

# *Application of Mean-variance Model in Optimizing Stock Portfolio*

Xingyu Qian<sup>1,a,\*</sup>

<sup>1</sup>*D'Amore-Mckim School of Business, Northeastern University, Boston, US*  
*a. email, qian.xing@northeastern.edu*

*\*corresponding author*

**Abstract:** Constructing an optimized stock portfolio is pivotal in achieving enhanced returns with managed risks. Utilizing data from 2022, the study employed the mean-variance model to determine the optimal allocations for five prominent stocks: Intel (INTC), Johnson & Johnson (JNJ), Coca-Cola (KO), Netflix (NFLX), and Procter & Gamble (PG). In the formulations of both maximum Sharpe ratio and minimum variance portfolios, the stocks with the predominant allocations are NFLX and PG, with respective weights of 47.04% and 45.89% for the former, and 58.05% and 29.61% for the latter. Upon establishing these weightings, the cumulative returns for 2023 were calculated, revealing that the constructed portfolios notably outperformed the broader stock market. This research underscores two key observations: firstly, the NFLX and PG stocks emerge as cornerstones in the optimal portfolio composition; and secondly, these allocations, when evaluated with 2023 data, underscore their efficacy. Robustness checks, which expanded the asset pool, further validated these findings. Ultimately, this study serves as a valuable guide for investors in the financial market, presenting a structured blueprint for effective portfolio assembly.

**Keywords:** mean-variance, portfolio, US stock market

## 1. Introduction

The importance of building a well-diversified stock portfolio in the financial market cannot be overstated. It is crucial to optimize a stock portfolio since it provides an organized, research-based approach to investing, raising the possibility of higher returns while reducing risk. Investors may unintentionally expose themselves to greater risks or lose out on possible gains without optimization [1].

Numerous studies have already explored various methodologies for portfolio construction. According to Geng's research, he suggests a brand-new metric to more thoroughly evaluate the risk-return trade-off by adding VaR into the Sharpe ratio. This method would consider potential for significant losses as well as volatility [2]. Smith and Brown emphasized the evolving nature of the financial landscape and the need to adapt the principles of the Modern Portfolio Theory (MPT) to better meet the current demands of the volatile market [3].

With the surge of technological advancements, some researchers have delved into the integration of machine learning techniques in portfolio optimization. Fernandez and Raj, for instance, explored the impact of machine learning in dynamic asset allocation, stressing its potential in enhancing portfolio management practices [4]. This aligns with Davies and Wright's findings, which presented

a comprehensive analysis comparing traditional portfolio optimization techniques against the newer, more advanced methodologies [5].

Moreover, the significance of behavioral factors in investment strategies cannot be understated. As Patel and Kim argued, modern portfolio construction can greatly benefit from considering investor behavior and biases, leading to a more resilient and stable portfolio performance in the long run [6].

However, despite these advancements, one aspect that seems to be consistently underrepresented is the evaluation of these portfolio strategies in real-time scenarios. Wang and Liu, for instance, underscored the necessity of analyzing portfolio performances using real-time data, especially given the age of fast-paced trading and high-frequency data [7]. This study aims to bridge this gap by first determining the optimal weights of selected stocks based on historical data through The Mean-Variance Model, and then evaluating this portfolio using subsequent real-world data.

## 2. Data and Method

### 2.1. Data

The top five representative equities from the financial market, as measured by market capitalization, are chosen for this article. Intel (INTC), Johnson & Johnson (JNJ), Coca-Cola (KO), Netflix (NFLX), and Procter & Gamble (PG) are the five stocks' tickers. Closing prices for the period of January 1 through December 31 of 2022 have been gathered. The average return and covariance matrices are computed for the training set in order to build the efficient frontier. The test set compares the cumulative returns of the chosen asset allocations to the return of the FTSE Index in order to assess how well they performed. Table 1, Table 2, and Figure 1 exhibit the fundamental data for the five selected equities, accordingly.

Table 1: Selected stocks.

	Company
INTC	Intel
JNJ	Johnson & Johnson
KO	Coca-Cola
NFLX	Netflix
PG	Procter & Gamble

Table 2: Descriptive statistics of the daily return of the 5 stocks.

