

Allowance-based Carbon Options Pricing Strategy and Asset Management Research Progress

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Abstract: Under the issue of global warming and environmental protection, low-carbon development has become an unshakable responsibility for governments worldwide. In China, the national carbon emission trading market established in 2021, which promotes the economic development of carbon derivatives. In order to facilitate the trading system, carbon options pricing theory has become a centerpiece of academic attention. This paper reviews the study progress in detail in the area of allowance-based carbon options pricing strategy carbon internationally, and makes an analysis of the application and management in international carbon markets. The mainstream pricing models include the Black-Scholes and the Monte Carlo option pricing models. On this basis, the carbon option prices estimated using the GARCH model and fractal Brownian motion have a higher degree of fitted value. In addition, this paper analyzes the application and management of carbon options in the carbon market. The carbon option helps reduce the carbon trading risk and the spendings on emission reduction for enterprises. This research intends to give a reference for the subsequent theoretical exploration of carbon options and promote the growth of the derivatives market.

Keywords: emission derivatives, carbon option pricing, carbon asset management, emissions permit market, asymmetric information

1. Introduction

Carbon emission allowance serves as the core asset in the permit market, which shifts government constraints on enterprise emissions in the form of its prices. Faced with the uncertainty of the permit price, carbon derivatives are expected to provide a feasible solution for these enterprises to better hedge risk under a cap-and-trade system. Carbon options, as one category of carbon derivative, are the standardized contracts formulated by futures trading venues, which stipulate that buyers have the right to buy or sell carbon allowance or offset credits (including carbon futures contracts) at a certain time and price in the future.

Currently, much financial research on carbon options has demonstrated their necessity for the carbon market. They explored the formulas of option pricing models from different theories and conducted empirical analysis to provide higher precision. However, as the beginning period of the permit market is prone to asymmetry, options can influence their underlying assets in multiple ways. There is still a lack of research on the comparison of various carbon option pricing models and the methods of investment decision-making.

This essay reviews the main research and viewpoints in the field of carbon options, and studies the market application of pricing strategies and asset management in regulated enterprises. This research aims to help to give some references for subsequent theoretical exploration and promote the carbon market. The remaining contents are organized as follows: in Section 2, carbon emission allowance and the existing permit market are introduced. Section 3 focuses on the fundamental theories of carbon options pricing strategy. Section 4 presents the management strategies for carbon options in the market. Section 5 concludes.

2. Carbon Emission Allowance and Existing Permit Markets

The economic theory of allowance trading can date back to the pioneering work by Coase [1], and Dales [2], which introduced the concept of the cap-and-trade system originally. His paper proposed the basic idea of a market model with tradeable permits, which can be used to gain emission targets with the minimal social costs. The first application was conducted by the U.S. Environmental Protection Agency (EPA) for sources of atmospheric pollution (such as SO₂ emissions) and river pollution. In 1997, more than a hundred countries in the world signed the Kyoto Protocol at the United Nations General Assembly (UNGA).

Montgomery [3] provided theoretical support for the marketization of pollution abatement. By tackling with the problem of optimal control, he obtained the efficient allocation of abatement costs under market equilibrium. Subsequent studies have further improved the theory. Tietenberg [4] and Cronshaw and Kruse [5] incorporated banking and borrowing. Rubin [6] further applied the results to a continuously temporal situation for dynamic allowance trading. Schennach [7] and Newell, Pizer and Zhang [8] predicted the optimization problem concerning uncertainty through an extended model. In January 2005, the European Union implemented the Emissions Trading System (EU ETS), and the cap-and-trade system enabled them to achieve more emission reduction through lower abatement costs.

Since then, more empirical studies have tested the gap between theoretical and market prices by exploring the historical time sequences of the permit price. Cetin and Verschuere [9] used a filtering method and hidden Markov models to derive the spot prices of carbon allowances. Paolella and Taschini [10] advocated a novel GARCH structure to analyze emission permits in EU ETS. Benz and Trück [11] analyzed the short-run spot price practice of CO₂ permits employing AR–GARCH models and Markov switching. They all provided a numerical simulation to demonstrate the effectiveness. After that, the influencing factors of the fluctuation of permit prices have attracted research attention.

