

# ***Should We Focus More On The Offline Retail When Constructing Financial Portfolios?***

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**Abstract:** Modern portfolio theory has been researched and applied widely. This paper shows an application of the modern portfolio theory to choose optimal weights for a composite stock s portfolio. It could help optimize investor's portfolio by increasing returns and decreasing volatility. Considering returns and volatility at the same time, Sharpe ration is a good indicator. To research portfolio optimization, this article uses Monte Carlo methods to acquire the weights of securities in portfolio that has the highest Sharpe ratio. Current studies about portfolio theory do not take safe securities into consideration. Therefore, this paper first concludes risk-free asset with other risky assets in portfolio. The portfolio in this study includes five stocks (Walmart, Amazon, Dexcom, Warner Bros., T. Rowe) in different sectors and risk-free asset (one-year U.S. Treasury Securities). The result shows that the weight of offline retail increases as the proportion of risk-free asset rises. This has implication that risk-averse investor might prefer offline retail stocks compared with other stocks in portfolio.

**Keywords:** Modern portfolio theory, optimization, risk-free asset.

## **1. Introduction**

Modern portfolio theory (MPT) was first introduced by Economist Harry Markowitz [1]. It opened up the quantitative analysis of financial economics. MPT provides the two important indicators expected returns and risk measured by standard deviation at a given weight. The use of MPT allows the construction of efficient frontier and optimization of a set of portfolios which provide the optimal weights of portfolio in terms of a given level of risk or volatility [2]. In finance, some investors may prefer low risk while others are more likely to accept a high-risk strategy. Therefore, investors could allocate their portfolio better according to their different risk preferences in the help of MPT. In addition, decision maker always chooses an investment portfolio, a collection of different types of investments, due to this diversification helping to minimize the investment risk. It is true of the saying goes: "Don't put all your eggs in one basket."

What we find currently is that numerous studies about portfolio optimization have been done. However, few studies take risk-free asset into consideration and research the optimized investment portfolio when adding risk-free asset. Portfolio only consists of risky asset in most current research. For instance, Cui and Cheng generated an optimal portfolio of the six largest stocks which are traded on the Australian stock market to prove MPT is still efficient tool in the context of volatile

markets [3]. Likewise, Biswas selected 6 securities listed on national stock exchange and came to conclusion that diversification leads to risk reduce in Indian capital market [4]. And He et al. picked 5 representative stocks from the A shares listed on the Chinese stock market and got the best preformed stock as the variance is the lowest [5]. Meanwhile, portfolio theory was also applied to research in other fields except finance, such as how to optimally meet future water needs [6], and some suggestions on increase healthcare service levels [7]. Other researchers on the topic have focused on portfolio theory itself. Hwang, Xu, and in showed naive diversification has better performance than the optimal mean–variance diversification because of its decrease in tail risk and concavity [8]. Regarding to the relation of the size of portfolio and portfolio volatility, debate on this issue lasted for a long time. In 1968, Evans and Archer first mentioned the relation of size and risk and put forward adequate portfolio sizes larger than 10 or so securities [9]. This was soon followed by 40 stocks which are sufficient and efficient for a well-diversified portfolio to reduce the risk [10], however, Statman found that at least 300 stocks could diversify the risk of portfolio [11]. Furthermore, Campbell, Chong, Jennings & Phillips used different methods to optimize the portfolio and discussed the resulting portfolios in terms of risk, return, and Sharpe ratio [12]. They also provided a guide about which was a better choice under different circumstances.

The paper is constructed as follow. First, we select five representative stocks in the area of offline retail, online retail, healthcare, electronic communications, and finance, as well as risk-free asset, U.S. Treasury Securities matured at one year. Second, we calculate the average returns of five stocks and treasury security. Third, we assume 10%, 20%, 30%, 40%, and 50% risk-free asset in the portfolio and people have less preferences toward risk as the proportion of risk-free asset increases. Fourth, we use mean-variance analysis to calculate the Sharpe ratio of portfolio. Fifth, we apply Monte Carlo simulation to get the optimized weights of five stocks with highest Sharpe ratio. The result shows that stock in financial sector has the highest weight and as the proportion of risk-free asset increases, the weight of stock in offline retail sector rises. Sixth, we do stability test to examine whether the result will change as we add other stock into the portfolio. Fortunately, the result remains the same. Thus, we could roughly see that the optimized weights of portfolio investors could hold and investor with different risk-preferences would prefer which stock. And this may also serve as a reference for people seeking different risks in the post-epidemic era to invest.

This investigation is summarized as follow. Section 2 refers to data and Section 3 refers to methods. Section 4 talks about results and then section 5 is stability test. At last section 6 shows the conclusion.

