Proceedings of the 7th International Conference on Economic Management and Green Development
DOI: 10.54254/2754-1169/44/20232231

Research on the Pricing Model of Derivatives

——Based on the Black-Scholes Model and Binomial Model

Yingxi Wang[1,a,*]

[1]School of Finance, Nankai University, Tianjin, China, 300071
a. 1814091@mail.nankai.edu.cn
**corresponding author*

Abstract: The pricing of financial derivatives plays a significant role in risk management and financial transactions. The models of pricing are mainly divided into Black-Scholes Model and Binomial Model. The two models have different characteristics and distinguishing the models is helpful for price makers. The paper aims to analyse the distinction between the two models on the Huaxia Stock Exchange 50ETF by using Python. This paper selected the stock price, the volatility and the option price from the Eastmoney database. First, edit the formulas of two models and then introduce the parameters into the formula by Python and get the model calculated price. Comparison between the calculated prices helps to compare the two pricing models. The result is that Black-Scholes Model is more accurate and Binomial Model is more simplified which is suitable for new officers.

Keywords: Black-Scholes Model, Binomial Model, financial derivatives, pricing model

1. Introduction

Black-Scholes Model is proposed by Fischer Black who is a mathematician and Myron Scholes who is an economist. The Black-Scholes Model can accurately measure the value of the European option. The Binomial Model is also a formula for option pricing which can apply to American options. In Amir Ahmad Dar and N. Anuradha's research, there is no significant difference between the means of the European options by using the above two models [1]. However, there are few researches using the data in China. By using the data in China, people can decide the proper model to price the option in the Chinese market. This paper aims to distinguish Black-Scholes Model and Binomial Model in the Chinese market. Data in the Easymoney database is selected in the paper. The result provides a piece of advice when people choose models to price their options and makes researchers pay attention to the Chinese market.

2. Construction of Model and Analytical Derivation of Option Pricing

2.1. Black-Scholes Model

Strict applicable assumptions are specified for Black-Scholes Model:

① Stock price is subject to geometric Brownian motion

$$dS = \mu S dt + \sigma S dz \tag{1}$$

z means standard Brownian motion, μ means the expected growth rate of stock, σ means volatility.
② Total income can be used to oversold derivative securities.
③ There are no transaction expenses and taxes in market, and all securities are highly divisible.
④ There are no bonds in the duration of derivative securities.
⑤ There is no chance of riskless arbitrage in transaction market.
⑥ Securities transaction is continuous.
⑦ Riskless rate r is constant and is the same for all due date.
The formula for European call option “c” and put option “p” is:

$$\mathrm{c} = \mathrm{S_t N(d_1) - Ke^{-rT} N(d_2)} \tag{2}$$

$$\mathrm{p} = \mathrm{Ke^{-rT} N(-d_2) - S_t N(-d_1)} \tag{3}$$

Where,

$$d_1 = \frac{ln(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{4}$$

$$d_2 = \frac{ln(\frac{S_t}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \tag{5}$$

“S_t is the present price of the price of an underlying asset, K is the strike price, r is the free risk rate interest, σ is the volatility of the stock and N(*) is the cumulative distribution function for a standard normal distribution, $S_t N(d_1)$ is the present value of the underlying asset if the option is exercised and Ke^{-rT} is the present value of the strike price if the option is exercised [2] ”.

2.2. Binomial Model

The Binomial Model divides valid time from call date to option expiration date into much little time interval. It assumes that stock price S only have two possibilities of variation in each time interval:
① current price S goes up by a factor of u, which is S_u
② current price S goes down by a factor of d, which is S_d
“u>1, d<1. The probability of the current price rising is P and the probability of declining is 1-P. Option price goes up from f times to u times which is f_u, price goes down from f times to d times which is f_d" [3].

The main idea of the Binomial Model is to divide price movement into massively discrete small amplitude binary motion and then simulate continuous asset price movement by the small amplitude binary motion. Node price of the path is calculated by discounting methods. It simplifies the calculation of option pricing. The Binomial Model can be divided into one-step Binomial Model and multi-step Binomial Model.