Environmental finance became a new field of study, and emission allowances promoted more trading of derivative financial instruments. The year 2021 witnessed the official establishment of China's national carbon market. Judging from the data released by the International Carbon Action Partnership (ICAP), in 2023, the amount of global carbon markets, which are in operation, has risen to 28, and the scale of emissions under control has accounted for 17% of global greenhouse gas emissions. The price evolution in six ETSs is shown below from 2018 to 2023. (See Figure 1).

There was a steep increase in the carbon price of EU ETS, which surged from less than 40 USD/tCO_{2e} in 2020 to approximately 90 USD/tCO_{2e} in 2023. Meanwhile, the carbon prices of New Zealand ETS also grew, but much less significantly from 20 USD/tCO_{2e} in 2020 to around 45 USD/tCO_{2e} in 2023. The carbon prices in California ETSs and Regional Greenhouse Gas Initiative (RGGI) rose moderately to 30 USD/tCO_{2e} and 10 USD/tCO_{2e} respectively. However, the Republic of Korea ETS's carbon price experienced huge fluctuations and declined to approximately 10 USD/tCO_{2e} in 2023. The latest started China National ETS has the least but steadiest carbon price, which remains around 9 USD/tCO_{2e} in 2023.

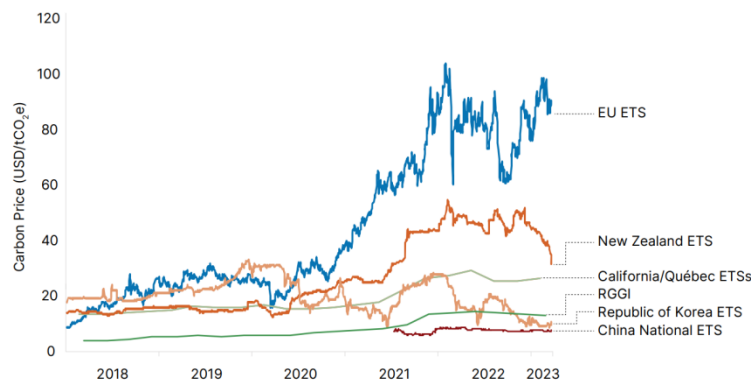


Figure 1: Price change in selected ETSs (2018-2023) [12].

Data source: State and Trends of the Carbon Market 2023 World Bank

(Note: The selected data comes from ICAP Allowance Price Explorer. The primary market provides prices for California and Québec CaT and for the RGGI initiative, whereas the secondary market provides for the prices of the other systems.)

3. Fundamental Theories of Carbon Options Pricing Strategy

Based on the rigorous research on CO₂ emission allowances, the discussion on the allowance-based carbon options was further developed. According to Carmona and Hinz [13], although the European options concerning allowance futures had been introduced and exchanged, no theoretical foundation for their pricing has been available yet. However, large numbers of theoretical studies have been conducted relating to carbon options as the cap-and-trade system has become widespread around the world for the past few years. Consequently, updating the overview of the theoretical research progress on carbon options becomes necessary.

3.1. The Necessity of Carbon Option

The research on carbon options began with the understanding of its necessity. Li et al. [14] encouraged enterprises of power generation to join in the carbon market for the existence of an evolutionary equilibrium strategy to improve environmental quality in this process. Carbon options are not only beneficial to the environment, but also helpful for businesses to hedge risks. Chevallier, Le Pen and Sévi [15] promoted that introducing options may ameliorate the emission permits' price volatility. Li et al. [16] pointed out that the existence of carbon options provides emission reduction companies with a new mechanism to avoid the uncertain spot prices in the future carbon emission trading market.

3.2. Black-Scholes Option Pricing Model

Afterwards, the research on carbon option pricing mostly adopts the Black-Scholes Model's closed-form formula proposed by Black and Scholes [17]. Further research continuously deduced the theoretical model to improve the accuracy of pricing. Chesney and Taschini [18], Daskalakis, Psychoyios and Markellos [19] and Kijima, Maeda and Nishide [20] studied the pricing issues about options on emission allowances. Wang and Wang [21], Wu and Qu [22] and He [23] proposed to discuss the pricing of carbon options in accordance with this option pricing method.

The Black-Scholes option pricing model is regarded as one of the most important research results in the financial field. This model assumes that all participants can participate in the financial market without cost. Venture capital follows a geometric Brownian process.

Due to the following properties, their pricing applies to the Black-Scholes option pricing formula: The carbon option does not pay dividends. The carbon option allows the holder to execute this option at any time. The market of carbon trading is frictionless. The carbon market's price volatility is

relatively stable. The carbon option is a public resource with clear property, which can prevent arbitrage opportunities in trading.

