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The Concept of Infinity under German Idealism and Modern Set Theory

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Abstract: The purpose of this paper is to compare and analyse the concepts of infinity in German idealism and Modern set theory. The fisrt part of this paper analyses Kant and Hegel's views on mathematical infinity, where Kant suggests a notion of potential infinity constructed by means of intuition, while Hegel that of actual infinity. The second part illustrates the characteristics of Cantorian transfinite sets and both the progress it has made and the limitations. Following the two main parts, not only a clear contrast between the notions of infinity under German idealism and Cantorian set theory can be made but their close link can also be shown. By analysing perspectives of Frege's logicism, Hilbert's formalism, Brouwer's intuitionism, and Badiou's comments, it is clear German idealism has lost its mainstream position in understanding the mathematical concept of infinity. However, the concept of infinity in German idealism can be supplied as a powerful facilitator for philosophers and mathematicians to understand this concept in the realm of mathematics.

Keywords: infinity, German idealism, set theory

1. Introduction

Infinity, as an important concept in the human intellectual enterprise, occupies a core position in many fields, such as philosophy, mathematics, logic, theology, natural science, and so on. The purpose of this paper is to discuss the meaning of the concept of infinity in German idealism and in modern set theory, their historical significance, and try to answer the question of whether the concept of infinity in set theory really surpasses the concept of infinity in German idealism.

This paper tries to clarify the concept of infinity and give corresponding answers to the above question through comparative analysis of texts, theories, and commentaries. This paper is divided into three parts: The second chapter expounds the concept of infinity from Kant to Hegel, and the development of infinite from potentiality to actuality under the context of German idealism; The third chapter mainly gives the definition and mathematical properties of infinity through Cantor's set theory, and also includes the improvement of the concept of infinity after Cantor's set theory, such as non-standard analysis; The fourth chapter attempts to critically evaluate the concept of infinity in German idealism and modern set theory through the perspectives of the three major schools of modern mathematics, logicalism, intuitionism, and formalism, and Badiou's comments. Finally, one modest conclusion can be drawn from the preceding procedure: while set theory for our

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understanding of the infinite provides an effective way and has largely shaped our understanding of the infinite today, German idealism in the infinite ideas still requires more attention, not as a replacement for the concept of the infinite set theory, but as a supplement and facilitator.

2. The Concept of Infinity in German Idealism

Kant's view of infinity relies heavily on the metaphysical expositions of the concepts of time and space in his Transcendental Aesthetic, in which space and time are redefined as the a priori forms of intuition that can be filled by empirical matter to form appearances [1].

We can certainly perceive appearances as well as the appearances of relations between them. No matter what the appearance is, it must be spatial and temporal in the sense that it must be perceived as external to us and as occurring either simultaneously or successively. In every appearance, externality presupposes space, while simultaneity and succession presuppose time. Space can be represented without any particular appearance, but no appearance can be represented without space, from which the apriority of space can be found. In the same manner, we can imagine a pure succession of time in which no appearance occurs but cannot imagine any appearance that can happen without time. Thus, space and time are not derived from the empirical matter of appearances but are the prerequisites for all appearances and the forms of all intuitions. Moreover, both space and time are one and the same since both of them are qualitatively homogeneous and the parts of them only have quantitive differences with each other. In this perspective, time and space, as the pure intuition of magnitude, are the apriori structures through which some concrete and particular appearances are limited to some particular parts of space and time, which have a concrete magtitude, but time and space themselves are quantitively unlimited or potentially infinite and given to us as the intuitive capacity prior to any appearance limited by some particular parts of space and time. And this is why Kant calls space and time "represented as an infinite magnitude" [1].

In contrast to the mathematical platonist view of infinity as a real being external to the subject, the intuitionistically infinite given magnitude in the Kantian sense is an a priori sensible capacity of the subject of knowledge, what is necessarily contained in time and space as intuition, rather than some kind of objective infinity in external reality [1, 2, 3]. However, it does not mean that infinity lacks reality. For Kant, following his thesis that space and time are empirically real because "we encounter them in all our sensory experience and there is no empirical reality that would not be spatio-temporal" and they are also transcendentally ideal because "they are part of the fabric of our sensibility insofar as it forms the empirical reality contained in intuitions," it can be concluded that infinity, as the way in which space and time can be represented, is also empirically real and transcendentally ideal [1]. According to this singular viewpoint, infinity is neither an artificial nor subjective fantasy, but rather a necessary condition without which neither our experience of the objects nor the objects of our experience can be formed.

