

Game Study on Morning Peak Road Congestion Problem

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Abstract: The morning peak road congestion problem has always been a hot issue in urban traffic management. Road congestion during the morning peak not only affects the travel efficiency of travelers, but also has a serious negative impact on the urban environment and economic development. This article starts from the perspective of game theory and focuses on the problem of road selection. This article considers government supervision and builds a game model from the two game subjects of the driver and the traffic controller. This article analyzes the relationship between the driver and the driver, the driver and the traffic controller. This paper studies the impact of traffic time, whether the driver investigates in advance, and whether the traffic commander conducts road command on the stable strategy of the driver's road choice game. The research results of this article show that drivers and drivers can achieve Nash equilibrium by choosing different roads, and drivers adopting collaborative strategies during the morning peak period can effectively alleviate road congestion. Under the condition of ensuring the cost of arranging traffic commanders for drivers and the government, drivers can investigate the road in advance and traffic commanders can direct road 2, which can effectively promote the smooth operation of the traffic system. At the same time, this article puts forward targeted policy suggestions, aiming to optimize the allocation of transportation resources, improve road traffic efficiency, reduce morning peak road congestion, and provide scientific reference and decision-making support for urban traffic management.

Keywords: Game theory, morning rush hour, road selection, traffic congestion

1. Introduction

With the acceleration of urbanization and the surge in the number of vehicles, the problem of road congestion during morning peak hours has become increasingly prominent. This phenomenon affects driving efficiency, traffic accident rates, and energy consumption and brings many challenges to urban transportation. The urban road traffic system is a stochastic dynamic system composed of people-vehicles-roads. Among them, the driver is in the dominant position of the system. What a vehicle must consider from its current location to its destination is not just the shortest path, but the best path that takes into account a variety of influencing factors. Therefore, studying driver route selection behavior can provide a theoretical basis for optimizing road transportation systems, and alleviating traffic congestion and many other issues [1]. Drivers need to consider factors such as their own time cost and the choice behavior of other drivers when choosing a road. At the same time, to alleviate the morning peak road congestion problem, the government will also add traffic commanders to maintain road traffic order. Traffic commanders need to consider costs and benefits,

which involve the application of game theory. The purpose of this article's research is to analyze the evolutionary rules of driver's route selection behavior through the method of complete information game and provide theoretical support and decision-making reference for solving the morning peak road congestion problem. In the study of factors affecting drivers' road choices, Qu Keqi proposed a static game analysis of driver's intersection choices. She believed that drivers should investigate road conditions in advance, and the government should also commercialize some public products to encourage passers-by to choose public road transportation [2]. Xiang Hongyang considered the factors of building new roads. He established a complete information dynamic game model and concluded that the path choice among travelers will reach a balanced state under ideal circumstances [3]. Xu Wei and Jia Yuanhua considered road network capacity and car ownership, and they proposed a charging strategy to alleviate traffic congestion [4]. Wang Jing et al. used travel time, early arrival/late delay, and in-car congestion as factors affecting road selection, and obtained the distribution of passengers' starting point train choice related to the distance between the starting point and the end point [5]. Based on the different psychology of drivers when choosing routes, Li Jianmin suggested to drivers that they should obey the traffic commander's instructions and not blindly choose the route according to their wishes [6].

In research on the issue of inducing factors affecting road choice, Li Jianmin influenced the driver's choice of route based on the traffic commander's release of inducing information [6]. Li Zhenlong considered the influence of traffic managers, drivers, and other factors on road choice. He established an evolutionary game model to explore the impact of whether drivers accept induced information on road choice [7]. Gong Yawen used parameters such as traffic volume and capacity to quantify the gaming benefits of two randomly paired travelers on a simple road network, which supported the establishment of practical induction strategies [8]. Cai Shengye took into account the trust of the driver's mobile phone navigation, the driver's attributes, daily travel attributes, and the tendency to change route behavior for analysis. He obtained the travel time of each factor through modeling and believed that the driver should choose a different travel path [9].

The driver's choice of different roads during morning rush hour is a kind of game. Under complete information, each driver knows the decisions of other drivers on the route. The driver maximizes his interests by making decisions on the travel route, that is minimizing his travel time. This is not possible in real life, so to achieve complete information, people need to enable drivers to investigate route conditions before making decisions and discuss whether the government should introduce traffic wardens to alleviate the morning peak traffic congestion problem.

2. Establishment of a Complete Information Game Model

2.1. Model One

2.1.1. Game Model Assumptions

The morning peak road selection problem is a game between the driver and the road. When converting the actual problem into a mathematical model, it is necessary to reduce the changes in some variables. Therefore, based on the road congestion and the driver's choice, the following assumptions are made.