Therefore, the Black-Scholes model uses ITO's lemma and hedging principle to obtain the following carbon option pricing formula under the above conditions:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (1)$$

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1) \quad (2)$$

where $d_1 = \frac{\ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$, $d_2 = \frac{\ln(\frac{S_0}{K}) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$, r = risk-free rate, σ^2 = volatility of return rate, c = theoretical price of European call options, p = theoretical price of a European put option, S_0 = current price, K = strike price, T = time to maturity, N = a normal distribution.

Since there was no mature national carbon market then, the initial trading market was very unstable. The transaction price was affected by many factors, and it was hard to obtain the carbon emission allowance's trading data to calculate the carbon option's value. However, these papers used such pricing model to study the initial allocation of carbon emission, which provided new ideas and directions for relevant theoretical research.

3.3. Monte Carlo Option Pricing Model

However, researchers studying carbon option pricing realized the fixed parameters of the geometric Brownian model for the underlying asset, which cannot show the external changes. More precise calculations were produced in order to obtain precise spot market prices. Zhao [24] compared the expected results of the traditional Black-Scholes method with the Monte Carlo option price simulation and obtained a more applicable result of the latter. In her study, the EUAs put option prices in the ECX trading market expiration was used as the comparison standard. She found that there was a big difference in accuracy between the actual price and predicted price, and the return rate of carbon financial products failed to conform to the normal distribution (see Figure 2). Therefore, it can be seen that the Black-Scholes model does not suit the pricing of carbon financial derivatives.

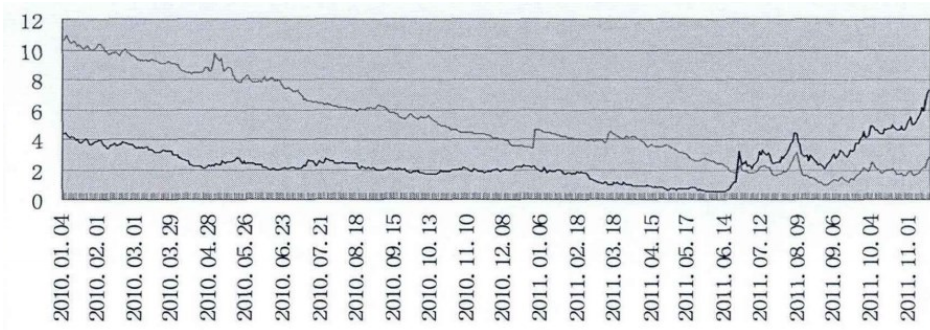


Figure 2: B-S option pricing fitting-figure by Zhao [24].

The Monte Carlo option pricing model is a data simulation method, which simulates the change path of the underlying asset value. It uses random number updates to determine the step asset price, and generates a large number of simulated estimates to obtain the expected value of the real price. Euler's Method was used to select an update path.

In the next period, the price of the financial asset will be:

$$S(t + \delta t) = S(t) + \delta S \quad (3)$$

$$\delta S = rS\delta t + \sigma S\sqrt{\delta t}\Phi \quad (4)$$

Therefore,

$$S(t + \delta t) = S(1 + r\delta t + \sigma\sqrt{\delta t}\Phi) \quad (5)$$

$$C = E(\max(0, E - S)) \quad (6)$$

where δt = interval step size, Φ = random number.

Zhao used the Crystall Ball 2000 to fit the same sample data. The fitting of Monte Carlo simulation price and real price of put options was obtained (see Figure 3). The fluctuation trend and price change were highly consistent. Therefore, she noted that the Monte Carlo method was more advantageous than the Black-Scholes model.

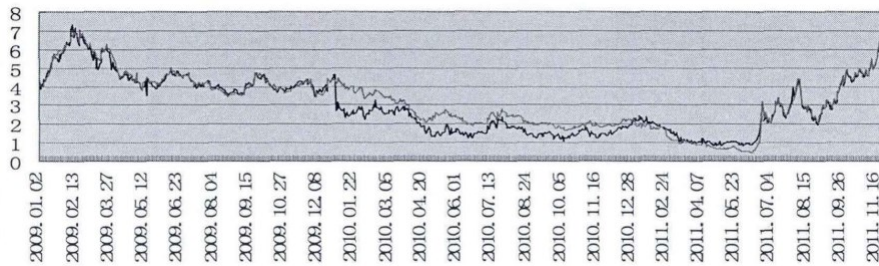


Figure 3: Monte Carlo pricing fitting-figure by Zhao [24].