Nonetheless, infinity, as mere intuition, does not form knowledge but only the matter of knowledge. According to Kant, it is only through pure understanding that infinity or a set that contains infinite elements can be established as a mathematical concept and that knowledge of the infinite can be made possible. In making this point, Kant takes time alone into account. The basic idea is that, based on the intuition of time as the subject of knowledge, we can quantitatively combine or organise a homogeneous unit through some creative association called transcendental imagination by Kant, and form a time sequence. Quantity has unity, plurality, and totality. Unity is reflected in the homogeneous unit, plurality is reflected in the superposition or combination of units, and totality shows that such a combination process based on the intuition of time is potentially infinite in principle, through which the mathematical concept of infinity can be possible. In short, on the basis of infinity as an a priori intuitive capacity, pure understanding can construct a mathematical concept of infinity through such a process [1, 2, 3].

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Both the intuitive infinite magnitude and the infinite mathematical concept are only applied in the realm of experience and are empirically real and transcendentally ideal. In other words, the concept of the infinite is valid only within the limits of all possible experience, beyond which the infinite has no meaning as soon as it reaches the reality of absolute objectivity, that is, the realm of the thing in itself. This is clearly indicated in Kant's first set of Antinomies: The thesis that the world has a first beginning in time and a limit in space and the antithesis that the world has no first beginning in time and no limit in space can both be justified by reason sufficiently [1].

With respect to time, the argument for the thesis can be summarised as:

(1) If the world had no beginning, the world would have been preceded by an infinite number of states.

(2) An infinite number of states can never be traversed.

(3) There cannot be an infinite number of states.

(4) Therefore, the world must have a beginning.

The argument for the antithesis can be summarised as:

(1) If the world had a beginning, then there would be empty time before the beginning.

(2) Nothing can happen without a reason.

(3) There is no reason in empty time.

(4) The world cannot be generated in empty time.

(5) Therefore, the world cannot have a beginning.

The basic idea is also contained in the second part of the antinomy with regard to space. What is implied in the antinomy of time and space is that the world has to be both infinite and not infinite, and the only way out of the dilemma is to restrict the notion of infinity within the limits of experience or knowledge rather than that beyond it. This suggests that the phenomenal world comprised by all the possible experience cannot be grasped as the totality of an independent object which is infinite or finite itself [1, 2]. Hence, in spite of the objective validity it can produce, the concept of infinity here is an ideal one in the sense that it has no independent existence in absolute reality. In other words, the subject of knowledge, with the aid of sensibility, understanding, and reason, can intuit infinite magnitude in space and time, construct an infinitue sequence, and conceptualise it as a mathematical concept, but cannot substantiate the infinity in things themselves by a misuse of pure reason. Therefore, for Kant, the infinity is never a closed and finished set but only a potentially infinite process.

This Kantian notion of infinity, interpreted through both empirical reality and transcendental ideality, has achieved great success in overcoming potential problems lurking under both traditional Platonic and Aristotelian notions of infinity. For Plato, infinity can be counted as an actual mathematical entity lying in the transcendent realm of forms beyond experience, but it is seemingly inaccessible for us to know such a transcendent entity [4]. The Kantion interpretation overcomes the inaccessibility problem by replacing the Platonic actuality of infinity with the transcendental ideality of infinity. On the other hand, Aristotle believes that all mathematical objects are only tools abstracted from perceptible and finite objects and exist only potentially [4, 5]. Infinity is no exception. He, along with other ancient mathematicians, noticed that there were certain mathematical calculations or procedures that could be repeated definitely, e.g., bisection of lines. For segments of lines, there are an infinite number of potential places where they can be divided [5]. By contrast, the Kantian notion of empirical reality of infinity, in which infinity is recognized as the precondition of all possible experience rather than a product dependent upon sensory perception, successfully prevents the danger of making mathematics lose its purity, abstractness, and universality, brought about by Aristotle's notion.

W. F. Hegel, as a great successor and opponent of Kant, breaks through the Kantian limit between appearances and the things in themselves and consequently rejects his notion of potential infinity, which hugely relies on this strict limit.

