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Comparative Analysis and Research of Investment Portfolio Management Models

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Abstract: Portfolio management plays a significant role in the world of finance and investing, offering benefits and contributions to both individual investors and institutions. Researchers use many mathematical models to make informed decisions about how to allocate and manage investments within a portfolio to optimize risk versus return trade-off based on the investor's preference and constraints. This paper primarily employs the literature review methodology and comparative analysis methodology. Firstly, it collects, summarizes, and analyzes multiple papers on portfolio management models, including Markowiz Mean-Variance Model, Capital Market Line (CML), Arbitrage Pricing Theory (APT) and Capital Asset Pricing Model (CAPM) with the origin of the models, their key assumptions, components, and applications. Additionally, this paper also utilizes comparative analysis by comparing the similarities, differences, and respective downsides and benefits of the four main research models. Through this comparison, the paper investigates the relationships and application distinctions among the models. As a result, these models evolve and refine over time, with some building on others. CAPM extends the Markowitz Model by introducing the risk-free rate and market portfolio as benchmarks, simplifying the risk-return relationship, and introducing systematic risk. The Capital Market Line (CML), derived from CAPM, illustrates efficient portfolios made up of both market and risk-free assets and shows the risk-return trade-off. APT, a later model, can be seen as an extension of CAPM.

Keywords: Portfolio Management, Markowiz Mean-Variance Model, CML, CAPM, APT

1. Introduction

Portfolio management describes the procedure for selecting investments allocations and strategies to achieve specific financial goals while managing risk. It involves the selection, allocation, and monitoring of various assets within a portfolio to optimize returns depending on an investor's investing goals, time horizon, and risk tolerance. During its early phases of development, portfolio optimization was often constrained by its static implementation. In contrast to dynamic portfolio optimization, which continually monitors optimal portfolio weights based on new market information, static optimization weights cannot respond to market fluctuations within the investment horizon. Despite its power, dynamic portfolio optimization proved to be a computationally costly challenge. Remember that stochastic dynamic programming was the most appropriate approach for handling dynamic portfolio optimization issues. However, this strategy

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was frequently hampered by a number of problems, Addressing the dimensionality problem when there were too many state variables [1]. This paper introduces and compares several portfolio management models, exploring their respective advantages, disadvantages, and relationships. This study mainly uses literature review and comparative analysis methods to examine portfolio management models, including Markowitz Mean-Variance Model, Capital Market Line (CML), Capital Asset Pricing Model (CAPM), and Arbitrage Pricing Theory (APT). It explores model origins, assumptions, components, and applications, while also investigating model relationships and application distinctions. Different portfolio management models offer varying perspectives on the relationship between risk and return. By comparing these models, Investors may learn more about their advantages and disadvantages, allowing them to choose the model that best suits their portfolio management needs. Additionally, comparative analysis helps reveal connections and differences between these models, fostering the development of new research and models.

2. Model Explanation

2.1. Markowitz Mean-Variance Model

In 1952, an article titled 'Portfolio Selection' was authored by Harry Markowitz and published. Within this paper, Markowitz introduced a risk metric, specifically the concept of standard deviation, and formulated an equation for computing this metric concerning a collection of assets. He provided examples of the value of selecting assets with various correlation coefficients, which successfully introduced diversity as a way to lower the portfolio's total risk [2].

The foundation of the Markowitz model rests on a set of assumptions pertaining to investment patterns, as outlined by Reilly and Brown. These assumptions encompass the following points:

a. A distribution of probabilities representing the expected returns over a certain holding period can be used to illustrate different investment possibilities.

b. Investors aim to maximize their utility within a single period.

c. Portfolio risk is assessed based on the degree of expected return fluctuations.

d. Investment decisions are solely guided by considerations of anticipated return and associated risk.

e. Investors exhibit risk aversion, thus when faced with two equally risky investments, they will opt for the one offering a higher return.

Mathematically, The following is an expression for the equation:

$$E(Rp) = \sum_{i=1}^{n} w_i \cdot E(R_i) \tag{1}$$

Where n is the number of assets in the portfolio, w_i denotes the weight of asset i within the portfolio, and $E(R_i)$ is the anticipated return of asset i. (Rp) is the expected return of the portfolio.

Additionally, the formula for the portfolio's variance (or risk) is:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \cdot w_j \cdot \sigma_{ij} \tag{2}$$

Where σ_{ij} is the variance of the portfolio, w_i and w_j are the weight of assets i and j in the portfolio, and σ_{ij} is the covariance between the returns of assets i and j.

