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Portfolio Optimization Strategy Based on Four Deep Learning Models

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Abstract: Deep learning techniques have provided a fresh outlook on the evergreen subject of portfolio optimization within the finance domain. This article selects the stocks of Google, Tesla, Tractor Supply Company, Analog Devices, and Duke Energy Corporation and deploys four deep learning models to estimate returns and covariance respectively. The mean-variance model is utilized to generate the target portfolio for each deep learning model, incorporating the predicted outcomes. Ultimately, the returns of each portfolio are compared to the market benchmark (S&P 500) returns. The findings demonstrate that the proposed target model outperforms the market benchmark (S&P 500) across multiple financial metrics. This study highlights the groundbreaking and promising applications of deep learning in the financial sector, providing valuable insights into innovative portfolio allocation strategies for risk-averse investors who aim to achieve stable and positive returns even in turbulent market conditions.

Keywords: RNNs, self-Attention, transformer, portfolio optimization, mean-variance

1. Introduction

Markowitz introduced the Mean-Variance model, which revolutionized portfolio optimization by considering both the expected returns and the risk associated with different asset allocations [1]. This seminal work laid the foundation for modern portfolio theory and become a focus of modern finance.

Since then, numerous research papers have focused on refining and extending the Mean-Variance model. Das et al. introduce a novel approach to portfolio management that incorporates psychological factors and the concept of mental accounting, allowing investors to make more informed and personalized investment decisions [2]. Over time, there has been a surge in enthusiasm for the application of state-of-the-art methodologies [3]. Furthermore, Laher et al. explores the use of deep learning models, including GRU and LSTM, for the optimization of portfolio rebalancing, contributing to the advancement of machine learning techniques in the field of finance [4]. In a subsequent study, Kisiel et al. introduce the innovative use of attention-based models, such as the Portfolio Transformer, to improve the effectiveness of asset allocation strategies [5]. However, there is still a lack of relevant research in this area, so this paper aims to further explore the optimization of portfolios using four deep learning methods.

By leveraging deep neural networks and self-attention mechanism, the paper presents an approach for portfolio optimization and contributes to the development of advanced techniques for optimizing investment portfolios using deep learning methods. To test the proposed methods, a selection of five

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representative stocks from the U.S. stock market was made. In order to train each model, the study employed weekly stock price data from the preceding 52 weeks to forecast the subsequent week's stock prices. Subsequently, the mean-variance optimization method was employed to determine optimal portfolio weights on a weekly basis. Throughout the test dataset, this process was repeated for each week, with portfolio weights being updated based on the most recent real stock prices. Upon completion of the testing period, the overall portfolio returns were computed and compared against the returns generated by the SP500 index as a benchmark [6].

The paper is organized as follows: Section 2 presents a detailed description of the data utilized in this research and discusses the methodology employed for stock selection, accompanied by a descriptive analysis of the chosen stocks. Section 3 elaborates on the methods employed in this research, providing detailed explanations. In Section 4, the effectiveness of the proposed approach is examined, comparing it to benchmark assets and other simplistic portfolios. Finally, Section 5 concludes the paper, highlighting key findings and suggesting potential directions for future research.

2. Data and Methodology

2.1. Data Source and Pre-process

This paper carefully selects 5 representative stocks based on several key factors. Firstly, emphasis is placed on identifying industry leaders that drive technological advancements and innovation. These stocks represent companies at the forefront of their respective sectors, showcasing their ability to shape and influence industry trends. Secondly, the selection process takes into account the financial performance of the chosen stocks, prioritizing companies with a track record of consistent growth, profitability, and efficient operations. The selected stocks are listed in Table 1 as follows:

Table 1: Selected stocks.

Stock Symbol	Company
GOOGL	Alphabet Inc.
DUK	Duke Energy Corporation
TSCO	Tractor Supply Company
ADI	Analog Devices, Inc.
TSLA	Tesla, Inc.

The study collected the adjusted closing prices of five stocks from July 2nd, 2013, to July 2nd, 2023, from Yahoo Finance (https://finance.yahoo.com/). Subsequently, the dataset was partitioned into a training set and a test set. The study performed data cleaning to align the timestamps. In total, the study obtained 523 data points for further research. The reason for selecting stock prices from the past decade for research is to analyze recent market trends and incorporate up-to-date information for more relevant insights. Table 2 and Figure 1 presents the descriptive statistics for five stocks. These statistics provide a concise summary of the data distribution and variability, offering insights into the average performance, range, and dispersion of stock prices for the selected stocks.