## 2. Data

The data of stock in this paper is selected from Nasdaq (<https://www.nasdaq.com/>), while the data of security is selected from Fred (<https://fred.stlouisfed.org/>). Six stocks in this paper are Walmart, Amazon, Dexcom, Warner Bros., T. Rowe, and Meta (used to do stability test). These six stocks are the representative of the sector of offline retail, online retail, healthcare, electronic communications, finance, and technology due to their nearly highest capital cap in respective sector. In addition, the reason why we choose Meta to do stability test is that it has the different sector from the stocks we researched. In the following text, we will use the abbreviation of the stocks, WMT, AMZN, DXCM, WBD, TROW, and META, respectively. These stocks cover a period from 30th may, 2021 to 27th may, 2022. Totally, 252 closing prices of one stock are collected. We use daily returns which is calculated through the formula that closing price today minus closing price yesterday and divided by closing price yesterday. Mean, standard deviation, mini, and maxim of daily returns are shown in Table 1.

Table 1: Descriptive statistics of the selected stocks.

	WMT	AMZN	DXCN	WBD	TROW
Mean (%)	0.049	0.164	0.136	0.262	0.174
Std	0.014	0.025	0.029	0.031	0.020
Min (%)	-0.039	-0.119	-0.115	-0.144	-0.054
Max (%)	0.128	0.163	0.124	0.101	0.071

Through simple data calculation, we can find that all five stocks have the positive average return, among them “WBD” have the highest average return, while “WMT” has the lowest average return. In terms of standard deviation, “WBD” has the highest, while “WMT” has the lowest. From the calculation above, the relation between average return and standard deviation is most negative as expectation. As for risk-free asset returns, we use the market interest rate of U.S. treasury securities matured at one year from the same time period as the stock above and calculate the average daily returns 0.0025%.

### 3. Methods

#### 3.1 Mean-Variance Analysis

The mean-variance analysis is a mathematical approach to asset allocation to achieve the two main decision objectives of the investor in question: the highest of return and the lowest risk as possible as they can. The investor should find the best choice between these two top goals, which are in balance with each other. According to previous investigations, mean-variance analysis has been a common method for measuring and constructing asset portfolios. The resulting investment model is the mean-variance model. The mean-variance model satisfies a strict restriction shown in equation (1).

$$\sum_i Weight_i = 1 \quad (1)$$

where the weight of the Asset<sub>i</sub> in the portfolio is represented by  $Weight_i$ . The return and risk of the overall investment can be calculated as follows,

$$E[Portfolio Return] = \sum_i Weight_i \cdot E[Asset Return_i] \quad (2)$$

where  $E[Portfolio Return]$  and  $E[Asset Return_i]$  signify the expected return of the chosen allocation and Asset<sub>i</sub> respectively,

$$Var[Portfolio Return] = \sum_{i,j} Weight_i \cdot Weight_j \cdot Cov[Asset Return_i, Asset Return_j] \quad (3)$$

where Var stands for the variance of portfolio return and Cov stands for the variance-covariance between Asset Return<sub>i</sub> and Asset Return<sub>j</sub>. The expected return and variance of one certain investment that contain both risky and risk-free assets can be realized by the following equations,

$$\sigma_c^2 = w_f^2 \sigma_f^2 + (1 - w_f)^2 \sigma_p^2 + 2w_f(1 - w_f) \rho_{f,p} \sigma_f \sigma_p \quad (4)$$

$$E(R_c) = w_f R_f + (1 - w_f) E(R_P) \quad (5)$$

where the expected return is shown as  $E(R_P)$ , the risk – free rate is shown as  $R_f$ , and  $\sigma_P$  is the risk of the portfolio.

### 3.2 Sharpe Ratio

Generally speaking, rational investors choose their investments with the hope that they can achieve the maximum return for a certain level of risk, or take the least risk for a given return. They will hold an efficient portfolio of investment assets. The Sharpe ratio, reflects the risk-return characteristics of capital markets in a comprehensive manner. It is currently widely used as evaluations of the performance of asset portfolios, judges of the operational efficiency of capital markets, and guide of investment decisions. The higher the ratio, the higher the excess return obtained for a certain amount of risk taken. Conversely, a small or even negative figure indicates little or no excess return for a given amount of risk. The Sharpe ratio of an investment can be shown as the following formula,

$$\text{Sharpe Ratio} = \frac{E(R_P) - R_f}{\sigma_P} \quad (6)$$

where  $E(R_P)$  is the expected returns of the assets,  $R_f$  is the risk-free interest rate, and  $\sigma_P$  is the standard deviation of the chosen assets.