The goal of the Markowitz model is to identify the set of asset weights w_i that optimizes expected return at a given level of risk or minimizes risk at a given level of expected return. This involves considering the cost-benefit analysis of risk and return, as well as the diversification benefits of combining assets with different correlation coefficients. The core of this approach is the idea of the effective frontier, which denotes the collection of portfolios with the maximum projected

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return for a certain degree of risk. A risk-free asset has zero variance, and there is no link between asset with zero risk and any other risky asset. As a result, the risk of a portfolio that contains a risk-free asset is reduced and begins to have a linear relationship with the portfolio's standard deviation that exclusively contains risky assets.

$$\sigma = \sqrt{\left(1-w_{rf}\right)^{2}\sigma_i(1-w_{rf})\sigma_i} \tag{3}$$

Where w_{rf} is the weighting of the risk-free asset.

2.2. Capital Market Line

A natural progression from Markowitz's concept is potential for the investor to allocate a portion of his budget to risk-free assets or to borrow in order to achieve a specific level of leverage. Tobin was the first to propose this expansion of the Markowitz model, followed by Sharpe and Lintner. In a well-functioning market, the dynamics in terms of supply and demand alone would lead to every investor having a portfolio composition identical to the portfolio of markets. Consequently, the optimal portfolio in this equilibrium state is the market portfolio itself. This equilibrium situation is represented by a line known as the Capital Market Line (CML)[3].

The Capital Market Line (CML) is not represented by a single equation but rather a graphical concept within the framework of the Capital Asset Pricing Model(CAPM). It represents efficient portfolios that combine the risk-free asset with the market portfolio, and it is a straight line that connects the risk-free rate of return to the projected return of the market portfolio.

The equation for the CML can be summarized as follows:

$$E\left(R_p\right) = R_f + \frac{E(R_m)-R_f}{\sigma_m}\cdot\sigma_p \tag{4}$$

$E\left(R_p\right)$ stands for anticipated return of a portfolio, R_f for risk-free rate, $E(R_m)$ for expected return of a market portfolio, σ_m for standardized market portfolio risk, and σ_p for standardized portfolio risk. For effective portfolios, the risk-return trade-off is visually represented by the CML. It demonstrates how expected returns change as you adjust the portfolio's risk by varying the allocation between the risk-free asset and the portfolio of markets .

2.3. Capital Asset Pricing Model

The Markowitz model forms the foundation of the CAPM, which is then extended by Sharpe and Lintner and strengthened by the addition of crucial assumptions. CAPM assumes that all investors hold a combination of risk-free assets and a single, perfectly diversified market portfolio. The market portfolio is a key concept in CAPM and represents all risky assets available in the market. This assumption simplifies the analysis by reducing the number of portfolios to consider. CAPM distinguishes between market-related risk and asset-specific risk. It makes the erroneous assumption that systemic risk, which is what drives asset values, cannot be completely avoided by diversification. Diversification can help to avoid irrational risk. The CAPM introduces the idea of beta, which stands for how sensitive an asset's returns are to market fluctuations. The asset's vulnerability to systematic risk is evaluated using beta. When the asset's beta is 1, a security follows the market exactly; when it is lower than 1, it is less sensitive [4].

The following equation serves as the foundation for the Capital Asset Pricing Model (CAPM):

$$E(R_i) = R_f + \beta_i\cdot(E(R_m) - R_f) \tag{5}$$

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$E(R_i)$ stands for anticipated return on asset i, R_f for risk-free rate of return, β_i for asset i's beta coefficient, which denotes how sensitive the asset is to market fluctuations, and $E(R_m)$ for expected return on the market portfolio.

Based on an asset's beta, the risk-free rate, and the anticipated return of the entire market portfolio, the CAPM calculates the expected return of each individual asset in this equation. The beta coefficient calculates how vulnerable an asset is to market fluctuations or systematic risk. Given the risk-free rate and the anticipated return of the entire market, the CAPM offers a framework for calculating the expected return needed by investors to justify taking on the additional risk associated with a particular asset.

In summary, the CAPM is an extension of the Markowitz Model, incorporating additional assumptions such as a risk-free rate, a single market portfolio, and the use of beta to quantify systematic risk. These assumptions help simplify portfolio analysis and make it more applicable to real-world financial markets.

2.4. Arbitrage Pricing Theory

The most widely used model for recent-year portfolio management has been the arbitrage pricing theory (APT), which was developed by Stephen Ross in 1976. Theoretically, using a few risk criteria, the APT model may efficiently price risky assets in a portfolio. It selects stocks from a pool of probable candidates after detecting three to five risk characteristics and compares the securities' relative market risks and returns. Heuristics are usually required when using APT alone to get over bottlenecks in getting a decent list of companies [5].