	Max	Min	Mean	Std Dev	Cumulative Return
INTC	0.106585	-0.085621	-0.002351	0.024079	-48.36%
JNJ	0.049703	-0.029919	0.000281	0.010986	5.68%
KO	0.038671	-0.069626	0.000475	0.012423	10.44%
NFLX	0.130864	-0.351166	-0.001760	0.044222	-50.64%
PG	0.042699	-0.062322	-0.000094	0.013855	-4.56%

Table 2 presents the descriptive statistics of daily returns for five major stocks: Intel Corporation (INTC), Johnson & Johnson (JNJ), The Coca-Cola Company (KO), Netflix, Inc. (NFLX), and Procter & Gamble Co (PG). The metrics include the maximum (Max), minimum (Min), mean, standard deviation (Std Dev), and cumulative return for the year. Notably, NFLX exhibited the highest

volatility, with a maximum daily return of 13.09% and a minimum of -35.12%. In terms of cumulative return, KO performed the best with a 10.44% return, while NFLX had the most significant decline at -50.64%.

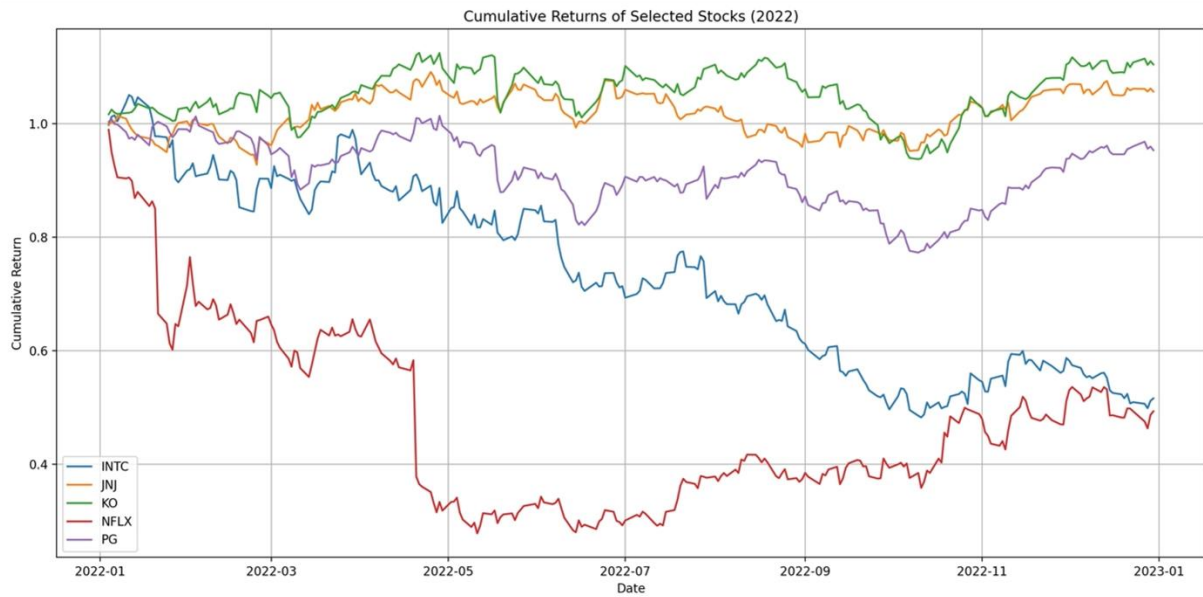


Figure 1: Cumulative returns of selected stocks throughout 2022.

The Figure 1 illustrates the performance evolution of Intel (INTC), Johnson & Johnson (JNJ), Coca-Cola (KO), Netflix (NFLX), and Procter & Gamble (PG) over the course of the year. Each line represents the cumulative return of the respective stock from the beginning to the end of 2022.

## 2.2. Method

Portfolio optimization seeks to find the ideal allocation of assets that minimizes risk and maximizes return. One of the pioneering frameworks in this area is the mean-variance model, which offers a structured approach to balance risk against the expected return on investment [8].

