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Empirical Research on Pension Investment Optimization Based on Three-factor Model

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Abstract: With the arrival of the aging era, retired elderly people are paying more and more attention to property planning. The elderly hope to improve the quality of life after retirement through pension investment planning. This article takes a firefighter who has worked in Florida for 25 years as an example. He selects a total of 10 stocks including ETD, CVGW, AAPL, ASML, UNH, LVMH, NNDM, SBUX, TM, and BUD, and calculates their respective weights based on Excel. And the Sharpe Ratio, which is optimal when investing with a pension or without a pension, uses the three-factor model to conduct empirical research on the two situations, and summarizes the advantages and disadvantages of the two Sharpe Ratios in terms of cycles and returns, and we get When optimizing pension investment, we will give the best investment advice according to different investor situations, so as to maximize the income of the elderly after retirement.

Keywords: Fame-French three-factor model, pension, portfolio

1. Introduction

Population aging is becoming one of the distinctive features of the current global economy. According to the "World Social Report 2023" released this year by the United Nations Department of Economic and Social Affairs, the global population aged 65 and over will be 761 million in 2021, and this number will increase to 1.6 billion by 2050.

According to the United Nations' definition of an aging society, "if the population over 60 years old accounts for more than 10% of the country's total population, the country is deemed to have entered an aging society." Taking China as an example, in the 1970s the government The family planning policy was enacted to limit the number of newborns, which led to China entering an aging society in 1999. In 2015, in order to cope with and slow down the impact of population aging, China implemented a two-child policy to stimulate an increase in the birth rate. Although the introduction of this policy has effectively slowed down the rate of population aging, the aging situation in China is still not optimistic. At present, at least 49 countries in the world have entered the aging process, and the number will continue to increase.

In the increasingly severe situation of global aging, more and more elderly people are beginning to pay attention to their pension issues. They hope to maximize their income and enjoy a happy and beautiful retirement life through reasonable planning and utilization of their property.

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The core content of personal financial management is to achieve the customer's needs and goals based on the customer's asset status and risk preference. The fundamental purpose of personal financial management is to achieve the economic goals among life goals [1].

Professional financial investment institutions and their managers are undoubtedly a good choice for these elderly people. They usually graduate from finance or actuarial majors in the world's top universities, have a solid foundation in mathematics and finance, and have keen investment talents. Financial investment managers use their abilities to help seniors invest in their property and find the best solution. This article will use the three-factor model and excel data processing tools, based on relevant data over the years, taking a firefighter in Florida, USA in 2017 as an example to analyze and explore, and draw research conclusions.

2. Literature Review

The capital asset pricing model (CAPM model) based on modern portfolio theory (MPT) is widely used by many researchers as a classic asset pricing model. The CAPM model is an asset pricing theory and model created and developed by American scholars William Sharpe, John Lintner, Jack Treynor and Jan Mossin in 1964. At first, it was mainly used to study the relationship between the expected return rate of assets and risky assets in the securities market, and how the equilibrium price is formed. This theory provides very good ideas and methods for studying the modern financial price market [2]. Therefore, CAPM theory has long been widely used in various aspects of academic finance and practice. Since the CAPM model was proposed, it has been widely studied by scholars. For example, in 2001, Chinese scholar Wu Lina theoretically discussed the feasibility of applying the CAMP model to real estate [3]. In 2017, Wang Nan added a unique risk adjustment coefficient to the theoretical basis of CAPM and applied it to the value assessment of mortgage-type commercial real estate [4]. In 2019, Li Taiyao explored the effectiveness of CAPM in the securities industry, and the analysis results showed that CAPM has a significant ability to explain the stock returns of securities companies [5]. In 2020, MJ Alhabeeb examined the applicability and validity of the CAPM as a technical tool for measuring the expected return on a stock investment and assessing the market risk associated with that investment [6]. In 2022, Li Kaixuan studied the applicability of CAPM to the Chinese stock market, and the research results showed that it has weak applicability to the Chinese stock market [7]. In the same year, Vergara-Fernández Melissa and others used CAPM to describe the model relationship within the family [8]. In 2023, Sri muyaningsih and Jerry Heikal applied CAPM technology to bank stock investment decisions in digital bank operating models [9].

