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Analysis the Nature of Logic: The Distinctions Between Logic and Mathematics

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Abstract: The paper aims to explore the distinctions between logic and mathematics. Logic and mathematics have always been important branches of human knowledge, closely related in many ways and with far-reaching consequences in areas such as science, technology and philosophy. Although logic and mathematics have much in common in terms of necessity, universality, a priori and a high degree of abstraction, leading to the belief that the two fields of study are identical, they are essentially two very different disciplines. While they do appear very similar owing to their universal necessity and independence from temporal and spatial constraints, logic and mathematics are, in essence, two very distinct disciplines. They can be strictly differentiated based on their focus and epistemological perspectives. This paper will first explain the fundamental concepts of mathematics and logic, then delve into the two main differences between logic and mathematics, and finally, point out the limitations in the study of mathematics and logic.

Keywords: Philosophy of Logic, Philosophy of Math, Kant, a priori knowledge

1. Introduction

Logic and mathematics have always been essential branches of human knowledge. They are closely related in many ways, profoundly influencing fields such as science, technology, and philosophy, and forming the foundation of various modern scientific disciplines. Logic, as an independent subject, has a long history, tracing back to ancient Greek philosopher Aristotle, whose work "Organon" is considered the cornerstone of logic. Over time, logic further developed during the medieval period, with important logicians, e.g., William of Ockham and John Duns Scotus emerging. Mathematics, on the other hand, predates logic, with its origins in ancient Egyptian and Babylonian times. Particularly during the ancient Greek period, mathematics experienced significant growth, with renowned mathematicians like Pythagoras and Euclid establishing mathematics as a rigorous discipline. The relationship between logic and mathematics has attracted scholarly attention since the late 19th century. In recent decades, many researchers have delved into the connection between the two fields. Logic plays a foundational role in mathematics, providing a rigorous framework for mathematical reasoning [1]. On the other hand, Carnielli have explored the close relationship between logic and mathematics by comparing the similarities and differences between various logical systems and mathematical structures [2]. Researchers have approached the relationship between logic and mathematics from multiple angles. Smith's work, focuses on the role of logic in mathematical proofs,

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emphasizing the crucial position of logical reasoning in ensuring the rigor and validity of mathematical proofs [3]. Additionally, Rosen has deeply investigated how to employ the methods and ideas of logic to solve problems in mathematics, particularly in areas such as set theory and model theory [4].

In recent years, the development of computer science and artificial intelligence has provided new perspectives for the study of the relationship between logic and mathematics. For instance, Awodey has explored the close connection between logic and computer science through the study of category theory and type theory, as well as its impact on the foundations of mathematics [5]. Simultaneously, Ferreirós has examined the application of Gödel's incompleteness theorems in computational theory, investigating the mutual influence between logic, mathematics, and computation [6]. Due to their similarities, logic and mathematics are often combined as the foundation for research in other scientific fields. They belong to the category of formal sciences and share characteristics such as necessity, universality, a priori nature, and high abstraction. Furthermore, both mathematics and logic are widely used as tools for researching other scientific disciplines, often referred to as instrumental sciences. Their properties and features have many commonalities and similarities, with such a close relationship that their content and methods can be used interchangeably and mutually permeate, leading to the misconception that these two fields of study are the same. As Alfred North Whitehead once said in his book, "Pure mathematics consists entirely of such asseverations as that, if such and such a proposition is true of anything, then such and such another proposition is true of that thing" [7]. Furthermore, proponents of logicism, such as Russell and Frege, argue that mathematical theories, or at least arithmetic theories, can be reduced to logical theories.

The main purpose of this article is to systematically study the differences between logic and mathematics. While they do appear very similar owing to their universal necessity and independence from temporal and spatial constraints, logic and mathematics are, in essence, two very distinct disciplines. They can be strictly differentiated based on their focus and epistemological perspectives. This paper will first explain the fundamental concepts of mathematics and logic, then delve into the two main differences between logic and mathematics, and finally, point out the limitations in the study of mathematics and logic.

2. Basic Descriptions of Logic

In the field of modern logic, scholars typically view logic as the study of reasoning and argumentation, focusing on uncovering the fundamental principles and rules of reasoning to determine the validity of an argument, rather than paying attention to specific facts obtained through empirical observation or experimentation [8]. The essence of logic lies in a priori reasoning and argumentation rules, which transcend the influence of any experience or fact. Basic logical rules encompass a series of specific principles and laws followed during the process of reasoning or argumentation, ultimately leading to valid conclusions [9, 10]. These rules may involve diverse concepts, e.g., the law of non-contradiction, the principle of bivalence, deductive reasoning, inductive reasoning, and abductive reasoning. Notable characteristics of logic include its universality, general necessity, and abstractness. Logical rules can be widely applied to reasoning and argumentation processes in various fields, without being constrained by domain-specific knowledge. Furthermore, the abstract nature of logical rules allows them to be employed in the analysis and regulation of different languages or modes of thinking [9]. As a result, the focus of logic lies in the validity of arguments, rather than the truth or falsity of their content. An effective argument is one where the conclusion can be logically derived from the premises, regardless of whether the premises themselves are true or false. Thus, as a tool for analyzing argument structure, logic is concerned with the form of argumentation rather than its content. This approach highlights the importance of understanding the structure and principles underlying reasoning processes, enabling the evaluation of argument validity across a wide range of disciplines