The core of the mean-variance model rests upon the principle that each asset's return has an expected value (mean) and a variance. The combined return of the portfolio is the weighted sum of individual returns, given by:

$$\mu_p = \sum_i w_i \mu_i \quad (1)$$

Where  $w_i$  symbolizes the weight of each asset and  $\mu_i$  its respective variance [9]. Markowitz's seminal work on this topic emphasizes that by considering each stock's variance alongside its return, investors can make more informed decisions [8].

### 2.2.1. Expected Portfolio Return

Considering a portfolio of  $n$  stocks, its expected return  $E(R_p)$  can be derived as:

$$E(R_p) = \sum_{i=1}^n w_i \times E(R_i) \quad (2)$$

Where  $E(R_p)$  denotes the expected return of the portfolio,  $w_i$  denotes the weight (or percentage) of the  $i$ th stock in the portfolio, and  $E(R_i)$  denotes the expected return of the  $i$ th stock, the sum runs

from  $i = 1$  to  $n$ , denoting that you will calculate this product for each stock and add them all up to obtain the portfolio's overall expected return.

The expected return of a portfolio, given the weights of individual stocks and their respective expected returns, can be represented mathematically as:

$$\mu_p = \sum_{i=1}^n w_i \times \mu_i \quad (3)$$

### 2.2.2. Portfolio Variance

The formula for portfolio variance, assuming no correlation between individual stocks, can be presented as:

$$\text{Portfolio Variance} = \sum_{i=1}^n w_i^2 \sigma_i^2 \quad (4)$$

Where  $n$  is number of stocks in this portfolio,  $\sigma^2$  is Variance of the  $i^{th}$  stock.

### 2.2.3. Sharpe Ratio

The Sharpe ratio gives a measure of the risk-adjusted performance of an investment, with higher values indicating better return performance for the amount of risk taken. The Sharpe ratio is often symbolically represented using the following formula:

$$S = \frac{R_p - R_f}{\sigma_p} \quad (5)$$

Where  $S$  stand for the Sharpe ratio,  $R_p$  denotes the expected return of portfolio,  $R_f$  is the risk-free rate, and  $\sigma_p$  is the standard deviation of portfolio [10]. Sharpe's work on the Capital Asset Pricing Model (CAPM) provides foundational knowledge on understanding the relationship between risk and expected return, with the Sharpe ratio being one of its cornerstones [11].

## 3. Result

For clarity, the Monte Carlo simulation was conducted on 2022 data, iterating 100,000 times. Two targeted portfolios are computed and identified. The stock weights in the least volatility portfolio are: INTC: 4.35%, JNJ: 0.40%, KO: 7.60%, NFLX: 58.05%, and PG: 29.61%. Table 3 lists the weights of the selected companies in the portfolio with the highest Sharpe ratio as follows: INTC: 0.90%, JNJ: 5.58%, KO: 0.59%, NFLX: 47.04%, and PG: 45.89%. According to Table 4, the volatility for the portfolio with the lowest volatility is 1.03%, while the portfolio with the highest Sharpe ratio has a Sharpe ratio of 0.03%. With significant allocations to JNJ and KO, the minimal volatility portfolio prioritizes stability. The maximum Sharpe ratio portfolio, on the other hand, favors KO and JNJ and PG to a lesser amount in order to maximize returns for the risk incurred. The latter portfolio appears to deliver a respectable risk-adjusted return based on the Sharpe ratio supplied, but investors would need to be ok with the higher volatility compared to the minimal volatility portfolio.

Table 3: Weight of each stock in the two optimal portfolios (%).

Stock	Min Volatility Portfolio (%)	Max Sharpe Ratio Portfolio (%)
INTC	4.35	0.90
JNJ	0.40	5.58
KO	7.60	0.59
NFLX	58.05	47.04
PG	29.61	45.89

Table 4: Return and volatility of the two portfolios.