## 4. Results

In this paper, we are interested in two issues in portfolio management. The first one is to analyze the function of risk-free asset in portfolio management, and the second one is to analyze which kind of asset is preferred by different investors. In this section, maximum Sharpe ratio portfolio with different weights of risk-free assets is selected. Using one-year U.S. Treasury Securities as risk-free assets with weights of 0%, 10%, 20%, 30%, 40% and 50%, the standard deviation, expected return and weights of five risky assets -- WMT, AMZN, DXCM, WBD and TROW, are shown as follows.

Table 2: Results for portfolio construction of maximize Sharpe ratio portfolio.

Portfolio	Weights (Risk-free Asset, WMT, AMZN, DXCM, WBD, TROW)	Expected Returns	Standard Deviation
0% risk-free asset	0, 0.236, 0.089, 0.002, 0.271, 0.402	0.421	0.252
10% risk-free asset	0.1, 0.213, 0.08, 0.002, 0.243, 0.362	0.379	0.227
20% risk-free asset	0.2, 0.191, 0.071, 0.002, 0.215, 0.321	0.337	0.201
30% risk-free asset	0.3, 0.168, 0.062, 0.002, 0.188, 0.28	0.295	0.176
40% risk-free asset	0.4, 0.146, 0.053, 0.001, 0.16, 0.24	0.253	0.15
50% risk-free asset	0.5, 0.123, 0.044, 0.001, 0.133, 0.199	0.212	0.125

Table 2 indicates that as the proportion of risk-free assets gradually grows, the weights of all five risky assets in the total portfolio decline, especially the downward trend of TROW is very obvious. But at the same time, TROW has the largest weight among the five spice insurance assets, followed by WBD and WMT. This also suggests that in the current post-epidemic era, people are going out less and stocking up on household items such as food or paper towels at home. This consumer behavior has maintained offline retail sales, and offline retail stores of daily necessities such as WMT have not been greatly affected. At the same time, with further liberalization in some areas, the offline retail industry is gradually picking up.

In addition, based on the previous formula for calculating the variance of the investment containing risk-free and risky assets, we can reason that the higher the proportion of risk-free assets in the whole investment, the lower the risk of it, since the standard deviation of risk-free assets is calculated by default to zero. This inference can be verified based on the calculation of the asset weights for each scenario in the table, i.e., the higher the percentage of one-year U.S. Treasury Securities, the lower the risk of the portfolio and the lower the expected volatility.

Since the proportion of risky assets all declined as the proportion of risk-free assets increased, it is not possible to determine the trend of their shares specifically. We have tabulated the changes in the proportion of these five risky assets in the overall risky asset portfolio, as shown in the Table 3.

Table 3: The weight of risky assets in risky portfolio.

The proportion of risky assets	WMT	AMZN	DXCM	WBD	TROW
100%	23.5711 %	8.9111 %	0.1798 %	27.0827 %	40.2553 %
90%	23.6667 %	8.8889 %	0.2222 %	27.0000 %	40.2222 %
80%	23.8750 %	8.8750 %	0.2500 %	26.8750 %	40.1250 %
70%	24.0000 %	8.8571 %	0.2857 %	26.8571 %	40.0000 %
60%	24.3333 %	8.8333 %	0.1667 %	26.6667 %	40.0000 %
50%	24.6000 %	8.8000 %	0.2000 %	26.6000 %	39.8000 %

As can be seen from Table 3, when the proportion of risk-free assets rises from 0% to 50% and that of risky ones in the portfolio drops from 100% to 50%, among the five risky assets in the overall risky asset's portfolio, only the figure of WMT is steadily increasing from 23.5711% to 24.6000%, while those of AMZN, WBD and TROW are gradually dropping. Apart from that, it can be noted that the proportion of DXCM is the lowest under six circumstances, while TROW always has the largest weight, remaining almost at 40%.

From these two tables, it can be concluded that when the proportion of risk-free assets increases, TROW remains the largest proportion of all risky assets in the overall portfolio, while WMT is the only asset that increases in proportion in the risky asset portfolio.

## 5. Robustness test

In this paper, robustness test was conducted to ensure the reliability of the results. Based on the original five risky assets, we added META as a risky asset of the technology field and re-performed the above model. The results of the stability test are displayed in the Table 4 below.

Table 4: The proportion of risky assets in robustness test.

The proportion of risky assets	WMT	AMZN	DXCM	WBD	TROW	META
90%	22.6667%	0.6667%	0.0000%	27.3333%	32.3333%	17.0000%
80%	22.8750%	0.6250%	0.0000%	27.2500%	32.3750%	16.8750%
70%	23.0000%	0.7143%	0.0000%	27.1429%	32.4286%	16.8571%
60%	23.3333%	0.6667%	0.0000%	27.0000%	32.3333%	16.6667%
50%	23.6000%	0.8000%	0.0000%	26.8000%	32.2000%	16.6000%

As can be seen from the table, after adding META, the share of WMT in the risky portfolio continues to gradually increases from 22.6667% to 23.6000% when the proportion of risk-free assets rises from 10% to 50%. Also, TROW still is the largest proportion of risk assets with the number around 32%, while the figure of WBD remains dropping.