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and contexts. By emphasizing these formal aspects, logic serves as a foundation for critical thinking and rigorous analysis, promoting the development of sound arguments and fostering intellectual growth.

3. Basic Descriptions of Maths

As a formal and systematic field of study, mathematics is dedicated to studying patterns, structures, and relationships. It uses sophisticated symbolic representations and equations to describe and analyses these concepts, ensuring rigor and precision in mathematics [11]. These symbols and equations are used extensively in mathematics, enabling mathematicians to deal with complex statistical problems and models, such as calculus, probability theory, and topology. Mathematics also involves analyzing various mathematical constructive properties, including numbers, shapes, and functions. Mathematicians can study these mathematical objects to understand how they relate to each other and how they are helpful in practical applications. In addition, mathematics is associated with developing mathematical models, enabling a better understanding of complex natural phenomena such as weather forecasting, biology, astronomy, etc. [12]. The precision and rigor of mathematics are one of its main characteristics. Every argument in mathematics must be based on a clear set of axioms and definitions, with theorems and conclusions drawn through a series of logical deductions. These reasonings are based on the inherent logic of mathematics itself, not on experience or facts. Mathematicians base their research on a set of strict mathematical axioms, and on the proven axioms, they then research other theories. As a result, mathematical argumentation processes are usually thorough and in-depth, with a high degree of credibility and repeatability [11]. In contrast, mathematics is more concerned with argumentative validity than logic. However, it is more concerned with the truth or falsity of propositions, the goal of argument and reasoning in mathematics.

4. Difference on Focus Objects

Logic and mathematics are two very different academic disciplines and differ significantly in their focus objects. Logic focuses on the study of thought and argument, and it is primarily concerned with analyzing and deriving logical principles. Logicians usually represent logical propositions and relations through symbols, such as "$A \wedge B$" or "$A \rightarrow B$". Such symbolic representations allow logicians to deal with abstract concepts without reference to content in order to examine the soundness of arguments and the correctness of reasoning [11]. Logicians also study the relations between logical propositions, such as implication, equivalence, and negation. Through these relations, they can derive new logical propositions and determine whether they are true or false from a purely abstract point of view [11].

In contrast, mathematics usually deals with specific objects and problems. While mathematicians also often use symbols to represent mathematical objects and relations, e.g., numbers, functions, and geometric shapes, each symbolic object contains content. Mathematicians study the properties and relationships of these objects and develop new methods and techniques to analyze and deal with them [13]. For example, mathematicians can use symbols to represent variables and equations to solve complex computational problems in algebra. In geometry, mathematicians can use symbols and graphs to describe and analyze shapes and spatial relationships. These symbolic representations allow mathematicians to work with specific mathematical objects and develop various mathematical tools to solve practical problems.

Wittgenstein believed that mathematics was a linguistic game with its own rules and principles. He argued that mathematical concepts are not derived from logic but from the way in which language is used. He thought that mathematical propositions were not determined by logical necessity but by examples in previous mathematical axioms [11]. This is different from logic, where the necessity of the rules of logical reasoning establishes logical propositions.

Overall, mathematics and logic are different in their focus objects. On the one hand, mathematics is concerned with specific objects. On the other hand, logic focuses on the nonspecific object more abstractly to find the logical principle to ensure validity. Logic is more abstract and concerned primarily with its general form, whereas mathematics deals with specific objects and problems based on axioms. Although both disciplines use symbols to represent concepts and relationships, their symbolic representations and usage are different. In logic, "$\neg(\neg A) \Leftrightarrow A$" no matter what "A" here stands for, the argument must be true due to the logical form, but in mathematics, "A+B=C" each symbol must represent the specific correct content to make this equation to be true. Understanding these differences is essential to gaining insight into the methods and goals of study in both disciplines.

5. Difference on Epistemological View

Although logical and mathematical knowledge is characteristic of a priori knowledge, they are essentially two very different kinds of a priori knowledge. In Kantian philosophy, a priori knowledge is a knowledge that does not depend on experience. It is not acquired from observation and experimentation. However, it is based on rational thought and reasoning, and a priori knowledge can be thought of as the innate capacity of the human mind. Kant argues that although mathematical knowledge and logical knowledge are reliable knowledge acquired through reflection, these two kinds of knowledge belong to the knowledge of a priori analytical judgment and the knowledge of a priori synthetic judgment [14]. Understanding Kant's work is significant for distinguishing logic from mathematics in epistemological terms (seen from Table. 1).