3.4. GARCH Model with the B-S Option Pricing Model

Zhao Shanshan's seminal paper made the latter research aware of the shortcomings of the Black-Scholes formula in the carbon market and made efforts for more reasonable carbon options pricing. After the controversial discussion of the distribution features of prices of carbon option and the prediction method of the volatility in the return rate, Liu and Huang [25] confirmed the role of the GARCH model in compensating for the volatile traditional B-S option pricing models.

The GARCH model (Generalized AutoRegressive Conditional Heteroskedasticity Model) combined with the B-S model can make up for the restriction of the fixed volatile traditional model. GARCH Model improved the defect that the autocorrelation coefficient of the ARCH model was too slow to be described, and established the correlation of the variance between time t and the previous q period. The conditional variance at time t can be expressed as:

$$\sigma_t^2 = \beta_0 + \beta_1\mu_{t-1}^2 + \beta_2\mu_{t-2}^2 + \cdots + \beta_q\mu_{t-q}^2 + X_1\sigma_{t-1}^2 + \cdots + X_p\sigma_{t-p}^2 \quad (7)$$

where $\beta_0 + \beta_1\mu_{t-1}^2 + \beta_2\mu_{t-2}^2 + \cdots + \beta_q\mu_{t-q}^2$ is the ARCH term, $X_1\sigma_{t-1}^2 + \cdots + X_p\sigma_{t-p}^2$ is the GARCH term, and β_0 is the constant term. The constraint conditions are:

$$\beta_0, \beta_1, \dots, \beta_q > 0, X_0, X_1, \dots, X_p > 0, \sum_{i=1}^q \beta_i + \sum_{j=1}^p X_j < 1 \quad (8)$$

Using the result of the GARCH model, we can predict volatility in the return rate. Compared with other models, the GARCH model is more accurate, easier to calculate, and less different from the real value. The GARCH (1,1) model can be used to detect the volatility of earnings. The existing research

shows that the correlation and volatility of financial data of time series may be predicted via the GARCH model with better results.

3.5. GARCH Model with the Fractal Brownian Motion

Liu and Huang [26] further confirmed that using fractal Brownian motion to predict option prices is more reasonable compared to geometric Brownian motion. This is a method to price options of carbon assets through fractal Brownian motion. In the actual operation process, the logarithmic return rate of carbon assets is larger, which does not apply to geometric Brownian motion, but to fractal Brownian motion. Therefore, the fractal Brownian motion obtains the following formula of carbon option pricing:

$$c = S_t N(d_1) - K e^{-r(T-t)} N(d_2) \quad (9)$$

where $d_1 = \frac{\ln(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})(T^{2H} - t^{2H})}{\sigma \sqrt{(T^{2H} - t^{2H})}}$, $d_2 = d_1 - \sigma \sqrt{(T^{2H} - t^{2H})}$, r = risk-free rate, σ^2 = volatility of return rate, c = theoretical price of European call options, S_t = current price at t ($0 \leq t \leq T$), K = strike price, T = time to maturity, N = cumulative standard normal distribution. Hurst index can be obtained using rescaled range analysis.

To measure the model's effect, Huang used mean square error (MSE), mean absolute error (MAE) and mean absolute percentage error (MAPE). The result was as follows (see Table 1):

Table 1: Error comparison of four models' pricing results by Huang [26].

Evaluation Index	Dec-20 EUA			Dec-21 EUA		
	MAPE(%)	MAE	MSE(%)	MAPE(%)	MAE	MSE(%)
B-S model (Historical Volatility)	7.38	2.040	0.82	7.32	2.059	0.81
B-S Model (GARCH)	7.46	2.039	0.8	7.48	2.068	0.79
Fractal Brownian motion (Historical volatility)	17.45	4.305	3.05	13.89	3.415	1.93
Fractal Brownian motion (GARCH)	1.03	0.275	0.19	1.08	0.291	0.2

It can be found that the three indexes get consistent evaluation results. The carbon option pricing model based on fractal Brownian motion and GARCH bears the lowest prediction error, and can be adopted to carbon option pricing with various maturities.

3.6. Regime-switching and LSM Model

Liu et al. [27] designed a European carbon emission option pricing model applying a geometric Brownian motion model together with regime-switching. They believe that the stochastic process of simulating prices should take into account changes in external factors such as temperature, economy and even carbon regulation policies, so the price dynamics of carbon emission rights should have switchable parameters reflecting changes in these aspects.