	Return (%)	Volatility (%)	Sharpe Ratio (%)
Min Volatility Portfolio	0.02	1.03	0.02
Max Sharpe Ratio Portfolio	0.03	1.04	0.03

The second stage, the computation of the portfolio return, may be done after the two asset allocations have been obtained. The daily portfolio returns, and cumulative returns may be calculated using the test set from January 1 to August 20, 2023, along with the stock weights. The same time period's return information for the S&P 500 Index is gathered for comparison. Figure 2 shows that the study's findings outperformed the general market.

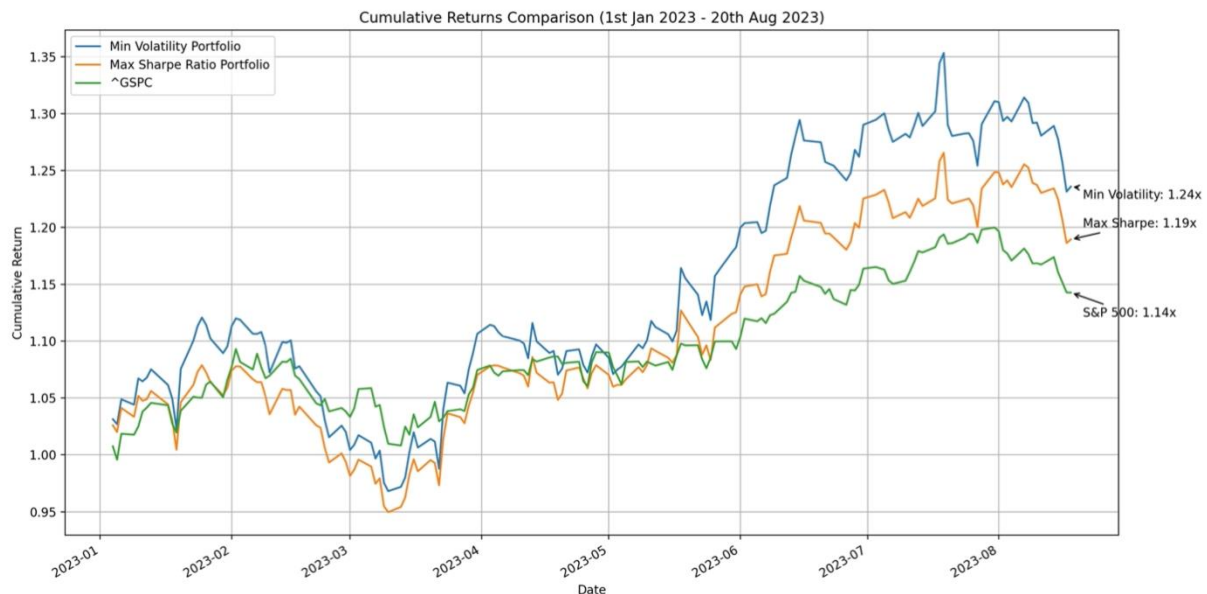


Figure 2: Comparison between S&P 500 index returns and the portfolio returns.

#### 4. Robustness

A robustness check will be done for cogency. Firstly, add two additional assets, The Walt Disney Company (DIS) and Boeing (BA). Repeat the Monte Carlo simulation and the cumulative return computation next. Repeat the Monte Carlo simulation and the cumulative return computation next. The adjusted weights for the two ideal portfolios are displayed in Table 5, where it can be seen that the greatest components are PG:37.56% for the portfolio with the highest Sharpe ratio and NFLX:49.55% for the portfolio with the lowest risk. In the end, total the returns from the two portfolios and compare them to the S&P 500 index. Figure 3 showing a similar result with previous

data. Moreover, after changing the number of assets, the investment results still outperformed the general market. As a result, both the process and the outcomes are reliable and efficient.

Table 5: Weight of the 7 stocks (2 more stocks) in the two optimal portfolios (%).