However, with the continuous development and improvement of the capital market, more and more scholars have tested the CAPM model and found that the model cannot explain financial "anomalies" such as the "small market capitalization effect" and the "book-to-market value ratio effect", so Fama Based on previous research and combined with their own findings, and French established the Fama-French three-factor model to explain stock excess returns in 1993 [10]. They found that the two factors of stock market value and book-to-market ratio can explain most of the stock price fluctuations, so they constructed three factors by simulating market risk, market value risk, and book-to-market ratio risk to explain changes in stock returns. After its establishment, the three-factor model has been widely used at home and abroad. For example, in 2002, Fan Longzhen and Yu Shidian found that the Chinese stock market can be explained by the three-factor model, and that the Chinese stock market has significant book-to-market ratio effects, scale effects, and price-to-earnings ratio effects [11]. In 2014, Zhang Hongliang and Zhao Yana used Shanghai and Shenzhen stock market data to test the effectiveness of each risk factor in the Fama-French three-factor model. The results showed that the size factor, book-to-market ratio factor, reciprocal price-earnings ratio factor and financial leverage factor can all be explained For the excess expected return of stocks, the explanatory power of the size factor is more significant than other risk factors [12]. In 2019, Intan

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Nurul Awwaliyah and Zaafri A. Husodo used U.S. monthly stock return data from January 1963 to December 2010 to test the effectiveness of the four-factor asset pricing versus the standard Fama-French three-factor model for investor understanding. Market conditions provide guidance [13]. In 2022, Wen Yiming and Yin Zongcheng applied the three-factor model to the study of excess stock returns of listed companies in China's pharmaceutical and biological industry. The results showed that the market premium factor, size factor and book-to-market ratio factor can well explain the excess stock returns of the sample. Yield [14]. In 2022, Xia Yu innovatively added the growth factor based on the Fama-French three-factor model to form the Fama-French four-factor model. Comparing the model fitting degree and significance with the three-factor model showed that adding the growth factor The formed Fama-French four-factor extended model has stronger explanatory power in China's securities market [15]. In addition, Li Yuanhong and Cheng Xianbao established an econometric regression model to test the capital asset pricing model (CAPM) and the Fama-French three-factor model's ability to explain and predict the phenomenon of excess returns in the stock market. The results showed that the three-factor model is better at explaining the phenomenon of excess returns. It is better than the CAPM model in terms of power and prediction power [16] [17].

3. Empirical Research

This section must be in one column.

3.1. Paper Title

This article selects a hypothetical case proposed by Cornell professor Justin Murfin as an analysis template. Client In retirement, he plans to fish and travel in the Florida Keys. He is not married and has no children. He rents a small two-bedroom house by the lake. He is in good health and expected to live a long life. In addition to the pension, this client also has a savings fund, and CDS of approximately US$600,000 will mature on the retirement date.

After retirement, the government promised to pay him $100,000 a year, plus 3% annual fee increases, until his last day. However, the state also offers pensioners a choice. A lump sum payment upon retirement in lieu of pension benefits. In his case, a lump sum payment, based on life expectancy and the pension plan's own discount rate, would be $ 145,000.

At this point, the client needs the help of an investment advisor to help him navigate two separate decisions: first, whether to take the money as a lump sum; second, how to invest his savings (either his savings or his savings as a lump sum, or simply as a supplement to his nest egg as a pension)

3.2. Model Selection

Regarding the selection of asset investment pricing methods, two investment models currently widely used in the financial community are the capital asset pricing model and the three-factor model. Both can be used to explain the relationship between asset returns and risk in capital markets. They both play an important role in theory and practice, but at the same time there are some significant differences.