In his Wissenschaft der Logik, Hegel separates "bad infinity" (das schlechte Unendliche), known for the conceptual infinity that metaphysics has been discussing since Aristotle proposed the surrounded infinity, from the infinity that mathematics considers, and gives the latter a meaning that can sublate itself. Hegel's eclectic method is to initiate from infinity, examine its limitations (Schranke), and ought (Sollen) when the specific infinity existed as being in itself [6].

To review Spinoza's concept of infinity (Epistles 12), Hegel conceived that the reality that affects the validity of the infinitum actu is that it can display itself as a simple and direct quantitative infinity in an integer or an infinite circular decimal, which concludes that this infinity is simply defined within the boundaries of letters or numbers, although operations will occur in it. But this infinite synthesis can manifest a potential and inexhaustible number through a limited number of symbols. On the other hand, as a special function with a positive integer set as its domain of definition, the number sequence in quantum has been explored hitherto only in recursion and general terms, which need to specifically describe its boundaries without thereby achieving its potential and the count it should become anyway. As Hegel said, "As long as mathematics discusses the infinite, it will no longer limit itself to the finiteness of its object" [4]. In Hegel's perspective, the essence of mathematics lies not only in the general laws summarized by the induction method under empiricism, such as the Fibonacci number sequence, but also in a certain imagination; that is, the prescribed daseiendes can be embodied in their various proportions, such as the negation of quantity in Kepler's solid geometry, the approximation line of the Cartesian tangent method, etc. All the evidence justifies an unshakable view that if and only if the proportional relationship is continued, the sequence can exist [7].

By this logical analogy, Hegel distinguishes between philosophical knowledge and empirical knowledge. Although knowledge is an important step for gegebenheit to become reliable knowledge, if this demonstrative nature is regarded as an a-priori structure and becomes a basis that can not be reflected through dialectics and rationality, then the infinity of this abstract category will be defined by its characteristic of canceling restrictions, so as to become a finite person limited by itself. Therefore, metaphysical infinity, or potential infinity, is regarded as the quality that is divorced from the relationship between quantity. It always tries to encapsulate something, but it can never be achieved because of its own failure in essence. Therefore, the narrative nature of scientific knowledge is reflected in the fact that it always presupposes the existence of indivisible knowledge. The primary goal of philosophy is to realize this knowledge from the inside or outside. More than randomness, example relations can emerge from this process of epistemology, while the smallest unit of knowledge can also be established through ontology. Eventually, the certainty of concepts can be cancelled through mutual negation, reaching what philosopher Zizek called "tarring with the negative" [7, 8].

Obviously, on the contrary, Leibniz's infinitesimal, according to what Hegel argued against, pays too much attention to philosophy and ignores the consequences of reality. Through this methodology, it seems to be able to grasp the trend of increment and decrement, but its accuracy is not satisfactory. Just like the transformation from natural science to natural philosophy, when people make hot water, they can obtain knowledge about what water is, how to boil water most effectively, and why to drink hot water. These are intuitive questions. But when such experience is accumulated in terms of physical concepts such as specific heat capacity and boiling point, brand-new conditions such as water carrier and atmospheric pressure will also be brought into consideration. In this way, natural science seems to be an endless linear accumulation, and we should nevertheless add whatever concepts are effective. Natural philosophy tries to reduce chaos

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by summing up some segments of universality, whether it is the universal formula familiar to the public or Newton's earliest ideas. Assuming that infinity is regarded as a quantitative entry proportion, its own conceptualization seems to become another more abstract carrier of knowledge, and the legitimacy of the system governing it is not directly related to its existence. Therefore, Leibniz's monadism does not seem to explain the relationship between indivisible points and continuity very well, just like a miscellaneous one can't relate to more in the count. Only "pruning" can give the meaning of "one" as the one associated with more [9, 10].

