Proceedings of the 3rd International Conference on Business and Policy Studies
DOI: 10.54254/2754-1169/67/20241289

# Poker's Paradox: The Interplay of Skill, Chance, and Game Theory

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***Abstract:*** The intricate dance between skill and chance in poker has been a topic of enduring debate, resonating not just among game enthusiasts but also in legal, economic, and cultural corridors. Drawing from historical underpinnings and foundational principles of game theory—pioneered by the likes of Emile Borel and notably advanced by John Von Neumann—this paper delves into the intricate interplay of skill and chance in poker. By introducing a comprehensive model that employs game theory principles, we navigate poker's multidimensional landscape, demonstrating that these elements are inextricably intertwined. Our analysis reveals that while the draw of cards introduces an inherent element of chance, a player's prowess in decision-making, strategizing, bluffing, and opponent interpretation is paramount to long-term success. This dual nature of poker – where both skill and chance operate in tandem – is what makes it a subject of such profound interest and complexity. By presenting poker in this nuanced light, our study seeks to break down traditional dichotomies and offers a more holistic understanding.

***Keywords:*** poker dynamics, skill-chance interplay, strategic decision-making, game theory

## 1. Introduction

Poker, a globally recognized and ardently debated game, straddles the intersecting realms of skill and chance. Serving both as a popular household pastime and a high-stakes professional endeavor, its intricacy stems from a blend of strategic planning, psychological acumen, knowledge of probability, and the unpredictable element of luck This blend sparks an enduring debate: Does skill or chance primarily drive poker? The answer holds weighty legal and economic ramifications, influencing regulatory frameworks, tax structures, and societal views on gambling. Furthermore, it offers intriguing insights into understanding the balance of skill and luck in areas like financial markets and economic predictions.

Understanding poker's delicate balance of skill and chance requires an appreciation for game theory. This mathematical discipline delves into decision-making within multi-player settings, where conflicting interests are at play. Originating with the insights of French mathematician Emile Borel in 1921 [1], it was John Von Neumann, a brilliant scholar from Budapest, who took the discipline to new heights [2]. As we delve into the intertwined dynamics of skill and chance in poker, this rich lineage of game theory serves as a pivotal lens, providing both mathematical rigor and strategic depth.

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This paper seeks to frame poker not as solely a game of chance or skill but as an intricate amalgamation of both. Though luck dictates the cards drawn, sustained success hinges on a player's strategic acumen, decision-making prowess, and capacity to read and bluff opponents.

## 1.1. Literature Review

Within the wider academic sphere focusing on game theory and decision-making, the prevailing view is that poker largely hinges on skill. It's important to acknowledge the depth and breadth of existing research in this arena. A standout study reinforcing this view is Steven D. Levitt and Thomas J. Miles' "The Role of Skill versus Luck in Poker: Evidence from the World Series of Poker," from the NBER Working Paper Series [3]. This research offers an empirical lens into the 2010 World Series of Poker, using a distinctive dataset that had pre-tagged certain players as exceptionally skilled. Notably, these skilled players achieved an impressive average return on investment of over 30%, vastly outperforming others who saw a -15% return. Such marked differences underscore the pivotal role of skill in poker, particularly at elite levels. Levitt and Miles' findings powerfully argue for recognizing poker as a realm where skill significantly overshadows luck.

Hannum & Cabot, in their study "Toward Legalization of Poker: The Skill vs. Chance Debate," undertook a comprehensive analysis [4]. They examined the issue from two angles: a mathematical study contrasting a random player against a skilled one in Texas Hold'Em, and a wider simulation of full-table games, covering both Texas Hold'Em and Seven Card Stud. Their findings were clear-cut. In the former setup, the skilled player won an overwhelming 97% of hands. Meanwhile, in broader simulations, experienced players consistently outperformed less skilled participants, underscoring the pivotal role of skill.

The rest of the paper is organized as follows: Section 2 introduces our model and provides a detailed explanation, and Section 3 applies our model to an extended scenario to further validate that poker is a combination of skill and chance.