The APT assumes that the $n * 1$ vector of asset returns, Rt, is created by a linear stochastic process with k components.

$$R_t = \bar{R} + Af_t + e_t \tag{6}$$

where ft is a $k * 1$ vector of k common factor realizations, A is a n k matrix of factor weights or loadings, and et is a n 1 vector of asset-specific hazards. Because ft and et are considered to have zero expected values, R is the $n * 1$ vector of mean returns. The model indicates that an asset's anticipated return is directly proportional to the factor loadings or volatility in a market with no arbitrage opportunities.

$$\bar{R} = R_f + A_P \tag{7}$$

Where Rf is an $n * 1$ vector of constants representing the risk-free return, and p is $k * 1$ vector of risk premiums.

The APT theory's central premise is that returns from a sizable—and ultimately infinite—Nondivergence, systematic risk, which may be measured as exposure to a few key common components, and idiosyncratic risk, which can be totally reduced in big, confirmed portfolios, can be separated into two categories. This premise, along with the assumption that investors prefer more value to less value, results in an appropriate theory of anticipated returns by eliminating riskless arbitrage opportunities. Regrettably, the surface-level straightforwardness of the APT masks significant challenges linked to its execution. Specifically, putting the theory to the test requires a method for gauging the shared elements. Many researchers have resorted to utilizing factor analysis to indirectly assess these shared elements. This strategy replaces the issue of upfront factor identification due to the computational challenges involved in doing maximum likelihood factor analysis on large data sets through standard software packages [6].

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3. Comparison and Analysis of Each Model

The Markowitz Model, also known as Modern Portfolio Theory, introduces the vital concept of diversification to manage risk effectively. It offers a quantitative framework for portfolio optimization, considering both risk and return concurrently. Despite its strengths, this model assumes a typical range of returns, which might not always be accurate, and relies on precise estimations of expected returns, variances, and covariances.

The Capital Market Line (CML) visually illustrates the risk-return trade-off for effective portfolios, involving both the risk-free rate and the market portfolio. It conveys the notion of systematic risk and diversification effectively. Nevertheless, the CML assumes a linear relationship between risky and risk-free assets, doesn't account for non-market-related factors impacting returns, and necessitates accurate estimation of portfolio parameters for reliable implementation.

The Capital Asset Pricing Model (CAPM) presents a straightforward a connection between risk and anticipated return and provides a clear benchmark through the market portfolio. Its simplicity contributes to its widespread usage. However, the CAPM assumes a linear risk-return relationship, which may not hold true in all scenarios. Estimating the market risk premium and risk-free rate is necessary, and it overlooks other potential factors impacting returns.

The Arbitrage Pricing Theory (APT) offers a unique advantage in not relying on the market portfolio as a benchmark and accommodating multiple factors that influence asset returns. Unlike the CAPM, APT doesn't impose a specific utility function on investors. However, APT's implementation requires accurate estimation of factor sensitivities, which can be challenging, and identifying and measuring these factors might pose difficulties (see Table 1).

Table 1: Comparison of each model

Model	**Advantage**	**Disadvantage**
Markowitz model	Diversification	Assumptions reliance
CML	Visualization	Linearity
CAPM	Clearness	Oversimplification
APT	Flexibility	Complexity

Ultimately, the selection of a model hinges on the specific context, data availability, and the objectives of investors or analysts. Moreover, the financial field has evolved to encompass newer models and approaches, addressing some of the limitations inherent in these classical models.

4. Conclusion

This paper explains different models of portfolio management and compares the advantages and disadvantages of them. Overall, these models involve a progression of development and refinement, with some models building upon the concepts of others. CAPM is an extension of the Markowitz Model. It builds upon the mean-variance framework by incorporating the risk-free rate and the market portfolio as benchmarks. CAPM offers a simplified relationship between risk and return and introduces the idea of systematic risk. The CML is not a standalone model, but a graphical representation derived from CAPM. It illustrates the efficient portfolios that combine the risk-free asset with the market portfolio. The CML illustrates the trade-off between risk and return for efficient portfolios. APT is a later model that can be considered an extension of CAPM. It broadens the approach by accommodating multiple risk factors instead of relying solely on the market portfolio. APT’s development aimed to address some of the limitations of CAPM, particularly by allowing for a more flexible set of risk factors. Future research can explore the possibility of

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merging individual models to apply to more complex scenarios, making them more adaptable to a wider range of market environments.

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