# The Golden Ration in the Renaissance Art: A Comparative Study on the Geometrical Layout of the Mona Lisa and the Annunciation

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*Abstract:* Nowadays, the golden ratio is involved in many fields, for instance, to pursue the perfect face shape and features, people advocate referring to the golden ratio of the human face to carry out cosmetic surgery. People also match different clothes and fashion accessories to better shape the golden ratio of the human body structure, the ratio can also be used as an aid in painting or photography. It is worthwhile to answer the questions like why and when was the golden ratio extensively applied in art, as well as what evidence of the evidence behind would support the utilization of the golden ratio. To answer these questions, this paper would engender the answer by appreciating the Renaissance artwork with a golden ratio. The Renaissance would be a flourishing of the application of the golden ratio and have become classics. The golden ratio can be correctly used because the existence of this ratio is invented in ancient Greek and is based on science, the ratio has stood the test of time and can be expressed by mathematics proportion.

*Keywords:* The Renaissance, Golden Ratio and Spiral, Art and Science, Mona Lisa, Annunciation

#### 1. Introduction

Renaissance has brought a historic turning point in many countries, especially the Renaissance movement in European countries, a landmark movement in the history of European culture and even the world culture. Western Renaissance is an intellectual and cultural movement that occurred between the 14th century and the 16th century, the emergence of the Renaissance was following with the "dark age" of the height of prosperity of Greek and Roman antiquity, which is a representation of the rebirth of science and culture [1]. Science is the field of study that verifies the world around people through experimentation and observation, it involves various fields, including natural science, math, geometry, anatomy, and other aspects [2]. The Renaissance's influence on the sciences can be reflected in many ways, especially in the science factors in the arts. Art can be regarded as the expression form of creativity or imagination, people could represent their feeling, artistic talent, and experimentation through music, painting, dance, and other artwork. Science and Art are two separate disciplines; however, a wealth of early documents showed that science elements involved in the artwork, they often indistinguishable from each other [3]. This paper would commit to explaining the

intimate connection between science and art in the context of the Renaissance by discussing the mathematics and composition skills in the portrait painting Mona Lisa by Leonardo da Vinci and the tempera painting Annunciation by Sandro Botticelli. Firstly, the essay would emphasize the importance of art to science. Secondly, there will be an introduction to the history and nature of the Golden ratio. Thirdly, the method to prove the science in art and their connection is to explain the mathematics ratio in the Mona Lisa and Annunciation.

#### 2. The Importance of Art to Science

There is a lot of evidence that could support that art is necessary for science. Art involves creativity or imagination, it can express professional competence, beauty, emotion, or conceptual ideas in different forms. An article from Anna Powers contributed that the creativity and imagination that are brought by art are needed by scientific breakthroughs, and art would become a way of expressing scientific knowledge, which ultimately promotes art and science are closely related [4]. Peter W. Parshall and David Landau proposed that the illustrations could serve as standard references for scholars in laboratories in many fields, and accurate visual representation could be a highly specialized form of observation [5]. From the perspective of the development of art, it is based on the use of mathematics to a large extent [6]. The mathematics here mainly refers to the knowledge related to geometry, picture structure, line perspective, and dynamic graphics. The Renaissance artists had already made great contributions to human knowledge, it should be mentioned that the distinction between art and science during the Renaissance was not as clear as it is now, many artists sometimes create works that involved both art and science [7]. Artists are also absolutely authoritative in the production and utilization of certain knowledge. For instance, the Italian Renaissance Leonardo Da Vinci promoted the study of anatomy by dissecting dead animals. He was asked to adjust and draw manuscripts for hydraulic engineering. Leonardo also researched and practiced the design of large crossbows and other large weapons [8]. In general, literary humanists have promoted the interaction of the two disciplines, invented mathematical perspectives through art, made it possible to depict physical space accurately and realistically, revive various classics, and have expanded rapidly the range of human knowledge.

### 3. The Golden Ratio

The golden ratio in mathematics and the arts can also be called the golden section or divine proportion. The ratio refers to the ratio of the long side to the short side is 1:1.618 or the Greek letter  $\phi$ , and it was first proposed in ancient Greece by mathematician Pythagoras and the "Father of Geometry" Euclid [9]. The golden section has strict proportion, rich artistry, and harmony, it can arouse people's aesthetic feelings. The ratio is considered the most aesthetically pleasing ratio to the eyes and the most ideal ratio in architecture and art [10]. The ancient Greeks recognized the nature of 'division' before the Renaissance and were eventually reduced to 'section', which refers to 'ratio' nowadays [11]. The Greeks also noted that the golden ratio provided the most aesthetically pleasing proportion of rectangular edges, and this concept was reinforced during the Renaissance [12]. Meisner proposed that the golden section was generally favored and used by Renaissance artists [13]. For example, during the Renaissance time, painters like Leonardo da Vinci and Michelangelo Buonarroti showed the use of the golden ratio in the composition of paintings and the structure of figure sculptures, and architect Leon Battista Alberti recombined the basic shapes of circles and squares in proportion to find the golden ratio of building.