Thus, after adding META as an additional risky asset, the results remain the same and the conclusions obtained according to the model are reliable.

## 6. Conclusion

Currently, most portfolio studies are based on the analysis of general market conditions or specific industries. The purpose of this study is to perform a portfolio analysis of a mix of risky and risk-free assets in different industries, including offline retail, online retail, healthcare, electronic communications, and financial sectors, in order to benefit potential investors when making investment decisions. In this paper, we use historical one-year data identifying asset allocations across different sectors to construct maximum Sharpe ratio portfolios using mean-variance analysis and Sharpe ratios for portfolio optimization. The results of the optimal portfolio allocation for these five sectors and the one-year U.S. Treasury Securities show that, based on the performance of asset of last year, given the proportion of risk-free assets, stocks in the online retail sector and the financial sector occupy the largest share of the risky asset portfolio. Meanwhile, for investors who are risk averse, they can try to increase their allocation to risk-free assets and assets in the offline retail sector such as WMT, while for investors who are not risk averse, they can reduce their risk-free asset allocation and increase their investment in financial sector assets such as TROW. Thus, investors seeking stability in the post-epidemic era, where the economy is still volatile, may turn their attention more to offline retail. Moreover, this paper verifies that the larger the proportion of risk-free assets in the portfolio, the less risky the portfolio amount is, i.e., risk-free assets play a role of risk diversification in the overall portfolio.

However, drawbacks also exist. For example, the Sharpe ratio, measures the fund's historical performance, so it's not a simple way to base future moves on that performance. In calculation, the results of this ratio are related to the choice of the time horizon and time interval for calculating the returns, which may also cause errors and thus a stability issue.

## Reference

- [1] Markowitz, H.: *PORTFOLIO SELECTION*. *The Journal of Finance*, 7, 77-91. (1952)  
<https://doi.org/10.1111/j.1540-6261.1952.tb01525.x>
- [2] dos Santos, S. F., Brandi, H. S.: *Selecting portfolios for composite indexes: application of Modern Portfolio Theory to competitiveness*. *Clean Technologies and Environmental Policy*, 19(10), 2443–2453 (2017)  
<https://doi.org/10.1007/s10098-017-1441-y>
- [3] Cui, Y., Cheng, C. *Modern Portfolio Theory and Application in Australia*. *Journal of Economics, Business and Management*. (2022).
- [4] Biswas, D.: *The Effect of Portfolio Diversification Theory: Study on Modern Portfolio Theory of Stock Investment in The National Stock Exchange*. *Journal of Commerce and Management Thought*, 6(3), 445(2015).  
<https://doi.org/10.5958/0976-478x.2015.00027>
- [5] He, W.: *An Empirical Research Based on Markowitz's Portfolio Theory*. *World Scientific Research Journal*, 8(3), 2472 - 3703. (2022). [https://doi.org/10.6911/WSRJ.202203\\_8\(3\).0028](https://doi.org/10.6911/WSRJ.202203_8(3).0028)
- [6] Beuhler, M.: *Application of modern financial portfolio theory to water resource portfolios*. *Water Science and Technology: Water Supply*, 6(5), 35–41(2006). <https://doi.org/10.2166/ws.2006.828>
- [7] Fagefors, C., Lantz, B.: *Application of portfolio theory to healthcare capacity management*. *International Journal of Environmental Research and Public Health*, 18(2), 1–9(2021). <https://doi.org/10.3390/ijerph18020659>
- [8] Hwang, I., S. Xu, F. In.: *Naive versus Optimal Diversification: Tail Risk and Performance*. *European Journal of Operational Research*, 265 (1): 372–388. (2018).  
<https://d.wanfangdata.com.cn/periodical/2003e0c63c0b251fc93f6825a93b678f>
- [9] Evans, J. L., S. H. Archer.: *Diversification and the Reduction of Dispersion: An Empirical Analysis*. *The Journal of Finance*, 23 (5): 761–767. (1968).
- [10] Statman, M.: *How Many Stocks Make a Diversified Portfolio?* *Journal of Financial and Quantitative Analysis*, 22 (3): 353–364. (1987).
- [11] Statman, M. *The Diversification Puzzle*. *Financial Analysts Journal*, 60 (4): 44–53. (2004).  
<https://doi.org/10.2469/faj.v60.n4.2636>
- [12] Campbell, S. J., Chong, J., Jennings, W. P., Phillips, G. M.: *Portfolio optimization strategy for concentrated portfolios: Models and Time Horizons*. *The Journal of Wealth Management*, 21(2), 39-54 (2018)..  
<https://doi.org/10.3905/jwm.2018.1.064>