# 2. The Toy Model

In the subsequent section, we explore a simplified poker model, the Toy Model, to articulate the nuanced interplay between skill and chance in poker. The model includes two players and three cards: Ace (A), King (K), and Queen (Q). In this game, Ace has the highest value, followed by King, and then Queen, which has the lowest value. Each player is dealt one card with equal probability: $\frac{1}{3}$. Crucially, the game operates under the stipulation that players cannot be dealt identical cards; hence, the hands held by each player will always be distinct.

Player 1 will initiate the game by either calling or raising. In a call, both players will reveal their cards, and the one with the better hand wins \$10 from the opponent. However, if Player 1 chooses to raise, Player 2 has the option to either fold, consequently losing \$10 to Player 1, or call, wherein both hands are exposed, and the player with the higher card wins \$20 from the opponent. To examine the strategic nature of this game, we will look for the *Perfect Bayesian Equilibrium* [5] in which each player's strategies maximize their expected payoff, given their beliefs about the other player's strategies. Besides, each player's beliefs need to be consistent with the strategies and are updated using Bayes Rule whenever it is possible.

## 2.1. Analyzing Player 1's Optimal Strategy

To start with, we examine Player 1's strategy. In this model, a Player 1 holding a King (Type K) would never opt to raise, as raising only leads to a potential loss. The reason being, the only

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circumstance in which Player 2 would call the raise would be if they held an Ace- a better hand than King.

On the other hand, Type A of Player 1 will always raise as they are assured a win of $20 with the highest possible hand.

Interestingly, while one might intuitively assume that Player 1 with a Queen (Type Q) wouldn't raise, this isn't necessarily true. We will explore this counter-intuitive strategy in the succeeding discussion, emphasizing that ruling out a raise for Type Q cannot be deemed an equilibrium strategy.

### 2.2. Analyzing Player 2's Optimal Strategy

Switching our focus to Player 2's strategy, it's evident that Player 2 with an Ace (A) will always see a raise, as holding the superior card guarantees a win of $20 irrespective of Player 1's card. On the other hand, a Player 2 with a Queen (Q) will never see a raise because they would invariably lose $20 due to the inferiority of their card.

### 2.3. Considering Mixed Strategies for Type Q Player 1 and Type K Player 2

The strategy becomes intricate for Type K of Player 2, primarily due to the nuances of poker's balance between information and misinformation. For Type Q of Player 1, raising isn't just a move but a bluff, a feint suggesting a stronger hand than actually. This act of bluffing is a classic strategic element to introduce uncertainty in the opponent's decision-making.

Suppose type K of player 2 never sees a raise. Then, by calling, type Q of player 1 will lose ten dollars and by raising she will get a fifty-fifty lottery between losing twenty dollars and winning ten dollars. The latter option is better than losing ten dollars. So, type Q of player 1 will raise for sure. Then, by folding, type K of player 2 is losing ten dollars while calling would yield, at worst (i.e., if type A of player 1 also raises for sure) a fifty-fifty lottery between losing twenty dollars and winning twenty dollars. The latter is better. So, folding for sure cannot be an optimal strategy for type K of player 2. Then, if type K of player 2 sees every raise for sure, type Q will never raise which means that type K of player 2 loses for sure whenever she sees a raise. This cannot be a part of an equilibrium strategy. So, type K of player 2 must randomize to ensure that type Q of player 1 has an incentive to randomize. When Player 2 with a King faces a raise, she will accept the raise with a probability p. To obtain the probability p with which Type K Player 2 should see a raise. That is,

$$0.5 * -\$20 + 0.5 * \mathrm{p} * -\$20 + 0.5 * (1-\mathrm{p}) * \$10 = -\$10$$

The right-hand side of the equation above is the payoff for Type Q of Player 1 if they call, while the left-hand side is Type Q of Player 1's payoff if they raise. The first term on the left-hand side reflects the possibility that Player 2 might have an Ace (A), and the second term reflects the possibility that Player 2 might have a King (K) and call her bluff. The last term reflects the possibility that Type K of player 2 will fold.