Table 1: Descirptions of logic and maths.

Logic	Mathematics
A prior knowledge	
Analytical Judgement	Synthetical Judement
True by the logic form	True by synthesize different concepts

Logical knowledge is true by virtue of its own sense, that is, a priori analytical judgment, which is a judgment based on the nature of the logical connection between the subject and the predicate, which in its own way specifies the subject, meaning that the predicate is already included in the concept of the subject, determined by the structure of the sentence itself or by the logical relationship between the meanings of the words that make up the sentence. For example, all objects are extended. This is a typical logical knowledge of a priori analytic judgment since the notion of extension is already included in the notion of object. Similarly, when one judges whether an argument is valid or not, one is also making an analytical judgment because the concept of validity already includes that if the premises are true, it is impossible that the conclusion is false. So, the scope of logical knowledge is a priori analytical knowledge.

On the other hand, mathematical knowledge is also acquired through reflection. It is a priori synthetic knowledge in which there is no entailment between the subject and the predicate, which is an addition to the subject of a concept it does not initially have. Kant points out that in mathematical knowledge, it is indeed making judgments that are merely a priori and synthetic, such as the judgment

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that 7 plus 5 equals 12, which is necessarily a priori because it is universally necessary knowledge, meaning that 7 plus 5 must equal 12 and always equals 12 everywhere on earth and. At the same time, such knowledge of arithmetic must be synthetic because the concept of 12 cannot be derived by analyzing the two numbers, 7 and 5, separately [15]. One needs to synthesize the concepts 7, 5, and plus intuitively in order to arrive at such a piece of prior knowledge so that mathematical knowledge belongs to the scope of synthetic prior knowledge. In general, both logical and mathematical knowledge is a priori, but there are still differences in their subdivisions. Logic is a priori analytical knowledge, which uses deductive reasoning to justify conclusions through the intrinsic meaning of a priori logic. At the same time, mathematics is a priori synthetic knowledge, synthesizing different concepts to justify conclusions. As two disciplines of a priori knowledge, logic, and mathematics have very different natures.

6. Limitations & Outlooks

Nonetheless, the present paper's discussion is not without its inherent limitations. While it is feasible to distinguish mathematics and logic to a certain extent based on their conceptual interpretations, the crux of the matter lies in the fact that the essence of both disciplines entails epistemological inquiries within the realm of metaphysics. Delving beyond the surface-level phenomena, our efforts are still centered around deducing the definitions and intrinsic nature of mathematics and logic. However, the aspects of their nature that elude direct human sensory perception are constrained by the limitations of human rationality, rendering them inaccessible for exploration. Consequently, the comprehension of mathematics and logic hinges upon the specific epistemological stance from which conclusions are drawn. In future research, one might further probe the applications of logic and mathematics in other domains, such as computer science and artificial intelligence, in order to derive more persuasive epistemological positions based on the outcomes of these investigations. Moreover, one can examine the interrelationships between various logical systems and mathematical structures, facilitating a deeper understanding of the connections between logic and mathematics. Finally, one can also concentrate on the influence of logic and mathematics on educational practices and scientific research methodologies, thereby offering valuable insights for the advancement of related disciplines.

7. Conclusions

To sum up, although logic and mathematics as formal sciences have some metaphysical similarities, they are still essentially different disciplines with different objects of focus and epistemological perspectives. Logic deals with principles of reasoning and argumentation, focusing on the validity of arguments in a general and abstract way. Mathematics, however, deals with specific objects and problems, using symbols and numbers to analyze and solve complex mathematical problems. In terms of epistemology, logic is a priori analytical knowledge based on the internal logic of thought and the logic of reasoning. In contrast, mathematics is a priori synthetic knowledge that requires synthesizing different concepts to reach conclusions. The idea of a straightforward and crude reduction of mathematics to logic, as logicism does, would erase the essential differences between them. Understanding these essential differences is necessary for gaining insight into the research methods and goals of the two disciplines to build on what is already available for more advanced exploration in their respective fields. Overall, these results offer a guideline for the future studies both in mathematics and logic in two ways. Firstly, there is theoretical development. An in-depth understanding of the differences between logic and mathematics helps to advance the theoretical development of both fields. By clarifying their respective characteristics and uniqueness, the respective paths of development can be better understood, thus contributing to the further development and innovation of both disciplines. Secondly, there is the philosophical exploration that the distinction between logic and mathematics involves issues in metaphysics, epistemology, and other areas of philosophy. The study of these two areas can deepen the understanding of philosophical issues and provide a richer resource for reflection in related fields of study.

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