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Table 2: Descriptive statistics of five stocks.

	TSLA	DUK	ADI	TSCO	GOOGL
count	523	523	523	523	523
mean	81.4269	71.1324	91.9360	101.4659	61.7481
std	101.8096	11.6284	41.2211	51.7639	31.5840
min	1.9786	41.4470	31.9675	41.7073	21.0934
max	401.3633	101.9304	191.2798	241.3927	141.9524

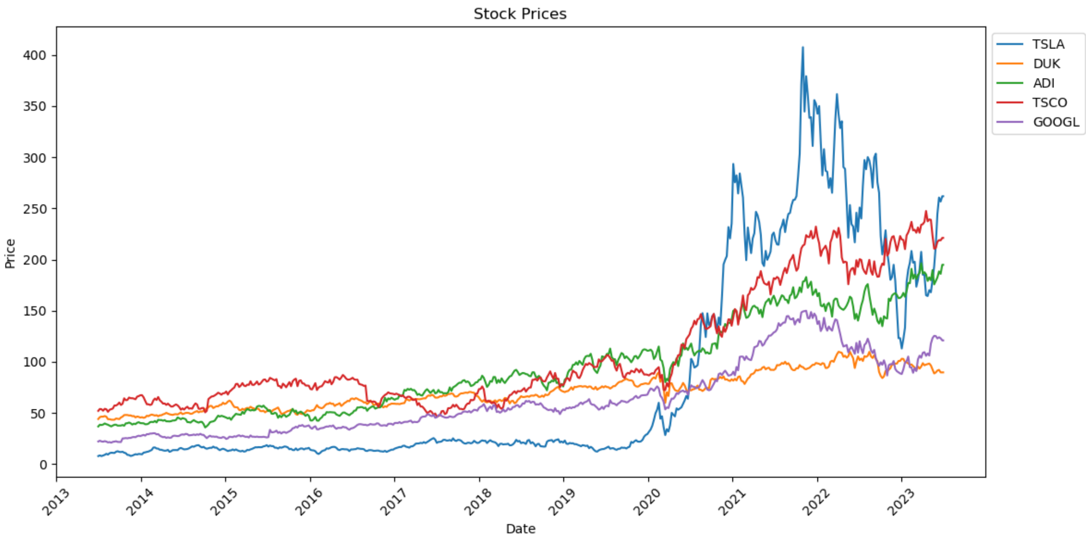

Figure 1: Weekly price of selected stocks.

2.2. Long Short-Term Memory (LSTM)

LSTM is a specialized recurrent neural network that addresses the challenge of capturing long-term dependencies [7]. It introduces a cell state c_t and three gates: the forget gate f_t, input gate i_t, and output gate o_t. These gates control the flow of information and help the model retain important contextual information over extended sequences. By overcoming the vanishing gradient problem, LSTM has become a powerful tool in various domains.

$$o_t = \sigma(x_t U^o + h_{t-1} W^o) \tag{1}$$

$$i_t = \sigma\left(x_t U^i + h_{t-1} W^i\right) \tag{2}$$

$$f_t = \sigma(x_t U^f + h_{t-1} W^f) \tag{3}$$

$$g_t = \tanh(x_t U^g + h_{t-1} W^g) \tag{4}$$

$$h_t = \tanh(c_t) \cdot o_t \tag{5}$$

$$c_t = c_{t-1} \cdot f + g \cdot i \tag{6}$$

2.3. Gated Recurrent Unit(GRU)

GRU is a variant of the LSTM model that simplifies its architecture by using only two gates: the reset gate r_t and the update gate z_t [8]. These gates play a crucial role in determining the behavior of the hidden state h_t. The reset gate controls how the current input is combined with the historical memory, while the update gate determines the extent to which the historical memory is retained in the node. By training the weights of the reset and update gates through backpropagation, the GRU model can effectively capture and utilize both short-term and long-term dependencies in the input sequence.

$$r_t = \sigma(x_t U^r + h_{t-1} W^r) \tag{7}$$

$$z_t = \sigma(x_t U^z + h_{t-1} W^z) \tag{8}$$

$$k = \tanh(x_t U^k + (h_{t-1} \cdot r) W^k) \tag{9}$$

$$h_t = (1 - z) \cdot k + z \cdot h_{t-1} \tag{10}$$

2.4. Self-Attention

The self-attention mechanism provides the advantage of capturing long-range dependencies and identifying relevant features in stock prediction tasks [9]. This abstraction highlights the selective focus on relevant information and the dynamic allocation of attention. It enables the model to capture important features by assigning varying weights to different values, regardless of the specific framework used.