Considering the one-step Binomial Model, asset price only has the possibility of rising and declining. This assumes that investors can use assets at riskless rates and do not need to pay bonds excluding transaction costs. So to get the normal formula people need to use the arbitrage-free

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principle. Assume that one underlying asset is stock and its present value is S, its derivatives are stock options and its present value is V. u means rising and d means declining. There is a riskless investment portfolio. To avoid risk, buying in Δ shares to hedge is needed while sell a stock option. So the value of the investment portfolio is $\phi_0 = V_0 - \Delta S_0$. Then deduce the pricing formula [4]:

$$\phi_T = V_T - \Delta S_T = e^{rT}\phi_0 \quad (6)$$

$$\phi_u = V_u - \Delta S_u \quad (7)$$

$$\phi_d = V_d - \Delta S_d \quad (8)$$

$$V_u - \Delta S_u = V_d - \Delta S_d \quad (9)$$

$$\Delta = \frac{V_u - V_d}{S_u - S_d} \quad (10)$$

$$V_0 = \Delta S_0 + e^{-rT}(V_u - \Delta S_u \quad (11)$$

Considering the multi-step Binomial Model, assume that there is no dividend and the ascension range of the stock is u, the descend range is d. Time interval [0,T] before option expiration date is divided into n time subintervals $\Delta t = T/n$. In each time subinterval $[t_i, t_{i+1}]$,(0≤i≤n-1), assume that the variety of stock price satisfies one-step Binomial Model, and then S will become a multi-step Binomial Model in [0, T]. So after going through n one-step Binomial Models, assume that the number of rise of price is i and the number of decline of price is n-i. The possibility of risk neutral is $C_n^i q^i (1-q)^{n-i}$ by using binomial theorem. The final profits is:

$$\max(S_0 u^i d^{n-i} - X, 0)$$

So n-step Binomial Model of call option is:

$$c_0 = e^{-rt}\sum_{i=0}^{n}[C_n^i q^i (1-q)^{n-i} max(S_0 u^i d^{n-i} - X, 0)] \quad (12)$$

Stipulate that h is the minimum non-negative integer which is satisfied for $S_0 u^h d^{n-h} \geq X$

So get the formula: $\max(S_0 u^i d^{n-i} - X, 0) = \begin{cases} S_0 u^i d^{n-i} - X, i \geq h \\ 0, i \leq h \end{cases}$

When the stock price goes up h times, the European call option will be in a real-valued state at the due date. So the pricing model is:

$$c_0 = e^{-rT}\sum_{i=0}^{n} C_n^i q^i (1-q)^{n-i} u^i d^{n-i} - Xe^{-rT}\sum_{i=0}^{n} C_n^i q^i (1-q)^{n-i} \quad (13)$$

Since $Xe^{-rT}\sum_{i=0}^{n} C_n^i q^i (1-q)^{n-i}$ is regarded as the expected revenue on the due date where the discount factor is e^{-nrT} in the condition of risk neutral. So promise $\Phi(n,h,q) = \sum_{i=h}^{n} C_n^i q^i (1-q)^{n-i}$ where $\Phi(n,h,q)$ means at least going up at n times in the n-step Binomial Model experiment. If regard $\hat{q} = uqe^{-r\Delta t}$, the n times European call option pricing formula will be:

$$c_0 = S_0\Phi(n,h,\hat{q}) - Xe^{-rT}\Phi(n,h,q) \quad (14)$$

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3. Parameter Estimation

When in the condition that European options have no dividend, the riskless rate is fixed and the price can only be executed on the expiration date, a comparison of pricing efficiency is of great significance to pricing accuracy [5]. This paper selects European call option data of Huaxia Shanghai Stock Exchange 50ETF on July 9, 2023 from the Eastmoney database. The due date is July 26, 2023, which means the expiration time is 17 days. The riskless rate r is 0.0427. Other data is in Table 1.

Table 1: Other data of huaxia shanghai stock exchange 50ETF.

Current price S(yuan)	Exercise price X(yuan)	Volatility σ	Real price (yuan)
2.5510	2.3000	0.2986	0.2547
2.5550	2.3500	0.2892	0.2059
2.5640	2.4000	0.2612	0.1560
2.5610	2.4500	0.2146	0.1090
2.5520	2.5000	0.1772	0.0670
2.5460	2.5500	0.1516	0.0359
2.5490	2.6000	0.1399	0.0161
2.5500	2.6500	0.1399	0.0070
2.5520	2.7000	0.1446	0.0026
2.5540	2.7500	0.1493	0.0010
2.5530	2.8000	0.1493	0.0005

Since in Black-Scholes Model, the pricing formula is formula(2), and in Binomial Model the pricing formula is formula(14), the paper can get the theoretical price of the two model by using Python which is in Table 2.