Therefore, if the beginning of mathematics is a quantity associated with its specifier as a closed totality, then quantification needs to rely on and ignore its own boundaries; that is, it becomes the basis of a quantity. The extension and intension of the being determine whether quantitative analysis needs to be closed in other quantities (or units) or includes them. Imagine the example given by Hegel, "3 times a 4 feet straight line leads to a 12 feet straight line, but 3 feet straight line times 4 feet straight line leads to 12 square feet" [6]. Among them, we can say that 4 feet extend out of 12 square feet, and we can also say that 12 square feet contain 4 feet. The difference is whether the result of arithmetic belongs to the given quantity or a brand-new quantitative result is obtained after the quantity is treated as quality. In this scenario, if we imagine infinity as a simple large or small quantity that has never and will not regard itself as an operational element, infinity itself will always lag behind its realization. In one of Zeno's paradoxes, the example of Achilles chasing a tortoise, infinity is defined as a tortoise that is always a little away from the chaser. The reason is that the tortoise, as the end point, contains all the steps to approach it, but it is like ignoring the unit of the square foot: the foot, so it can't contemplate the quality produced by the interaction of quantity and quantity, such as the existence of the simple proportional relationship between the points and lines stipulated by the extension of the three-dimensional space.

Generally speaking, for Hegel, what is important is never how infinity is perceived, but how infinity is included in all kinds of existing intensional and extensional relations and brought into their own calculable logic for the more macroscopic and developable world. Therefore, we can generally call Hegel's infinity a real infinity, that is to say, the das wahrhafte Unendliche lies in how it shows its quantitative comparative relationship under various special circumstances, rather than as a simple totality, for which the whole is equal to the combination of parts.

3. The Concept of Infinity under Set Theory

Traditionally, the common mathematical definition of infinity is: a conceptual number that is larger than all natural numbers (or larger than the largest natural number). Infinity is not actually a specific number: it represents the concept of having no limitations in magnitude or range. Our concept of infinity was greatly expanded in the nineteenth century, thanks to the great success of set theory in mathematics and logic, particularly when Georg Cantor used set theory to make a crucial contribution to the concept of infinity: that the cardinality of sets of infinity is not identical [4, 1].

The cardinality of a set means the number of elements contained by the set. Usually, people use counting to confirm the cardinality of a set. Cantor, instead, noticed that if every element in set 1 meets a one-to-one correspondence to set 2, then their cardinality must be identical [11, 12].

A and B are equal in cardinality, or become "isomerous" when and only when there is a one-to-one correspondence between A and B [12].

In this way, we can confirm that two sets are equal in their number of elements without knowing their actual cardinal number. This method, which now acts as one of the basic ideas of set theory, is known as bijection. However, instead of using bijection on finite sets, Cantor realised that the method could be used to compare the magnitude between infinite sets. Examples are the natural number set (N), the rational number set (Q), and the real number set (R).

As an example, for N {1, 2, 3, 4,... n}, it can be mapped against the set of square numbers.

0→0, 1→1, 2→4, 3→9, 4→16, 5→25⋯

From the mapping above, it is clear that N and the square number set are equal in cardinal, yet the latter should only be a small part of the former. Isomerous to a part of themselves is the nature of infinite number sets [12].

To prove the difference in magnitude between N and R, Cantor first specified that the cardinality of N is countable (since 1,2,3,4,…,n has a countable n number). He then assumes that the real numbers in (0,1) are countable. By mapping every natural number from 1 to n to every real number between 0 and 1 (the latter is marked as x1, x2, x3,…, xn), Cantor gets the mapping table below:

1 → x1 = 0.1298123⋯

2 → x2 = 0.9348931⋯

3 → x3 = 0.1928312⋯

…

n → xn = 0.a1a2a3a4⋯an⋯

According to his assumption, since all real numbers in (0, 1) are on this table, he shouldn't find any other real numbers besides these. And to prove R is larger than N, what he needs to do is to find a number b that is in (0,1) yet not on the table. An easy method is used by Cantor to find b: add 1 to the first decimal place of x1 and use it as the first decimal place of b. It's obvious that b is not identical to x1. Then add 1 to the second decimal place of x2 and use it as the second decimal place of b. b cannot be identical to x2. By iterating this to the nth decimal place, a number b that is not on the mapping table is found. This proves that the initial assumption is incorrect, which indicates that R is uncountable. Furthermore, R is larger in magnitude compared to N as infinite sets, where Cantor denoted the cardinality of the former as ℵ0 and the latter ℵ1 [12].