3.2.1. CAPM Model

CAPM was proposed by William Sharpe, John Lintner and Jan Mossin in the 1960s. It is a single-factor model designed to explain the expected return of assets through market risk factors. The basic formula of CAPM is as follows:

$$E(R_i) = R_f + \beta_i \cdot (E(R_m) - R_f) \quad (1)$$

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Among them, $E(R_i)$represents $i$the expected return of the asset, $R_f$which is the risk-free interest rate, and $\beta_i$is $i$the beta coefficient of the asset, $E(R_m)$which is the expected return of the market portfolio.

3.2.2. Three-Factor Model

The three-factor model was proposed by Eugene Fama and Kenneth French in the 1990s and is an extension of the CAPM. The three-factor model considers three factors: market risk, market value factor (SMB), and book-to-market factor (HML). Its basic formula is as follows:

$$E(R_i) = R_f + \beta_i \cdot \left(E(R_m) - R_f\right) + s_i \cdot SMB + h_i \cdot HML \quad (2)$$

Among them, $s_i$is $i$the market value factor beta of the asset, $h_i$and is $i$the book-to-market ratio factor beta of the asset.

3.2.3. Reasons For Choosing the Three-Factor Model

The following are the main differences between the CAPM and the three-factor model and the reasons for choosing the three-factor model:

(1) Difference In Number Of Factor

CAPM only considers market risk factors, and it assumes that the market portfolio is the only source of risk. The three-factor model considers three factors, including market risk, market capitalization factor and book-to-market ratio factor, thus more comprehensively explaining the differences in asset returns.

(2) Difference In Explanation Ability

The CAPM has some limitations in explaining differences in asset returns in real markets, especially when it comes to explaining the performance of small-cap stocks and high book-to-market stocks. The three-factor model, by introducing additional factors, more accurately explains the performance differences of these stocks and improves the explanatory power of the model.

(3) Empirical Support Differences

A large body of research has shown that the three-factor model performs better at explaining asset returns in real market data, especially in long-term investing.

Research since 2018 has also continued to support the validity of the three-factor model. For example, the study "An Alternative Three-Factor Model" by Chen, Novy-Marx, and Zhang (2019) further verified the three-factor model.

(4) Practical Application Differences

The three-factor model has become a standard tool in finance and investment practice, and is used by many investors and fund managers to evaluate and optimize portfolios. The three-factor model is widely used in fields such as quantitative investment, risk management and asset allocation.

3.3. Data Selection

Over the long term, stocks provide higher investment returns than any other investment vehicle. Because of its low investment threshold, low transaction costs and high transaction convenience, it has become the most common investment tool for individuals and even families.

This article selects data from December 1, 2012 to December 1, 2017, including market risk, market capitalization factor and book-to-market ratio factor. Ten stocks were selected as research targets in Yahoo Fiancé, they are ETD, CVGW, AAPL, ASML, UNH, LVMH, NNDM, SBUX, TM, BUD. In the selection of stocks, different industries are deliberately involved, including automobiles, beverages, clothing, medicine, technology, food and household products, etc. In terms of market

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capitalization of stocks, they are also widely distributed, ranging from small market capitalization to large market capitalization.

In the study, we first collected the monthly returns of these ten stocks from December 1, 2012 to December 1, 2017, and subtracted the monthly risk-free return from this return to obtain the monthly risk-free return of each stock. Excess returns were provided in table 1. Then use the excess return of each stock and the three factors to conduct linear regression analysis to obtain the three-factor coefficients of each stock. For the four data of Mkt-RF, HML, SMB, and RF, their average values from 1926.7 to 2023.7 are used respectively. The expected return of each stock can be obtained by adding RF to the sum of the products of the three factors and each stock. Integrate the returns of ten stocks from December 1, 2012 to December 1, 2017 with the known Florida GO Bond, use VARP to calculate the variance based on the entire sample population, and create a VCV table, as shown in table 2.

Table 1: Expected returns of ten stocks.

