Merton extended it to a combination of jumps and small continuous movements, when extending from the B-S model, where there are price jumps. This is a well-known version of the original model, which is known as the Merton Jump Diffusion Model (MJD model) [10].

However, it is important to note that there are some risks associated with the valuation of volatility derivatives. When selecting models for pricing volatility derivatives, particular attention needs to be paid to the limitations of individual models. The selection of an unsuitable model can lead to mispricing. In addition, volatility indices are subject to allegations of manipulation and powerful financial institutions may use their capital or information advantage to manipulate volatility. All of the above risks can have an impact on the pricing of volatility derivatives and their credibility. Therefore, to some extent, there is a need for governments or relevant regulators to step up supervision and risk management measures.

5. Conclusion

In conclusion, volatility derivatives hold significant importance in risk management and investment strategies as a unique financial instrument, offering investors opportunities to manage and speculate on market volatility.

Hedgers can utilize volatility derivatives to hedge against price fluctuations in the underlying asset, reducing their exposure to risk. Speculators, on the other hand, can use these derivatives to capitalize on their expectations of future volatility and potentially generate profits.

Valuation of volatility derivatives involves various models, including the widely used Black-Scholes model, stochastic volatility models like Heston and GARCH and Merton Jump Diffusion Model. However, the valuation of volatility derivatives continues to pose challenges that necessitate further development and research. It is crucial for regulators to strengthen their supervision and implement effective risk management measures in order to mitigate potential risks associated with these derivatives.

This paper only provides a qualitative analysis, comparison and summary of several most popular valuation models and does not involve a quantitative study. The model in this paper can then be optimised or new models introduced to make the pricing model more adequate and relevant to the development of financial markets and the characteristics of volatility derivatives, ultimately contributing to the further development of the volatility derivatives market.

References

[1] *Gangahar, Anuj Volatility Becomes an Asset Class Financial Times, May 2006*

[2] *Carr and R. Lee, "Volatility Derivative," Annual Review of Financial Economics, vol.1, no.1, pp. 319–39. (2009).*

[3] *Naik, N.K., and Pradeep Yadav. "Risk Management with Derivatives by Dealers and Market. Quality in Government Bond Markets." Journal of Finance, Vol. 58, No. 5 (2003), pp. 1873-1904.*

[4] *Scholes, M., & Black, F. The pricing of options and corporate liabilities. Journal of Political Economy, 81(3) pp.637-654. (1973).*

[5] *RUSSELL RHOADS. Trading VIX Derivatives: Trading and Hedging Strategies Using VIX Futures, Options, and Exchange —Traded Notes[M]. Hoboken: Wiley, 2011.*

DOI: 10.54254/2754-1169/42/20232114

[6] E. Szado, "VIX Futures and options: A Case Study of Portfolio Diversification During the 2008 Financial Crisis," The Journal of Alternative Investments, vol. 12, no. 2, pp. 68-85. (2009).

[7] J. Detemple and C. Osakwe, "The Valuation of Volatility Options," European Finance Review, vol. 4, no. 1, pp. 21-50. (2000).

[8] Haug, Espen G. et al. GARCH and Volatility Swaps Wilmott 'Magazine, 2002

[9] Cox, John C. & Ross, Stephen A. The Valuation of Options for Alternative Stochastic Processes Journal of Financial Economics, 1976

[10] Merton, Robert C. Option Pricing When Underlying Stock Returns Are Discontinuous Journal of Financial Economics, 1976