This is a revolutionary discovery in the field of infinitas: although both R and N have infinite objects, R has more objects than N. The cardinality of two infinity sets can be different. This discovery acts as a tremendous contribution to the research of infinity: before Cantor, no one had ever conducted a mathematical proof on any of the characteristics of the concept of infinity. Cantor became the first one to attempt to understand certain mathematical aspects of infinity instead of logical speculation, and he made progress. Though this discovery only refers to a limited aspect, this initiative thought of "attempt" itself is quite progressive. However, Cantorian set theory is based on a naive notion of actual infinity: he supposes that all transfinite sets, such as the set of natural numbers and the set of real numbers, exist as closed and finished [5]. On the one hand, for example, for N used in his set theory, N is {1,2,3,…,n}, which stops at n, which assumes that the infinity of N is already closed and finished and will never extend or change. Only through such an assumption, the larger cardinalities of transfinite sets, which are dependent upon the smallest cardinality, ℵ0, be possible and conceivable. On the other hand, there seems to be a huge gap between all the finite and the first infinity, that is, by whatever operation can be in the realm of the finite, the sequence of natural numbers cannot be ended and ℵ0, hence, cannot even be reached at all [5, 13]. Hilbert, as a proponent of Cantor, reforms the Cantorian concept of actual infinity into a much clearer one: a set can be "thought of as finished" "if it is possible without contradiction to think of all its elements as existing together". By turning the notion of actual existence into that of non-contradiction, the concept of infinity as a transfinite set only means a set with the possibility of the totality of its members actually existing together [5]. According to such a view, in the following development of ZFC axiomatic set theory, the axiom of infinity can be stipulated as the basic principle: There exists an infinite set, which, in particular, is the set of all natural numbers. Moreover, with the aid of the axiom of replacement, we can endlessly construct new and larger ordinals, taking any given ordinal until it reaches ℵ0, the cardinality of the power set of the set of all natural numbers. Hence, the axiom of replacement and repeated power sets can retain this process forever [5, 12, 13].

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However, this can be controversial as well. Firstly, Cantor believes such a process cannot last indefinitely and therefore supposes the existence of the so-called "absolutely infinite", by which he means it transcends any possible transfinite sets so that they cannot be reached. It is argued that such a notion of infinity has already slipped into the realm of potential infinity rather than actual infinity. Secondly, the previous problem still lurks behind this process. That is, it is still disputed whether or not the axiom of infinity can be legitimate, that is, whether or not the smallest cardinality of transfinite sets, ℵ0, under the notion of actual infinity can be accessible at all [5, 12, 13].

In an inference through further study of the axiom of extensionality, Quine made an inference: because the extensionality itself manifests the logical identity, the logic infrastructure closes itself with some transcendental truth. But when he could not give up the corresponding relationship between predicate logic and logical truth value, he still relied on the reduction theory from atomism, and he could not correctly realize that set theory represented not only neutral empirical science but also more radical discussions [14].

4. Does the Concept of Infinity in Set Theory Go beyond the Concept of Infinity in German Idealism?

In the 20th century, in a thoroughgoing contrast between the vigorous development of mathematical logic and set theory and the disappearance of German idealism, philosophers and mathematicians were faced with the question of whether set theory really taught us more about infinity than German idealism did. For logicists like Frege and Russell, German idealism has lost all possible ground. They argued that mathematics could be reduced entirely to logic and that mathematical propositions were analytic logical propositions [4]. The establishment of mathematics does not need Kant's a priori intuition and categories, nor does it need Hegel's dialectical logic, but only needs pure logic. In the same way, the property of infinity can be defined logically through set theory.