Expected return											
FF3Finfo	Average	ETD	CVGW	AAPL	ASML	UNH	LVMH	NNDM	SBUX	TM	BUD
Mkt-RF	0.0066	1.1707	0.4127	1.2352	1.0860	0.6072	0.8733	1.2252	0.7308	0.6479	0.9928
SMB	0.0020	1.4619	1.0263	-0.6702	0.0258	0.4025	-0.2010	-0.7489	-0.1814	-0.0427	-0.1475
HML	0.0038	0.4242	-0.0730	-0.7785	0.2813	-0.1082	-0.4362	-0.8590	-0.2144	0.1115	-0.7583
RF	0.0028										
Expected return		0.0151	0.0073	0.0066	0.0111	0.0072	0.0065	0.0061	0.0064	0.0074	0.0061

Table 2: VCV table of ten stocks.

VCV table _										
	ETD Return	CVGW Return	AAPL Return	ASML Return	UNH Return	LVMH Return	NNDM Return	SBUX Return	TM Return	BUD Return
ETD Return	0.0085	0.0027	-0.0001	0.0004	0.0017	0.0007	-0.0020	0.0004	-0.0002	0.0005
CVGW Return	0.0027	0.0085	-0.0006	0.0016	0.0001	-0.0003	-0.0006	-0.0001	0.0001	0.0013
AAPL Return	-0.0001	-0.0006	0.0047	0.0006	0.0007	0.0018	0.0006	0.0008	0.0004	0.0016
ASML Return	0.0004	0.0016	0.0006	0.0051	0.0006	0.0012	-0.0002	0.0009	0.0015	0.0013
UNH Return	0.0017	0.0001	0.0007	0.0006	0.0020	0.0006	0.0007	0.0007	0.0002	0.0003
LVMH Return	0.0007	-0.0003	0.0018	0.0012	0.0006	0.0036	0.0007	0.0010	0.0006	0.0007
NNDM Return	-0.0020	-0.0006	0.0006	-0.0002	0.0007	0.0007	0.0609	-0.0007	-0.0006	0.0020
SBUX Return	0.0004	-0.0001	0.0008	0.0009	0.0007	0.0010	-0.0007	0.0020	0.0007	0.0006
TM Return	-0.0002	0.0001	0.0004	0.0015	0.0002	0.0006	-0.0006	0.0007	0.0021	0.0007
BUD Return	0.0005	0.0013	0.0016	0.0013	0.0003	0.0007	0.0020	0.0006	0.0007	0.0029

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3.4. Model Construction

According to the statement in the case, Florida's pension annualized rate of return is 7.75%, and through the compound interest calculation formula, the monthly rate of return is 0.624%. On December 1, 2017, the annual interest rate of government bonds was 2.47%, and the monthly interest rate was 0.2035%. The next step is to find the optimal asset allocation without a pension. The expected return of the optimal allocation is expressed as the weight of each stock multiplied by the expected return of each stock. To calculate the variance, first multiply the weight of each stock by the two matrices of the previous VCV table, and then multiply the result by the two matrices of the weight of each stock. The standard deviation is the arithmetic square root of the variance. When seeking the optimal solution, the factor this paper considered was Sharpe Ratio, which is expressed as the difference between the expected return of the optimal allocation minus the monthly interest rate on government bonds divided by the standard deviation. This article uses the solver tool and draws conclusions. For the case of attached pensions, on the expected return of the optimal allocation, the expression is the weight of each stock and pension multiplied by the expected return of each stock.

Table 3: Portfolio without pension.

Portfolio without pension	
Portfolio Optimize r	
Weight ETD	0.2480
Weight CVGW	0.0013
Weight AAPL	0.0988
Weight ASML	0.1591
Weight UNH	0.0874
WeightLVMH	0
Weight NDM	0.0215
Weight SBUX	0.0838
Weight TM	0.3000
Weight BUD	0
Sum	1
Expected Return	0.0097
Variance	0.0013
Standard Deviation	0.0362
Sharpe Ratio	0.2114

Table 4: Portfolio with pension.