Table 2: Computational results.

Black-Scholes Model(yuan)	Binomial Model(yuan)	Real price(yuan)
0.2526	0.2573	0.2547
0.2066	0.2107	0.2059
0.1658	0.1708	0.1560
0.1140	0.1194	0.1090
0.0621	0.6840	0.0670
0.0264	0.3188	0.0359
0.0098	0.1434	0.0161
0.0026	0.5852	0.0070
0.0005	0.2238	0.0026
0.0001	0.8453	0.0010
0.0001	0.3444	0.0005

4. Error Analysis

There is an error between the model calculated price and the real price. Assume that the mean percentage error is simplified as MPE:

$$\mathrm{MPE} = \frac{\sum \frac{(P_{sj} - P_{mx})}{P_{mx}}}{n} \tag{15}$$

P_{sj} means the real price and P_{mx} means model calculated price.

Substitute data in Table 2 into formula(15), and get the Black-Scholes Model MPE is 1.806, the Binomial Model MPE is -0.623. Draw a broken line graph of the two model calculated price and real price as Fig. 1.

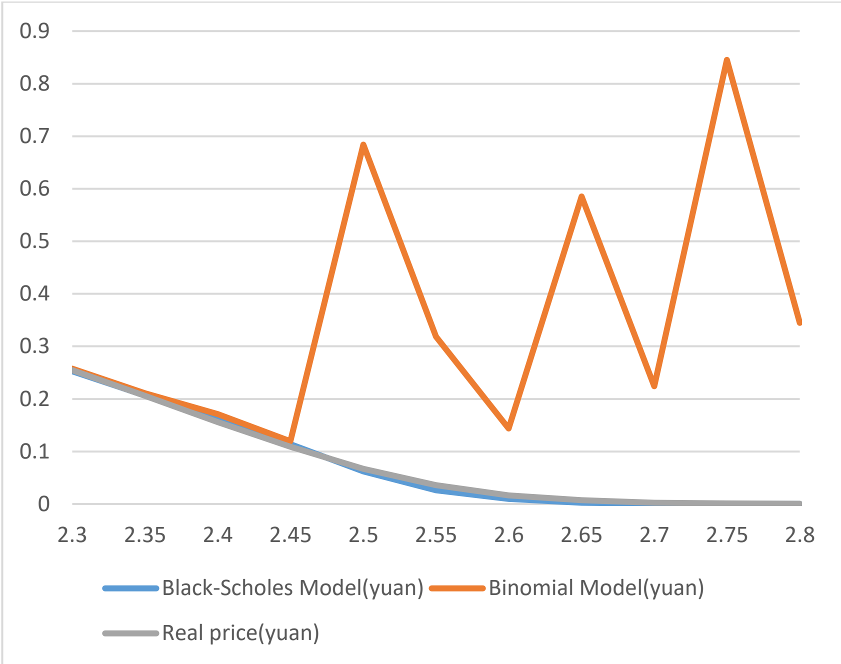

Figure 1: Comparison of price.

According to the trend in Figure 1 and the comparison of MPE, the paper gets the result that there is a bias in different degrees from different pricing models. On the whole, the MPE of the Black-Scholes Model is smaller than that of the Binomial Model. On a graph, the fitting degree of the Black-Scholes Model is higher than that of the Binomial Model. The paper concludes that Black-Scholes Model is better than Binomial Model.

5. Conclusion

This paper aims to research financial derivatives pricing model comparison by data analysis. By the data from Huaxia Shanghai Stock Exchange 50ETF on July 9, 2023, the paper gets the analysis that the MPE of the Black-Scholes Model is 1.806 and the MPE of the Binomial Model is -0.623. The former is lower than the latter which means that Black-Scholes Model is better than Binomial Model. However, Binomial Model has more simplified calculations and is valuable to junior practitioners. This paper only analyses two pricing models. There are other pricing models that can be compared such as the Constant Elasticity of Variance Option Pricing Model and the Heston Model. In addition, researchers can use international data to compare pricing models.

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DOI: 10.54254/2754-1169/44/20232231

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