$$\text{Attention (Query, Source)} = \sum_{i=1}^{L_x} \text{Similarity(Query, Key}_i) * \text{ Value}_i \tag{11}$$

In the initial phase, the weight coefficients associated with each Key, corresponding to the given Value, are determined by evaluating the relevance between each Query and the Keys. In this context, K represents the Key, Q represents the Query, F denotes a function, V signifies the weight value, Sim denotes similarity, a represents the weight coefficient, and A represents the attention value. These calculations determine the attention weights assigned to different Key-Value pairs, allowing the model to selectively focus on relevant information. This flexibility in computing the attention coefficients enables the model to adapt to different scenarios and capture important features effectively.

$$\text{Similarity}(Q, K_i) = Q \cdot K_i \tag{12}$$

In the second phase, the weights are normalized using a function similar to SoftMax, as shown in Equation (13).

$$a_i = \text{SoftMax}(\text{Similarity}_i) \tag{13}$$

In the third stage, the attention values are computed by taking the weighted sum of the attention weights (a_i) and the corresponding values (V_i), as shown in Equation (14). This step combines the relevance of each key-value pair to generate the final attention value.

$$\tilde{a}(Q, S) = \sum_{i-1}^{L_x} a_i \cdot V_i \tag{14}$$

2.5. Transformer

The Transformer model has the advantage of capturing long-range dependencies and effectively modeling sequential data, making it well-suited for predicting stock prices [10]. The Transformer model adopts an encoder-decoder architecture to effectively capture global dependencies between the input and output using attention mechanisms. Unlike traditional recurrent structures, it eliminates the need for recursion and achieves efficient parallel processing of data. The encoder is composed of six identical layers, which consist of a multi-head attention layer and a feed-forward layer. In contrast, the decoder has a more complex architecture that includes masked multi-head attention layers. By employing self-attention mechanisms, the Transformer model effectively preserves long-distance information between data and enhances the efficiency of training.

The Transformer architecture utilizes linear transformations of the input data to derive the matrix of queries (Q), the matrix of keys (K), and the matrix of values (V), with computations following the formulas.

$$\text{Attention}(Q,K,V)=\text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \tag{15}$$

$$\text{MultiHead}(Q,K,V)=\text{Concat}(\text{ head }_1,\text{ head }_2,\ldots,\text{head}_h)W^0 \tag{16}$$

$$\text{head }_i=\text{Attention}\left(QW_i^Q,KW_i^k,VW_i^v\right) \tag{17}$$

These parameters matrices (W_i^Q, W_i^k, W_i^v) are utilized to apply linear transformations to the input data.

2.6. Mean-Variance (MV)

MV model offers a mathematical framework for determining the optimal weights assigned to each asset in an investor's portfolio. The MV model aims to maximize portfolio returns while considering the associated level of risk. Let w_i represent the weight assigned to the i-th asset, satisfying the constraint $\sum_i w_i = 1$, and μ_i denote the anticipated yield of the i-th asset. The expected returns of the portfolio can then be expressed as follows, where R_p signifies the overall expected portfolio return:

$$\mu_p=\sum_i w_i\,\mu_i \tag{18}$$

Denote σ_i as the standard deviation of the i-th asset and ρ_{ij} as the correlation between the returns of the i-th and j-th asset. Then the portfolio return variance is

$$\sigma_p^2=\sum_i w_i^2\,\sigma_i^2+\sum_i\sum_{j\neq i} w_i\,w_j\sigma_i\sigma_j\rho_{ij} \tag{19}$$

Ideal portfolios lie on the efficient frontier. The equations can be transformed into matrix form, making implementation and efficient frontier calculations more convenient. The objective function for the Mean-Variance (MV) model is expressed as:

$$min(w^\top \Sigma w-qR^\top w) \tag{20}$$

In the Mean-Variance (MV) model, the portfolio weights vector, denoted by w, and the sample covariance matrix of asset returns, denoted by Σ, play crucial roles. The sample covariance matrix represents the historical data used for computation. To enhance the performance of the covariance matrix, certain adjustments can be made. The parameter q quantifies the investor's risk tolerance.