Portfolio with pension	
Portfolio Optimizer	
Weight ETD	0.1142
Weight CVGW	0
Weight AAPL	0
Weight ASML	0.0471
Weight UNH	0.0444
WeightLVMH	0
Weight NDM	0.0035
Weight SBUX	0.0143
Weight TM	0.0692
Weight BUD	0
Weight Pension	0.7073
Sum	1
Expected Return	0.0076
Variance	0.0004
Standard Deviation	0.0209
Sharpe Ratio	0.2664

To calculate the variance, first multiply the weights of each stock and pension fund by the two matrices of the VCV table, and then multiply the result by the two matrices of the weights of each stock. The calculation of standard deviation and Sharpe Ratio is consistent with the method without pension. For the convenience of comparison, this paper studied how to achieve the same Sharpe Ratio as without a pension when it comes with a pension, and what the annual interest rate of the pension should be at this time. In order to explore more of the relationship between annualized return and firefighters' lifespan, this paper also paid attention to the changes in annualized return at different lifespans.

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3.4.1. Figures

This article calculates the optimal weight of each stock and the corresponding expected returns and Sharpe under two different situations: with and without pensions. Ratio, as shown in table 3 and table 4. By comparison, I found that the Sharpe Ratio is 0.266 with a pension and 0.211 without a pension. It is obvious that the asset allocation with a pension has a better Sharpe Ratio. In terms of expected returns, those with pensions are 0.0076 and those without pensions are 0.0096. Asset allocation without pensions has higher expected returns. However, the standard deviation in the case of attached pension is 0.0209, and in the case of no pension, the standard deviation is 0.036. The former will be more stable and less susceptible to market influence.

According to the above description, it seems that it would be better to choose the investment portfolio with pension because it has a higher Sharpe Ratio. However, to analyze the above situation, we used an annualized rate of return of 7.75%, which is the rate of return 25 years after the firefighters retired. This sample data is not enough to explain the situation, so next I will continue to explore the age and pension of firefighters. relationship with annualized returns. Since the pension did not reach the government's promised one-time payment of $1.45 million 12 years ago, as seen in Figure 1, it starts in 13 years and continues to 35 years until the firefighter reaches the age of 90. And from Figure 1, we can intuitively see that the curve always keeps growing but the speed gradually slows down.