Solving for p gives $p = \frac{1}{3}$. This means that Player 2 of Type K should see a raise with a probability of $\frac{1}{3}$ to make Player 1 of Type Q indifferent between calling and raising.

Next, the strategic depth of poker can often be analyzed through the lens of probability and game theory. A critical element in these considerations is the application of Bayes' theorem, which relates the probabilities of events accounting for prior knowledge. For Player 2 of Type K, we want to derive the probability q with which Player 1 of Type Q should raise to ensure indifference in Player 2's decision. To do this, we equate the expected payoff from seeing a raise to the guaranteed payoff from folding.

Firstly, Bayes' rule is articulated as:

$$P[A|B] = \frac{P[B|A] * P[A]}{P[B]}$$

Applying the above rule to our scenario:

- Let A be the event that Player 1 has an Ace.
- Let B be the event that Player 1 raises.

Expanding the terms using Bayes' rule:

$$P[A|B] = \frac{P[B|A] * P[A]}{P[B|A] * P[A] + P[B|Q] * P[Q]}$$

Where:

- $P[A|B]$ is the conditional probability that Player 1 has an Ace given that they raised.
- $P[Q]$ is the prior probability that Player 1 has a Queen

Substituting this understanding into our original equation, for Player 2 of Type K, the expected payoff from seeing a raise with that from folding, to obtain the probability q with which Type Q Player 1 should raise. That is,

$$-\$20 * \left[\frac{1 * 0.5}{1 * 0.5 + 0.5 * q}\right] + \$20 * \left[\frac{0.5 * q}{0.5 * q + 0.5}\right] = -\$10$$

The right-hand side of the equation above is the payoff for Type K of Player 2 if they fold. The left-hand side is Type K of Player 2's payoff if they see a raise. The two terms on the left-hand side represent the two possible outcomes when Player 2 of type K sees a raise:

1. Player 1 has an Ace: Player 2 loses \$20.
2. Player 1 has a Queen: Player 2 wins \$20.

Solving for q gives q = $\frac{1}{3}$. Hence, Player 1 of Type Q should raise with a probability of $\frac{1}{3}$ to make Player 2 of Type K indifferent between seeing a raise and folding.

## 2.4. Conclusion of the Toy Model

With the preliminary strategy analysis, we arrive at the *Perfect Bayesian Equilibrium*, where both players need to randomize their strategies. For Player 1, Type A always raises, Type K never raises, and Type Q raises with a probability of 1/3. Meanwhile, for Player 2, Type A always sees a raise, Type Q never sees a raise, and Type K sees a raise with a probability of $\frac{1}{3}$. This equilibrium strategy hinges on both skill and chance. It becomes clear that the strategic actions of Player 1 are a blend of chance (the card they draw) and skill (their decision-making based on the card drawn). Player 2's strategy is also a combination of skill (how they respond to Player 1's actions) and chance (the card they hold).

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## 3. Extending the Toy Model to Further Prove the Dual Nature of Poker

In this section, we extend our initial Toy Model to further solidify the position that poker is a combination of skill and chance.

Consider a version of poker where each player independently draws a card from a uniform distribution over the interval [0,1]. Player 1, upon observing her card, can either "call" or "raise". If Player 1 "calls", both players reveal their cards, with the player holding the higher card winning $10, while the other loses $10. If Player 1 chooses to "raise", Player 2 can either "fold" or "call". If Player 2 "folds", they lose $10 and Player 1 wins $10. Conversely, if Player 2 "calls", the player with the higher card wins $20 and the other player loses $20. Assuming both players are risk-neutral, we aim to find a Perfect Bayesian equilibrium of this game.