But this does not mean that German idealism, especially Kant's philosophy of mathematics, has really lost its influence. The mathematical view and the concept of infinity under Kant's transcendental idealism system have been revived in the formalism and intuitionism of the 20th century. Hilbert, as a representative of formalism, affirms part of Kantian idealism; that is, he affirms the positive role of the subject's intuitionistic factor in constructing the concept of infinity. This idea is mainly embodied in the finitistic principle of mathematical reasoning. On the other hand, he accepts the concept of infinite sets under Cantor's set theory, that is, the existence of a closed and complete infinite set, though he restricts this Cantorian notion of actual infinity into the realm of "ideal mathematics" that is compatible with "actual mathematics", so that in his program, the notion of actual infinity with ideality can be compatibly and validly applied to a mathematics governed by finitistic procedure [4]. According to this basic idea, some commentators on Hilbert argue that there is some Hegelian spirit of dialectic under his program. In contrast, Brouwer, as the successor of Kant's philosophy and the founder of intuitionism, on the one hand, directly denies the existence of actual infinity through a Kantian notion of potential infinity, and on the other hand, rejects the pure logical way of understanding infinity in set theory. Under Cantor's system of set theory, the whole of natural numbers can be regarded as a set of natural numbers {0, 1, 2, 3...}. Although Cantor's concept of infinity is a very naive notion of actual infinity, such a set of natural numbers can still be regarded as closed, completed, that is, actual. Brouwer argues that such an actual infinite set is impossible. A potentially infinite set of natural numbers can exist only because the knower, as a subject, can construct it through intuition and understanding [4]. Brouwer abandons Kantian space and uses only time as the a priori forms of intuition, trying to construct the mathematical concept of natural numbers through sequences of time. In his intuitionistic system, the direct result of this move is that mathematical logic and set theory can no longer be the basis of mathematics, and they can no longer have the power to define what infinity is. Based on intuition, only natural numbers can be constructed first, which are constructed absolutely prior to set theory. And the infinite sequence of natural numbers in time is a kind of potential infinity itself, so infinity cannot be closed and completed, but can only be in the process of being constantly generated and constructed [4]. In this way, the actually infinite set of natural numbers under Cantor's set theory is rejected utterly by Brouwer.

In addition to the three main schools, Alain Badiou, a philosopher who accepted the inheritance of Kant and Hegel and is also a greater thinker within the context of set theory, may provide a different perspective. People can only suggest that, in set theory, infinite arithmetic should not be underestimated. As for what Alain Badiou demonstrated, Cantor's intuition of "optimism" led him to use Aleph numbers to try to construct the concept of infinite sets, just like the ambiguity of the mapping of individual truth values in first-order predicate logic. Relying on preset real number lines, Cantor discusses sub-multiplicity, that is, derivatives belonging to direct pure many. Therefore, the infinite of reason is regarded as a universal that can never be touched and used in all language systems. We use the symbol of the universal quantifier, "∀", to express it. When multiplicity is confined to an empty form due to each proper noun, logicians will use the symbol of the existential quantifier, "∃", to define it. The division between universality and existence also symbolises the opposing tradition since Parmenides, from existence to nonexistence, and then uses some attributes that can be shared by the two to mediate to maintain the synchronic reality of being and non-being. At least, Cantor found the simplest way, in terms of "gathering" infinity together to form an infinity that is greater than infinity in counting, which is the so-called absolute infinity [9, 15].

Through Badiou's perspective, tracing back to what Kant remarked on, schema, a transcendental paradigm in some degree, is the relationship between perceptual objects and categories, and grasped synthesis by some kind of linear structure inherent in the senses. As mentioned above, this method has had a great impact on his thinking. It seems that only through backtracking and accumulation, increment and decrement, can an individual grasp something bigger or smaller than the established one in the elapsing of time. The Manifold can be conceived as the absolute infinity of the field because it is the source of knowledge and the foundation and basis of all structures. When people make an overall plan, only Manifold, as an object, has always been in contact with the subject. Although it is so important for Kant, he rarely thought of understanding the arcane manifold from the inner part and at the same time turning it into a multiplicity for different entity-subject duals, or that the sub-multiplicity can never be turned into multiplicity in amorphous essence, vice versa. This is the limitation of Cantor. He cannot ensure infinite computability because infinity still depends on the superposition of the four infinite possibilities with a schematic method. The accumulation of these four also becomes intuition itself. The subsequent Zermelo-Fraenkel axiom system also made many attempts to visualize it. For instance, the substitution axiom ensures that the consistency and overall performance of a set change through the changes of elements on the premise that the hierarchy of the set does not exist. This is crucial for the multiple meanings of elements and sets, in that operations must act as intermediaries to communicate the set as an expression and the element as an object to be expressed. Concerning what a set is, this problem is not more important than how to operate the set. Just like being, set theory is self-evident [15, 16, 17].