In 2001, Longstaff and Schwartz made their choice by using the least square method to determine whether each node in the pricing process should be held or immediately exercised, and named the method the least square Monte Carlo Simulation Method (LSM). This approach not only inherits the advantages of the Monte Carlo simulation method in terms of efficiency, but also improves the

problem of optimal choice of option value for each node. Zeng [28] pointed out that the use of least squares Monte Carlo simulation (LSM model) combined with the GARCH model exerts a good pricing impact on carbon emission quota options, and the model fitting effect can reach over 90% (see figure 4).

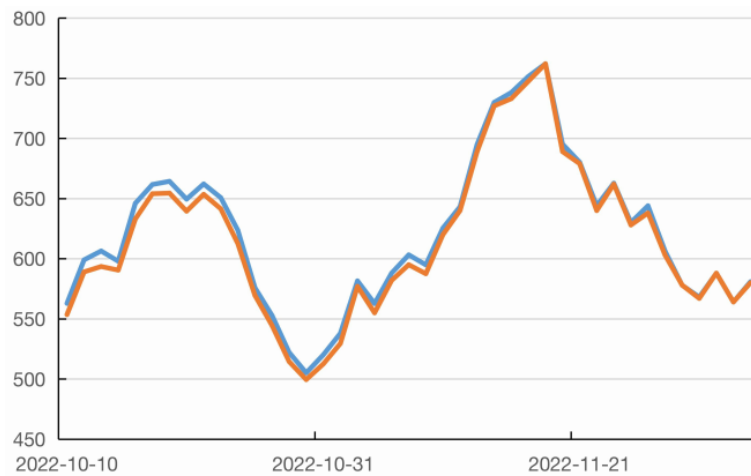


Figure 4: SR301C5000 option contract LSM simulated price and actual price comparison [28].

(Note: Blue line: Simulated prices; Red line: Actual prices; Unit: CNY.)

The M carbon emission rights' price path is simulated on the basis of the current price, and the option value at the initial time point of each path is obtained by turning back one-time point to the initial time point, and the average value is taken to get the simulated price. The white sugar option is selected to verify the pricing effect of the model. In the simulation process of option pricing, the variance between the actual price and the simulated one of an option becomes not significant, which has high accuracy and reliability.

4. Management Strategies in Carbon Options in the Market

4.1. Policies of Carbon Options in Different Countries

The building of the global carbon emission trading market is accelerating. Since 2011, China has launched a carbon emission trade exchange in seven places, including Tianjin, Beijing, Chongqing and Shanghai. In 2021, the initial compliance period of the Chinese carbon market in the energy sector was officially launched. Since the national carbon market was operated for one year, the cumulative transaction volume of carbon emission allowances (CEA) has been 0.194 billion tons, with an 8.492 billion RMB cumulative transaction value, and the demand for carbon trading and finance products has become increasingly strong. In addition to the steady development of traditional instruments such as carbon spot trading and carbon assets pledge financing, the carbon market is also actively exploring financial derivatives based on carbon emission allowances. The local governments recently proposed their support for the innovation and exploration of carbon financial derivatives. In 2017, an environmental and energy company entered into an over-the-counter option transaction for carbon emission allowances with a foreign energy company. Although the exchange has assumed the function of regulating the options contract, such trading is still not standardized OTC trading.

The EU Emissions Trading System, which began operation in January 2005, covers about 45% of their greenhouse gas emissions, with the European Union Allowances set according to the 'cap-and-trade' principle. According to publicly disclosed data, the transaction volume of the EU emissions trading system in 2022 is approximately 751 billion euros, with an increase of 10% compared to 2021.

The carbon option market's development also makes information transmission more effective and improves the completeness of the carbon financial market. Alongside the operation of the European carbon market, carbon derivatives' development has also encountered some disputes. The most typical concern is whether there is excessive speculative trading. However, EU regulators still believe that the carbon market is operating smoothly and that too many restrictions will affect the market function's effectiveness. Generally, the European carbon financial derivatives market is priced by attracting different market players to participate in the carbon market. Investors with different trading purposes fully participate and generate clear prices, which successfully helps enterprises better manage their carbon risk.