### 3.1. Establishing the Threshold for Player 2's Decision

To begin the analysis, let's assign random variables X and Y to represent the card values of players 1 and 2 respectively. Guided by intuition from the initial toy model, let's propose the existence of a threshold value y* such that, in response to Player 1 raising, Player 2 folds if Y < y* and calls if Y > y*. Technically, the expected payoff for Player 2 when holding card y and deciding to call is computed as follows:

$$E[Payoff|Call] = \$20 * P[Y \geq X|Player\ 1\ raises] - \$20 * P[Y < X|Player\ 1\ raises]$$

The first term on the right represents the payoff of Player 2 if Player 2's card value is greater than or equal to Player 1's card value knowing that Player 1 raised. On the other end, the second term on the right represents the payoff of Player 2 if Player 2's card value is less than Player 1's card value knowing that Player 1 raised. Given the guaranteed loss of $10 when Player 2 folds, using the standard monotonicity and continuity argument, we can establish that there exists a threshold y* where Player 2 folds if Y < y* and calls if Y > y*.

### 3.2. Calculating Player 1's Payoff from Calling

Next, we seek to understand Player 1's strategy by calculating the expected payoff when they choose to call with a card value X. Considering that the cards are drawn uniformly from [0,1], the probability of Player 2's card Y being less than or equal to X is simply X, and conversely, the probability of Y being greater than X is 1−X. Given these probabilities, Player 1's expected payoff from calling with a card value X is calculated as follows:

$$E[Payoff|Call] = \$10 * P[Y \leq X] - \$10 * P[Y > X] = \$10 * X - \$10 * (1 - X) = \$20X - 10$$

### 3.3. Evaluating Player 1's Strategy in Response to Player 2's Decision Threshold

Having established Player 1's expected payoff from calling, we process to determine Player 1's expected payoff from raising when holding a card value X. This can be expressed as:

$$E[Payoff|Raise] = \$10 * P[Player\ 2\ Folds] + \$20 * P[Player\ 2\ Calls] * P[Y \leq X|Player\ 2\ Calls] - \$20 * P[Player\ 2\ Calls] * P[Y > X|Player\ 2\ Calls]$$

The equation can be further developed as follows:

$$E[Payoff|Player\ 1\ Raise] = \$10y^* + \$20(1-y^*) * P[Y \leq X|Y > y^*] - \$20(1-y^*) * P[Y > X|Y > y^*]$$

The first term on the right represents the expected monetary gain for Player 1 when Player 2 folds. The $10 prize is multiplied by the probability that Player 2 will fold (y*). The second and third terms capture the potential payoffs when Player 2 decides to call. The second term represents the payoff when Player 1 wins, given that Player 2 has called. The $20 prize is multiplied by the probability of Player 2 calling (1−y*), which is then further multiplied by the conditional probability that Player 2's card value Y is less than or equal to Player 1's card value X, given that Player 2 has decided to call. The last term represents the potential loss for Player 1, given that Player 2 calls and Player 2's card value Y is greater than Player 1's card value X.

Now, we can explore the conditions under which Player 1 should call or raise, given the threshold y*.

If X ≤ y*, Player 1's payoff is defined as:

$$E[Payoff|Player\ 1\ Raise] = \$10y^* + \$20(1-y^*) * (P[Y \leq X|Y > y^*] - P[Y > X|Y > y^*]) = \$10y^* - \$20(1-y^*) = \$30y^* - 20$$

So, it is optimal for Player 1 to raise if $30y*−20 ≥ $20X-10 (Player 1's expected payoff from calling with a card value X), translates to raise if $X \leq \frac{3y^*-1}{2}$, and to call if $y^* \geq X \geq \frac{3y^*-1}{2}$.

For the case where X > y*, the payoff for Player 1 raising with a card value X is defined as:

$$E[Payoff|Player\ 1\ Raise] = \$10y^* + \$20(1-y^*) * (P[Y \leq X|Y > y^*] - P[Y > X|Y > y^*]) = \$10y^* + \$20(1-y^*) * \left(\frac{X-y^*}{1-y*} - \left(1 - \frac{X-y^*}{1-y^*}\right)\right) = -\$10y^* + 40X - 20$$

The optimal strategy is for Player 1 to raise if −$10y* + 40X – 20 ≥ $20X – 10, which is $X \geq \frac{1+y^*}{2}$, and to call if $y^* < X < \frac{1+y^*}{2}$.