Two portfolios of particular interest in the MV model are the minimum volatility portfolio and the maximum Sharpe ratio portfolio (MSP). The minimum volatility portfolio (MVP) corresponds to q=0, indicating complete risk aversion by the investor. The Sharpe ratio is a widely used measure for evaluating risk-adjusted return. It is calculated as follows:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p} \tag{21}$$

where R_f is the current risk-free rate of the market.

3. Results

The study initially assessed the fitting performance of GRU, LSTM, Self-Attention, and Transformer models MSE as the evaluation metric. Each model was trained for 10 epochs. Through extensive parameter tuning, the study obtained predictions for asset prices using these four models. Notably, the models demonstrated a certain level of accuracy in predicting future values of the asset price, as evidenced by the close alignment observed between the predictions and the validation values of the test set. The following Table 3 presents the mean squared error (MSE) values for each company corresponding to the four models:

Table 3: MSE for each corporation-model pair.

	GRU	LSTM	SELF-ATTENTION	TRANSFORMER
GOOG	141.7510	101.9332	51.4835	331.3277
DUK	81.8855	141.3416	61.7227	551.6717
ADI	221.6820	351.5012	41.7681	1981.2112
TSCO	791.3846	551.1798	121.9086	991.4045
TSLA	3061.9929	1111.9360	701.6178	3601.7982

This study proposes a trading strategy based on a weekly time frame. Throughout the 52-week testing period, the asset weights for each week are determined using the predictions generated by four distinct models: GRU, LSTM, Self-Attention, and Transformer. Each model corresponds to a set of weights that are utilized to construct portfolio strategies aimed at maximizing the Sharpe ratio and minimizing the variance. As a result, a total of eight portfolio strategies are developed, representing different combinations of the predictive models and optimization objectives.

To assess the performance of each model, the study obtains the historical returns of the S&P 500 index during the test period as a benchmark for the market. Subsequently, an ex-post analysis is conducted to determine the actual returns of each portfolio. The weights presented in Table 4 represent the allocations of the five companies for each respective model during the initial week, while Figure 2 illustrates the cumulative returns of each portfolio over time.

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Table 4: Initial Week Weights.

	MSP(LSTM)	MVP(LSTM)	MSP(GRU)	MVP(GRU)
GOOG	0.0716	0.1134	0.1048	0.0628
ADI	0.5846	0.2348	0.2229	0.5905
DUK	0.1640	0.1995	0.1654	0.1579
TSCO	0.1776	0.2064	0.2650	0.1875
TSLA	0.0022	0.2459	0.2419	0.0014
	MSP(Self-Attention)	MVP(Self-Attention)	MSP(Transformer)	MVP(Transformer)
GOOG	0.0267	0.0520	0.0856	0.1565
ADI	0.0720	0.5464	0.3655	0.6179
DUK	0.0478	0.1460	0.0815	0.1151
TSCO	0.5676	0.2535	0.2576	0.1028
TSLA	0.2860	0.0021	0.2099	0.0076