## 3.1. Mathematics Ratio in the Mona Lisa

Mona Lisa is a half-length portrait of Leonardo Da Vinci, a famous Italian Renaissance painter, natural scientist, and engineer [14]. This artwork is an oil painting and has been on permanent display at the Louvre Museum in Paris since 1797. There is much debate about the painting's creative time, some experts suggested it was painted between 1503 and 1506, but some argued that Leonard painted it in 1513 [15]. Mona Lisa is not only considered one of the most valuable paintings in the world but has also been described as the most mystical [16]. Mona Lisa's smile has been described as the "eternal smile", there are mystical, elegant, and engaging elements involved in her smile [17]. No matter what angle people look at the Mona Lisa, they will find Mona Lisa in the painting smiling at them. Many researchers are willing to attribute the reason why Mona Lisa's smile is mystical and attractive to the use of the golden ratio in the structure of the Mona Lisa, the golden ratio of 1:1.618 is maintained in many parts of Mona Lisa [18]. There is no direct evidence supporting that Leonardo consciously used the  $\phi$  in the Louvre Mona Lisa. However, Italian mathematician Luca Pacioli published a three-volume treatise about the golden ratio in 1509. Leonardo provided illustrations and some views for those treatises as Pacioli's close friend, which makes some scholars speculate some of Leonardo's paintings are embedded with a golden ratio [19]. Additionally, many articles claim that the ratio of 1:0.618 is embedded in the Louvre Mona Lisa with the composition proportion of rectangle, triangle, and spiral [15-17].

The composition of the Louvre Mona Lisa is closely related to the golden section. First, the golden rectangle and triangle are embedded in the Louvre Mona Lisa. Drawing a rectangle on the surface of the Mona Lisa, the theoretical and effective dimensions of it are similar. Theoretically, 53.14:76.77 is the ratio of the golden rectangle [17]. Compared with the actual wide and length, 53cm and 76.8cm, which is very close to the theoretical golden ratio. As Figure 1 shows, the golden triangle outlined by the blue line is located in the middle of the whole painting [18]. The width and height of this isosceles triangle are the same as the width and length of the golden rectangle, in other words, the proportion of the Mona Lisa was close to both the golden triangle and rectangle ratio.



Figure1: Golden triangle in the Mona Lisa, Gyan Bahadur Thapa and Rena Thapa, 2018. Followed by the golden spiral ratio, which can be represented by placing a square inside the golden rectangle, the remaining rectangle space will be used to form a smaller small golden rectangle. Then, putting another square inside the new golden rectangle, repeat the same operation, ultimately getting a patterning as Figure 2 shown [19]. Next, drawing a quarter circle inside each resulting square, connecting the quarter circle gets together would get a spiral shape, namely the gold spiral. The golden spiral is shown in Figure 3, points 1 to 5 are the golden section points in the golden rectangle they are located at [20]. Each radius of the quarter circle in Figure 3 is the edge of the square in which they are located [21].

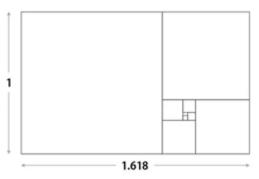


Figure2: Golden rectangle, Jun-Sheng Duan, Discrete Dynamics in Nature and Society, 2019.

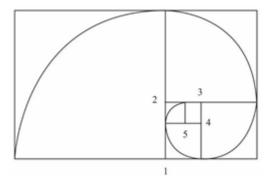
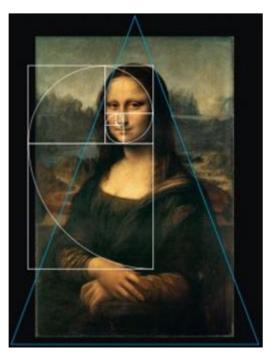


Figure3: Golden rectangle, Jun-Sheng Duan, Discrete Dynamics in Nature and Society, 2019.

As Figure 4 shows, it can be seen clearly that the spiral shape almost gave a perfect outline for Mona Lisa's face, with the quarter circle of the spiral on the right side of Mona Lisa's face and the vertical side of the golden rectangle is on the left side [22]. Moreover, the spiral line just started from the tip of Mona Lisa's nose, grazed the bottom of her chin, the edge of her forehead, all the way to her right arm, and then to the thumb of her left hand, the whole character was surrounded by the golden spiral.





# 3.2. Golden ratio in Annunciation

Sandro Botticelli, a pioneer of Italian portraiture, painted the tempera painting on board Annunciation around 1485 [23]. Annunciation is one of the most celebrated artworks in the Robert Lehman Collection and is now preserved in the Metropolitan Museum of Art in New York [24]. This tempera painting was rendered in a classic architectural interior. Botticelli's Annunciation integrated the golden ratio in its composition of it, it applied a one-point perspective with a vanishing point to express the illusion of depth [25]. Moreover, Botticelli used the scale system of the golden rectangle to set the dimension and composition of his painting. As Figure 5 shows, the size of the Lehman Annunciation is  $19.1 \times 31.4$  cm, and the ratio of length and width of this rectangular painting is 0.608, which is very similar to the perfect golden ratio of 0.618 [18].