The two strategies detailed above establish the best response for Player 1 if Player 2 adopts a cutoff strategy based on y*. To completely describe the Perfect Bayesian equilibrium, we need to find a y* such that using the cutoff strategy "Fold if Y < y*" and "Call if Y ≥ y*" is a best response for Player 2.

### 3.4. Finding the Optimal Cutoff Strategy for Player 2

If Player 1 adopts the above strategies, then after Player 1 raises, Player 2 believes Player 1's card is uniformly distributed on $\left[0, \frac{3y^*-1}{2}\right] \cup \left[\frac{1+y^*}{2}, 1\right]$. We then need to find the value of y* that makes Player 2 indifferent between folding and calling.

When Player 2 observes Player 1 raising, they update their beliefs about Player 1's card using Bayes' theorem. Let's calculate $P[X \leq y^*|Player\ 1\ Raise]$:

$$P[X \leq y^*|Player\ 1\ Raise] = \frac{P[Player\ 1\ Raise|X \leq y^*] * P[X \leq y^*]}{P[Player\ 1\ Raise]}$$

The various components of this expression are:

DOI: 10.54254/2754-1169/67/20241289

1. $P[Player\ 1\ Raise|X \leq y^*]$ is the probability that Player 1 raises given that their card is less than or equal to y*, which is $\frac{3y^*-1}{2}$ from our previous calculation.
2. $P[X \leq y^*]$ is simply y* since card are uniformly distributed in [0,1].
3. $P[Player\ 1\ Raise]$ is the unconditional probability that Player 1 raises. Player 1 will raise if and only if $X \in \left[0, \frac{3y^*-1}{2}\right] \cup \left[\frac{1+y^*}{2}, 1\right]$, this probability is $(\frac{3y^*-1}{2}) + \left(1 - \frac{1+y^*}{2}\right)$

Putting it all together we have $P[X \leq y^*|Player\ 1\ Raise] = \frac{3y^*-1}{2y^*}$.

### 3.4.1. Determining Y* and the Indifference Point

By the similar monotonicity and continuity argument, we know player 2 with card $y^*$ should be indifferent between folding and calling, which gives us:

$$\$20 * \left(\frac{3y^* - 1}{2y^*}\right) - \$20 * \left(1 - \frac{3y^* - 1}{2y^*}\right) = -\$10$$

Solving for y* we get $y^* = \frac{2}{5}$.

### 3.4.2. Finding the Perfect Bayesian Equilibrium

Let's substitute the equilibrium value we found for y* into Player 1's strategy, which is as follows:

1. If X ≤ y*, Player 1 should raise if $X \leq \frac{3y^*-1}{2}$ and call if $y^* \geq X \geq \frac{3y^*-1}{2}$.
2. If X > y*, Player 1 should raise if $X > \frac{1+y^*}{2}$, and to call if $y^* < X < \frac{1+y^*}{2}$.

Substitute $y^* = \frac{2}{5}$ into Player 1's strategy and the complete strategies for Player 1 and Player 2 are as follows:

- Player 1 raises if her card is such that $X \leq \frac{1}{10}\ or\ X \geq \frac{7}{10}$; otherwise, she calls.
- Player 2 calls if and only if her card is $Y \geq \frac{2}{5}$

## 4. Conclusion

Our extended model re-emphasizes that poker is an intricate blend of skill and chance. A player's decision to call, raise, or fold isn't merely based on their card's value, but also on an astute understanding of the opponent's likely moves and potential outcomes of each choice.

While our model distills poker's core strategic elements, real-world poker introduces further intricacies, including repeated plays, varied bets, and diverse hand rankings. Still, the model is instrumental in dissecting poker's foundational dynamics, highlighting how skill and chance are tightly woven in determining outcomes.

In summary, a seasoned poker player knows how to marry strategic insight with the unpredictable nature of the game, making poker perpetually captivating and challenging.

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DOI: 10.54254/2754-1169/67/20241289

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