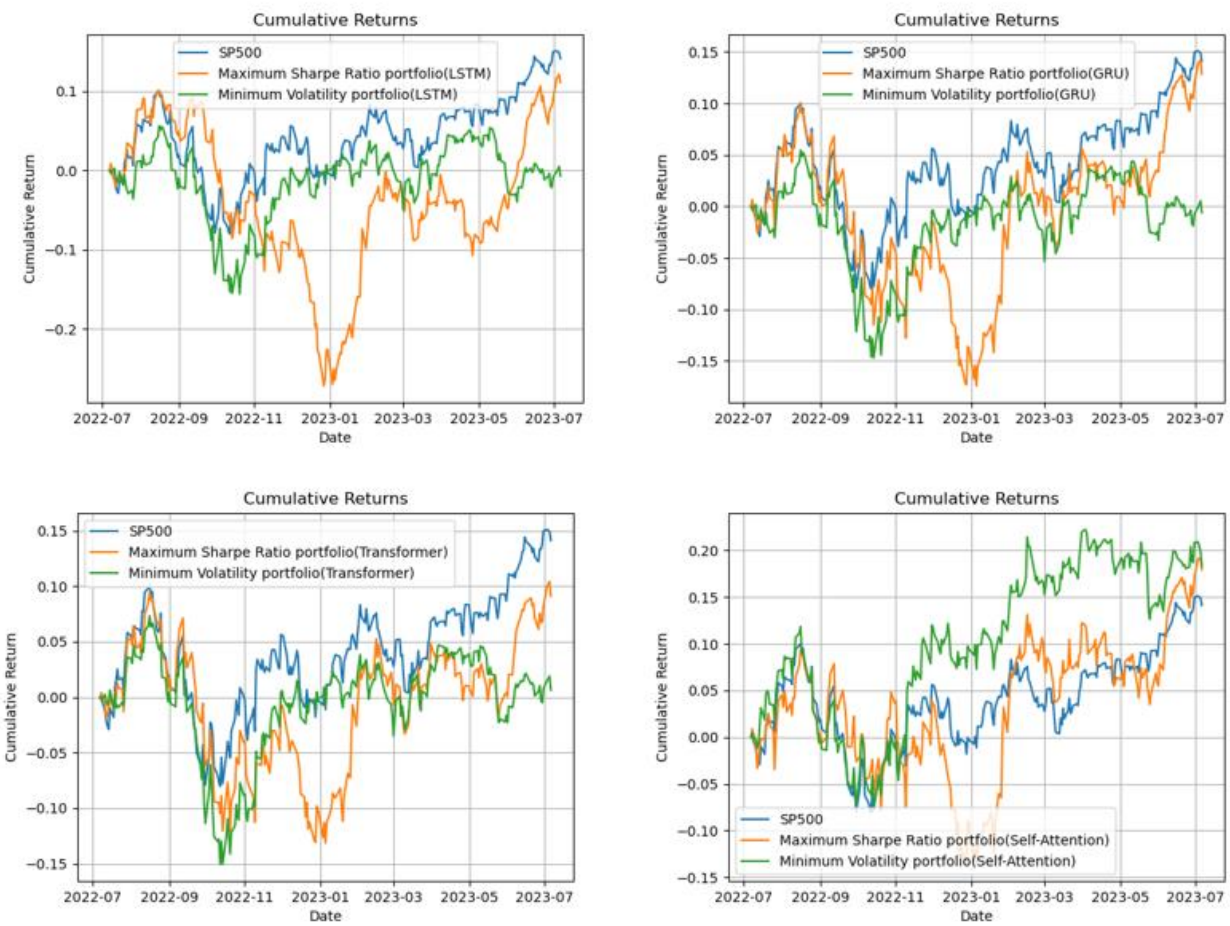

Figure 2: Comparison between SP 500 returns and alternative portfolios.

Based on the findings, the following characteristics can be observed:

The cumulative returns of the two combinations (MVP, MSP) corresponding to LSTM, GRU, and Transformer models are lower than the cumulative return of the S&P 500 index (14.2%); The cumulative returns of the two combinations (MVP, MSP) corresponding to the self-attention model (18.3%, 18.0%, respectively) were slightly higher than the cumulative return of the benchmark(14.2%); The cumulative returns of the benchmark are more stable compared to the cumulative returns of the MSP and MVP portfolios corresponding to each model; The target model (MVP corresponding to Self-Attention) has consistently outperformed both the S&P 500 and MSP corresponding to Self-Attention since November 2022. Furthermore, it has exhibited greater stability compared to the MVP portfolio, indicating a more consistent and favorable performance.

4. Conclusion

Deep learning techniques have revolutionized portfolio optimization in the finance domain, offering new perspectives and opportunities. This study focuses on the selection of stocks from Google, Tesla, Tractor Supply Company, Analog Devices, and Duke Energy Corporation. Leveraging the power of four deep learning models, returns and covariance estimates are derived for these stocks, respectively. The mean-variance model is then utilized to construct target portfolios for each deep learning model, incorporating the predicted outcomes. Subsequently, a comparative analysis is conducted between the returns of these portfolios and the market benchmark. The findings unveil the superior performance of our proposed target model across various financial metrics, indicating its potential for innovative portfolio allocation strategies. This research sheds light on the groundbreaking and promising applications of deep learning in the financial sector, paving the way for advancements in portfolio optimization through the integration of deep learning methodologies.

GRU, LSTM, Self-Attention, and Transformer models have proven effective in predicting stock prices. However, they do have limitations. These models may struggle to capture abrupt market changes and unpredictable events. They can be sensitive to extreme market conditions and outliers, impacting their accuracy. Additionally, overfitting can occur when the models are trained on limited or noisy data. Another challenge is their lack of interpretability, as they operate as complex black-box models. Despite these limitations, these models offer valuable insights and can be enhanced by addressing their weaknesses and combining them with other approaches in stock price prediction.

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