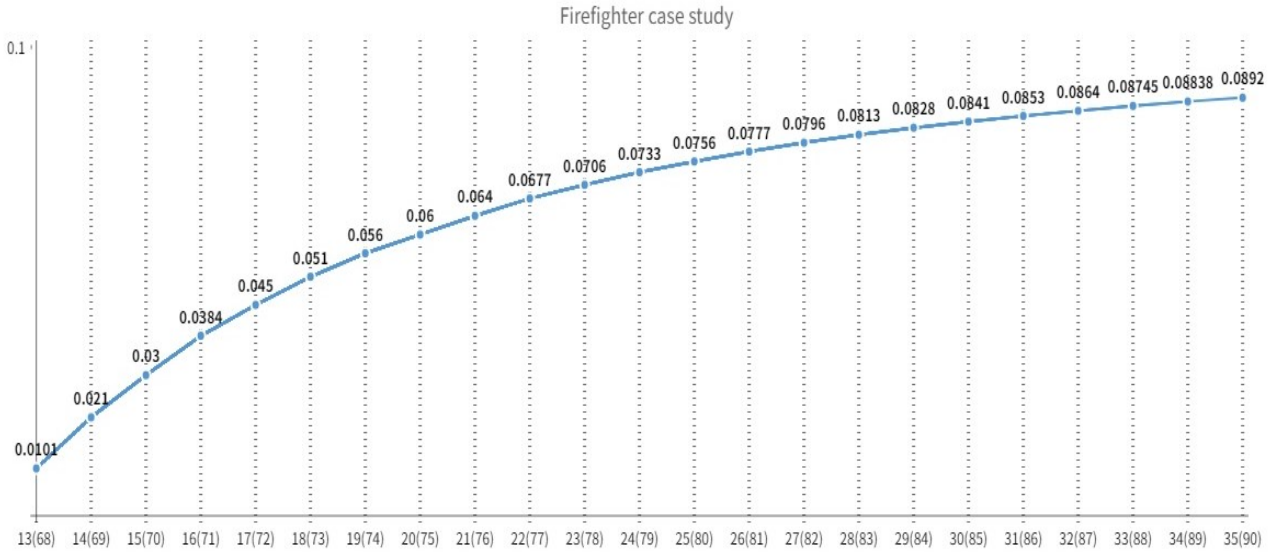

Figure 1: Firefighter living years after retirement and its pension return graph.

Figure 2 shows the comparison of the Sharpe Ratio between those with pensions and those without pensions at different ages. We can see that before the firefighter turns 74, which is 19 years after retirement, the Sharpe Ratio of the portfolio with a pension will be lower than that of the portfolio without a pension, but after the age of 74, the Sharpe Ratio of the portfolio with a pension It will overtake and be higher than the Sharpe Ratio without pension.

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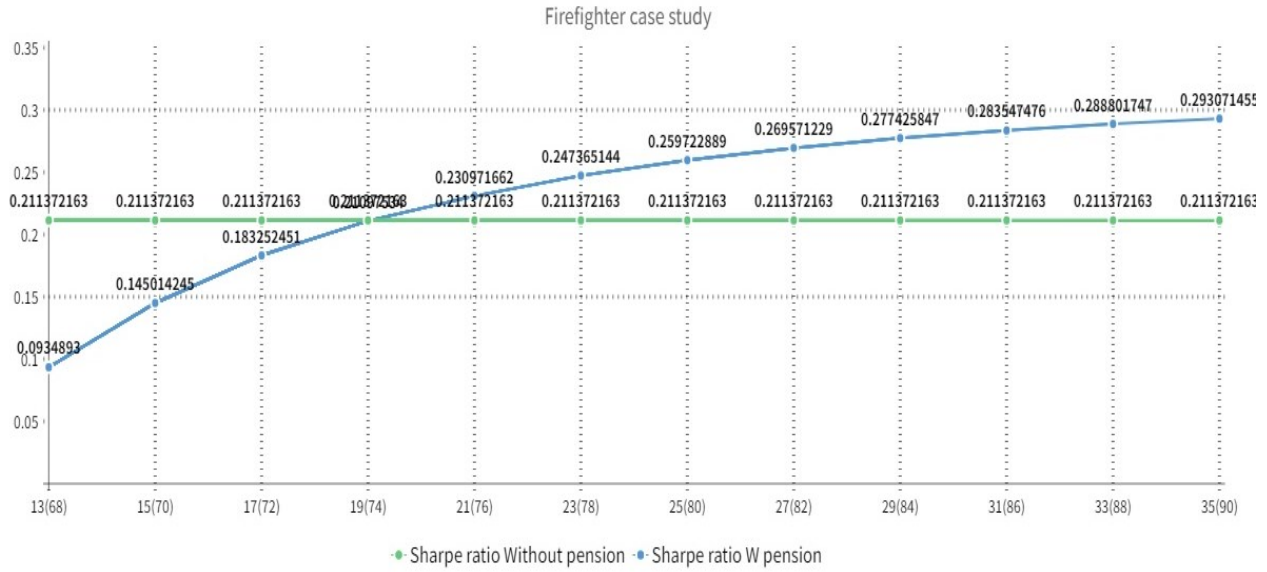

Figure 2: Sharpe ratio with pension and without pension comparison.