Figure 5: Golden ratio in Annunciation, The Met, no date.

Botticelli's Annunciation pursues a visual balance, showing the beautiful aristocratic courtyard, which is broadened by the accurate use of perspective science. This tempera painting's use of

rendering, the relationship between light and dark, and other techniques. As Figure 6 shows, in the left half of the corridor where the angel was, Botticelli applied the composition technique of parallel perspective and one-point perspective, so that the audiences would be conscious of the space between the long shot and close shot and the relationship between them. The audience would see the background outside the corridor through the round-arched window, including the ceiling of the corridor and the wall carvings on the left side of the corridor. Moreover, in terms of proportion, Figure 7 shows the hypotenuse of Angle 123 is right on where the angel's wings are raised, and the hypotenuse of Angle 123 at the angel's wings and the hypotenuse that is formed by the woman bend down is just diagonally on the large rectangle, which is shown as the Hypotenuse 14 in Figure 7. These fine-tuned composition proportions pave the solid foundation for the whole painting to present the golden ratio.



Figure6: Perspective in Annunciation, Photo credit: Original

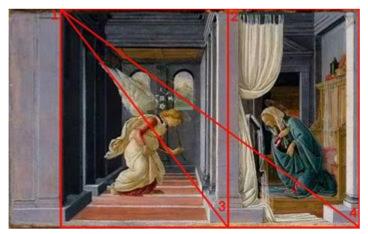


Figure7: Composition techniques in Annunciation, Photo credit: Original

It is worth noting that Botticelli integrated the geometric proof of the rectangle's dimensions in his Annunciation. To achieve the golden ratio, Botticelli tested the ratio and imply in the Lehman Annunciation [26]. As Figure 8 shows, while creating this painting, Botticelli drew a square ABCD with the same width as the whole Annunciation painting, with a side length of 19.1 cm [18]. Then he divided the square ABCD vertically into two smaller rectangles and get the smaller rectangle EFDC, and drew a diagonal ED in the rectangle EFDC. Using the diagonal ED as a radius, Botticelli drew a quarter circle and get a 31.4cm long rectangle, consummating a golden rectangle. As can be seen in

Figure 8, the center line of the wall or bridge pier located at the right of the center of the painting is just on the golden dividing line of the whole painting [18]. While the left corner of the corridor behind the angel corresponds to the vertical building line dividing the square where the angel is in half.

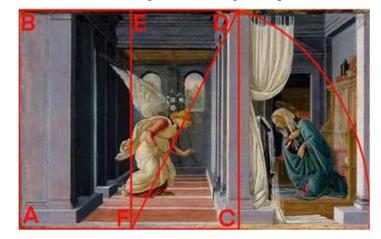


Figure8: Golden ratio verification of Annunciation, Richard, Artibus et Historiae, 2014. Arnheim proposed that artists can not only allocate the composition of the described object to take into account the influence of color on the center of the composition but also skillfully arrange the attention center of the picture [27]. To achieve a new force structure, In Annunciation, the geometric proportion that had been theorized to be the most aesthetically pleasing to the eyes was integrated into the painting, so that the painting would reflect the quiet and activity effectively, creating a kind of subtlety and tension of the whole painting.

It is obvious to see that the golden composition proportion, perspective, and a series of mathematical knowledge were constructed and emphasized in the architecture and space in the artwork. By comparison, the Mona Lisa's attainments in picture composition are higher than the Annunciation. Mona Lisa is a portrait of a single person, Leonardo presented the portrayal of Mona Lisa and the distance relationship of the background through golden ratio directly, without any clear moving line of geometry or the emphasis on perspective structure. However, the Annunciation painting needs parallel lines, corridors, floor tiles, windows, and other reference objects to effectively hint at the structural relationship and perspective relationship in the painting.

#### 4. Conclusion

In conclusion, there are many examples of the intersection of art and science, this essay focused on the Louvre Mona Lisa by the art master art in the Renaissance---Leonardo Da Vinci. The essay first emphasized the close relationship between art and science with support from the literature. The Louvre Mona Lisa was a portrait and oil painting painted by Leonardo; the divine proportion of the mathematics involved in many parts of the Mona Lisa. Divine proportion (golden ratio) was created in ancient Greek and prevailed in the Renaissance, it approximately refers to 0.618. There are 4 figures in this essay that verify the golden rectangular, triangle, and spiral are embedded in the Mona Lisa, and certify that the Mona Lisa contains mathematical proportion. The fascinating pictorial effects of the Mona Lisa are strongly associated with the golden section in math, as associated with science. The two figures used to explain the golden ratio created a comfortable viewing experience for audiences. The aesthetic value of the golden section has gone far beyond the scope of mathematics and art. Many objects in the world are closely related to the golden section. It has formed the law of the world and even the universe, not only the combination of mathematics, art, and architecture, believing in beautiful things can always bring some faith and awe to the world.

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