Hegel opposed infinite isolation and polarity (Polarität). Sein moves towards its own opposite (Gegenteil) and converts to Nichts. Similarly, ambiguous infinity also needs to abolish its one-sidedness when finiteness is taken as the direct thing. In Badiou's solution, infinite overflow happens to occur on the ambiguous object, not on the structure of metalanguage. As an absolute reality, infinity has the "right above existence" and coexists with various events, while mathematics and ontology are below it, trying to define infinity in every repetition and operation. Only set theory can do that. It has broken up unity and integrity. For the first time, Hegel's hard pursuit of real infinity has reached the public from a rarely visited area with the luculent method. Although Cantor's attempt is not satisfactory, set theory has been competent with the continuous attempts of later generations [9, 18].

In general, Badiou asserts that set theory employs a more specific methodology to ponder infinite concepts at the genesis of idealism. Although it is not benign at the beginning, with continuous refinement, it conspicuously attempts to underpin or even overthrow German classical philosophy, just as Marx did to Hegel, leading us to understand the world with a new dialectics.

5. Conclusions

Following a synthesis of the perspectives of logicists, formalists, and intuitionists and Badiou's comments, and through an analysis of the influences of both German idealism and modern set theory on people's understanding of infinity in the realm of modern mathematics, it is not difficult to find that set theory undoubtedly has made great progress by introducing cardinality, denumerability, and other mathematical concepts on the basis of more advanced mathematical and logical methods, which has hugely broadened people's view on the mathematical properties of infinity. These new discoveries could not even be imagined in the age of German idealism. However, as soon as mathematicians and philosophers try to bring the concept of infinity into the debates about the foundations of mathematics, German idealism remains vibrant and can still be used as a powerful philosophical pillar in these debates or revived in fresh theories as an inspiration and heritage. This is because people have gradually realised the limitations of set theory and some paradoxical and problematic impediments when understanding infinity under set theory. Therefore, it has sufficient reason to believe that German idealism, if possible, can still be a supplement and facilitator for set theory in the realm of human intellectual enterprise of understanding infinity by providing a reasonable philosophical basis.

References

[1] Kant, I., Critique of Pure Reason, Penguin Classics, London, 2007.
[2] Liu, F.J., Kant's Conception of Real Infinity, Philosophical Research, 2020.
[3] Shabel, L., Kant's Philosophy of Mathematics, Stanford Encyclopedia of Philosophy, First published Fri Jul 19, 2013, substantive revision Wed Aug 11, 2021. Received from https://plato.stanford.edu/entries/kant-mathematics/.
[4] Shapiro, S., Thinking about Mathematics: The Philosophy of Mathematics, Oxford University Press, Oxford, 2000.
[5] Shapiro, S., Linnedo, Ø., Actual and Potential Infinity, NOUS, 2019.
[6] Hegel, G. W. F., Die Wissenschaft der Logik, China Remin University Press, Beijing,2019.
[7] Zambrana, R., Hegel's Theory of Intelligibility, The University of Chicago Press, Chicago, 2015.
[8] Zizek, S., Tarrying with the Negative: Kant, Hegel, and the Critique of Ideology, Nanjing University Press, Nanjing, 2015.
[9] Hausman, A., Kahane, H., Tidman, P., Logic and Philosophy: A Modern Introduction, Wadswords Cengage Learning, Boston, 2010.
[10] Why Psychology is "Not a Science": Base on A Hegelian Perspective
[11] Papineau, D., Philosophical Devices, Oxford University Press, Oxford, 2012.
[12] Hao, Z. K., Yang, Y., Set Theory: Exploration to the Concept of Infinity
[13] Eawaran, K., Hájek, A., Mancosu, P., Oppym, G., Infinity, Stanford Encyclopedia of Philosophy, 2021. Received from https://plato.stanford.edu/entries/infinity/.
[14] Wang, H., Beyond Analytic Philosophy, Zhejiang University Press, Hangzhou, 2010.
[15] Badiou, A., Being and Event, Nanjing University Press, Nanjing, 2018.
[16] On Kant's Time Graphics and its Phenomenological Interpretation, Yang, Y. F., Wuhan University, Wuhan, 2004.
[17] Caygill, H., A Kant Dictionary, Wiley-Blackwell, Malden, 1995.
[18] Yan, Z. S., Badiou on Being and Event, Journal of Tsinghua University, 2013.