Regarding the life span of firefighters, we can see some useful information from Figure 3. For example, 71% of firefighters can live to the age of 74, that is, the situation is equal between those with pensions and those without pensions. More than 50% of people can live to the age of 80, which means they can achieve an expected annualized return of 7.75%

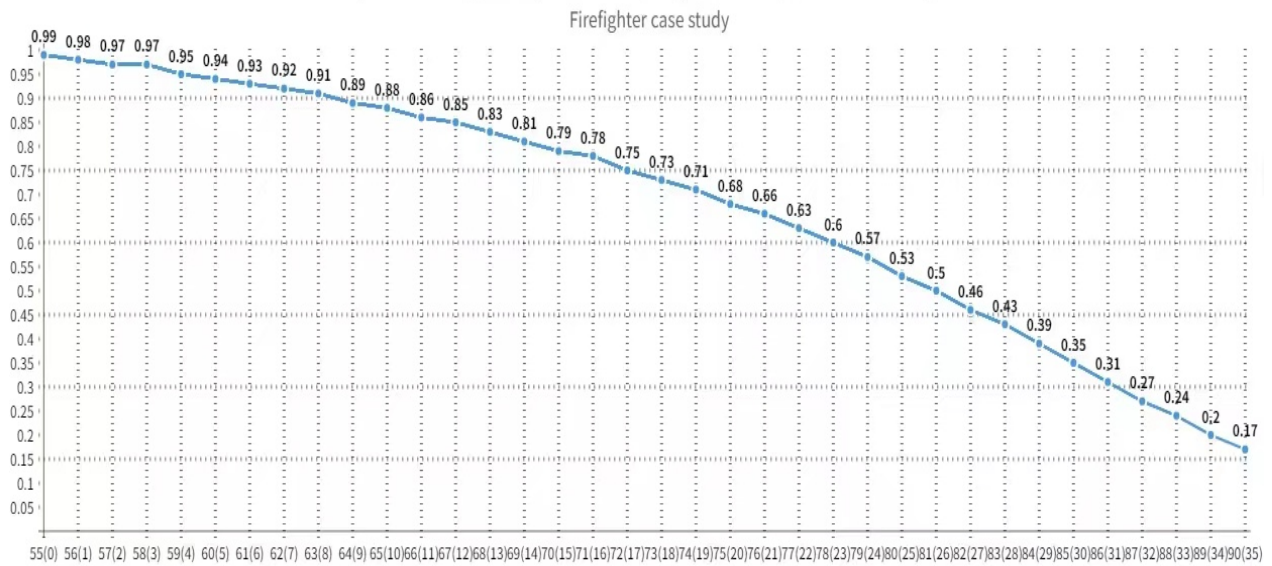

Figure 3: Percentage of people that stay alive and their age.

After careful analysis, both situations have their own advantages and disadvantages. For the investment portfolio without pensions, its advantage is that it has a constant Sharpe Ratio, which will reassure the elderly and have better performance in the first 19 years. But its disadvantage is that the low Sharpe Ratio for a long time will inevitably make people a little disappointed. For a pension portfolio, its advantage is that it has continued rising returns and has a higher Sharpe Ratio in the later period. But its disadvantage is that the revenue cycle is long and the early returns are not obvious.

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4. Research Conclusion

Based on the above research on the optimal portfolio of asset allocation, we found that there are advantages and disadvantages to the two investment portfolios without pensions and those with pensions. The advantage of the former lies in its stable Sharpe Ratio, while its long-term low Sharpe Ratio is its shortcoming. The advantage of the latter is that it has continuously rising income and a higher Sharpe Ratio in the later period, but its income cycle is longer and the early income is not obvious. In view of this, we recommend that retirees conduct a detailed examination of their physical condition at a regular medical institution before choosing an investment portfolio to determine whether the physical condition is good. If the health is good, it is recommended to choose an investment portfolio with a pension. If you are in poor health, a non-pension investment portfolio is recommended.

It should be emphasized that the conclusion of this article is based on the three-factor model. In the future, we can further expand it to a four-factor or even a five-factor model by introducing new factors, so that the data we can cover will be more comprehensive. This more comprehensive analysis will not only further enhance the reliability of our research conclusions, but will also help improve the predictive power of our models.

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