4.2. Carbon Options and Corporate Profits

The beneficial role of carbon options in carbon asset management and investment decision-making has been demonstrated. Zeng [29] pointed out that carbon options can achieve Pareto improvement in the supply chain and improve social welfare. Compared to other financial products, carbon options may be a product that brings profits to multiple participants. Yu et al. [30] built a carbon option pricing model under the maximization of the expected utility of Choquet, reflecting the changes in investors' attitudes toward carbon option investment risks.

In addition to the social benefits, carbon options also contribute to stabilizing the price of carbon permits. Ding et al. [31] proposed with high demand for carbon quotas, remanufacturers are more willing to purchase carbon options to cope with risks. Carbon financial instrument investment portfolio contracts enable companies to achieve better returns. Liu et al. [32] proposed applying carbon options to digital decision-making in carbon management to optimize controls over risk for all companies with carbon consumption. (See Figure 5). Ma and Chen [33] demonstrated the positive part of carbon option contracts playing in increasing corporate profits and addressing demand uncertainty.

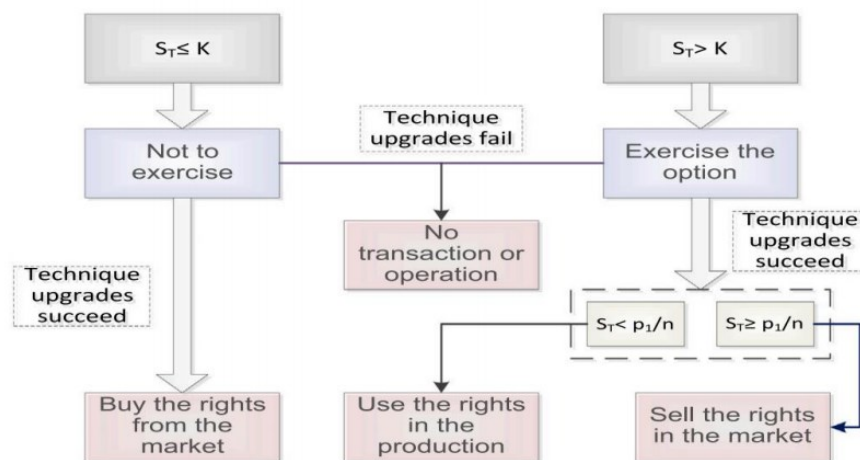


Figure 5: Carbon asset operation guidance and decisions by Liu [32].

For enterprises, holding call carbon options can promote the degree of confidence in carbon trading between enterprises. Companies that buy call options exercise them when the carbon price exceeds the exercise price. Whether to purchase or sell carbon allowance relies on the technological upgrading result, which also provides methods for environmental protection.

4.3. Carbon Options and Pollution Abatement

As the International Swaps and Derivatives Association (ISDA) points out, carbon derivatives are indispensable in the following ways:

(1) Emission controlled enterprises can use carbon financial derivatives to better fulfill their implementation, adjust the cost of emission control work and manage their risks.

(2) Investors can apply the price signals giving by carbon financial derivatives to measure risks of climate transition in their portfolios and benefit from energy transition opportunities.

(3) Carbon financial derivatives markets contribute to greater transparency in carbon markets, and provide policymakers with regulatory signals on carbon prices, which can promote to achieve the long-term sustainable development goals globally.

Carbon options have enriched carbon trading products and facilitated the carbon trading market. It helps buyers avoid adverse risks caused by carbon price fluctuations and has the function of hedging. In a well-established carbon market, carbon options are applied to allocate market resources reasonably and provide participants with more choices. Ultimately, carbon options will help increase companies' willingness to reduce emissions and enhance the effectiveness of pollution control.

5. Conclusion

This paper reviews the research progress of allowance-based carbon option pricing strategies, and finds that the mainstream pricing models include the Black-Scholes and the Monte Carlo option pricing models. On this basis, the carbon option prices estimated using the GARCH model and fractal Brownian motion have a higher degree of fitted value. More optimization methods, including Regime-switching and LSM model, are being further explored. In addition, this paper analyzes the application and management of carbon options in the carbon market. The carbon option helps reduce carbon trading risk and enterprises' emission reduction costs. This paper finds that although carbon options held by enterprises are worth investment and insurance, there are no widely circulated derivatives in the carbon market at present. The future markets need to strengthen supervision and cooperation among various sectors to provide a reliable environment for the development of carbon options. This research can give some references for the subsequent theoretical exploration of carbon options and promote the development of the derivatives market.

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