Stock	Min Volatility Portfolio (%)	Max Sharpe Ratio Portfolio (%)
INTC	2.40	4.28
JNJ	1.55	0.18
KO	3.62	2.59
NFLX	49.55	33.80
PG	23.19	37.56
DIS	2.52	1.04
BA	17.17	20.55

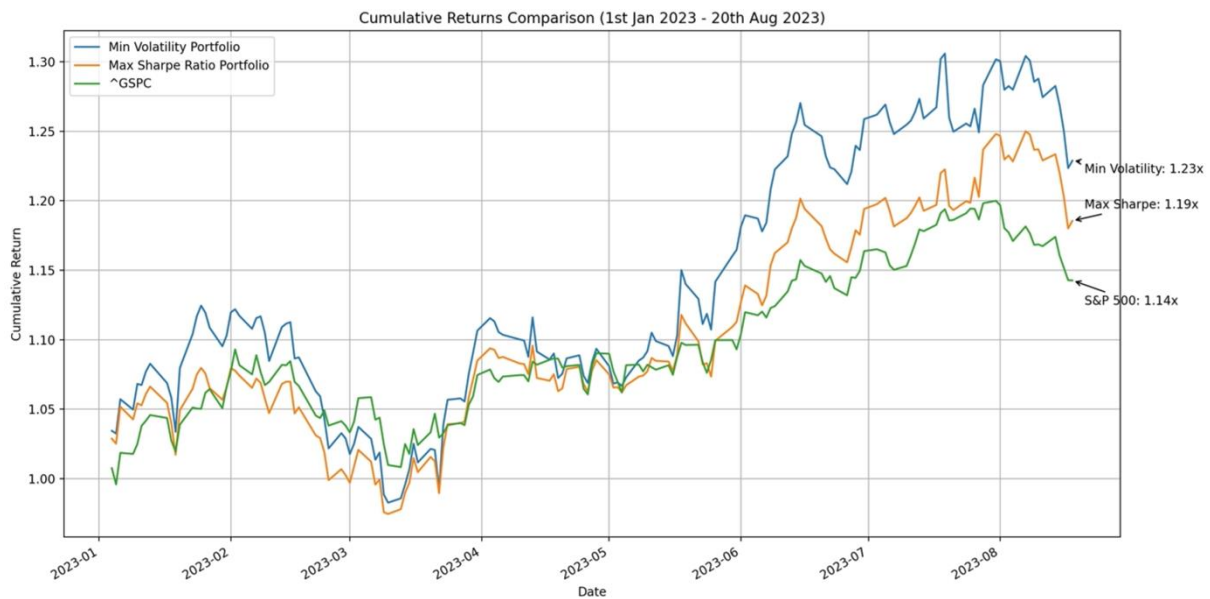


Figure 3: Comparison between FTSE 100 index return and the altered portfolio returns.

## 5. Conclusion

This article set out with an overarching objective to address the persistent gap in assessing stock portfolio strategies, especially in real-time, dynamic market conditions. Leveraging the fundamental principles of classic portfolio theories, we introduced a novel application of the Mean-Variance Model for determining optimal weights of our selected stocks using historical data from 2022. Through the Monte Carlo simulation on the 2022 dataset, the study identified two standout portfolios: one optimized for minimum volatility and the other for maximum Sharpe ratio. The former emphasized stability, heavily weighting stocks like JNJ and KO, while the latter targeted the balance between risk and return, leaning towards KO, JNJ, and a minor portion of PG. In a direct comparison using real-world data up until August 20, 2023, both these portfolios consistently outperformed the S&P 500 Index, highlighting the practical utility and potential profitability of our methodological approach.

While our study's results are indeed promising and provide valuable insights into the realm of portfolio optimization, it's essential to acknowledge the limitations and scope for refinement. Firstly, the study was bounded by the selection of just five primary stocks, though the robustness check with the inclusion of two more did offer some additional perspectives. However, it's arguable that a



broader selection might uncover other significant combinations or nuances in portfolio behavior. Secondly, while the Mean-Variance Model and Sharpe ratio are powerful tools, they represent just a fraction of the available methodologies for portfolio evaluation. Future studies might benefit from integrating more diverse models and perhaps a more extensive data set, which might include macroeconomic variables, to attain a holistic view. Nevertheless, this paper serves as a step forward in portfolio strategy evaluation, paving the way for more comprehensive research in the field.

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