Assumption 1: The actual road selection problem is simplified into the two roads shown in Figure 1. Road 1 is long, but the road is often open. Road 2 is short, but the road is often congested.

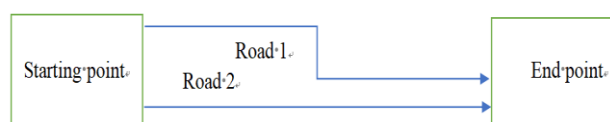


Figure 1: Simplified route map of road selection

Assumption 2: For the convenience of research, this article stipulates that there are two drivers A and B.

Assumption 3: This article considers that in real life, all drivers do not start simultaneously. Therefore, this paper stipulates that driver A chooses the travel route first, and driver B chooses the travel route later.

Assumption 4: In the process from starting point to end point, road 1 is longer than road 2, the travel time of road 1 is T1, the travel time of road 2 is T2, and $T2 < T1$.

Assumption 5: If two drivers A and B choose road 1 at the same time, their travel time is recorded as T3. If two drivers A and B choose road 2 at the same time, their travel time is recorded as T4.

Assumption 6: Drivers' road selection habits remain unchanged.

Based on the above assumptions, Table 1 provides a systematic symbol description.

Table 1: Symbol description

| meaning | symbol |
|---|--------|
| Only one person chooses the travel time of road 1 | T1 |
| Only one person chooses the travel time of road 2 | T2 |
| The driver also selects the travel time of road 1 | T3 |
| The driver also selects the travel time of road 2 | T4 |

2.1.2. Game Model Establishment

According to the analysis, it can be known that drivers A and B have two road choices:

- (1)When driver A chooses road 1, driver B also chooses road 1.
- (2)When driver A chooses road 1, driver B chooses road 2.
- (3)When driver A chooses road 2, driver B also chooses road 2.
- (4)When driver A chooses road 2, driver B chooses road 2.

Based on the results of the above analysis, it can draw the game matrix, as shown in Table 2.

Table 2: Game matrix of drivers A and B

| | | Driver B | |
|----------|--------|----------|--------|
| | | Road 1 | Road 2 |
| Driver A | Road 1 | T3, T3 | T1, T2 |
| | Road 2 | T2, T1 | T4, T4 |

2.1.3. Game Model Solution

According to the above assumptions and the BPR function $t_i = t_{i0} \times \left(1 + \alpha \left(\frac{Q}{C}\right)^\beta\right)$ [3], it can get $T2 < T1 < T4 < T3$ [2]. The results obtained using the scribing method are shown in Table 3.

Table 3: Analysis results of game matrix using line drawing method

| | | Driver B | |
|----------|--------|----------|--------|
| | | Road 1 | Road 2 |
| Driver A | Road 1 | T3, T3 | T1, T2 |
| | Road 2 | T2, T1 | T4, T4 |

2.1.4. Result Analysis

Through the line drawing method, it can get (Road 2, Road 1), (Road 1, Road 2). These two strategies can achieve Nash equilibrium, that is drivers choose different roads and form cooperation to reduce traffic congestion. But according to the actual situation, the driver cannot know in advance what path the driver will choose. Only when Assumption 6 is true, the driver can choose the accessible road based on his or her own experience to achieve equilibrium.

2.2. Model Two

Drivers have two options when making road decisions: one is to make personalized decisions based on the driver's own experience and habits as mentioned above, and the other is to follow the optimal road strategy given by the traffic commander [10]. Therefore, the factor of traffic controller will be introduced as the main body of the game below.

2.2.1. Game Model Assumptions

Assumption 7: The driver's cost of investigating road traffic conditions is C1, and the cost of having a traffic conductor investigate at the intersection is C2.

Assumption 8: The benefit (time saved) brought by the driver by passing the smooth road 1 is P1, and the cost (wasted time) incurred by the driver by passing the congested road 2 is C3.

Assumption 9: The additional income brought to the government by the driver via road 1 is P2, and the additional income brought to the government by the driver via road 2 is P3.

Assumption 10: For road 1, no matter whether the driver investigates the road conditions or not, there will always be a driver choosing road 1. For Road 2, the driver may choose Road 2 only before the investigation. After the investigation, the driver will not choose Road 2 after discovering that the road is congested.

Based on the above assumptions, Table 4 provides a systematic symbol description.

Table 4: Symbol description

| Meaning | symbol |
|--|--------|
| Cost of driver survey of road traffic conditions | C1 |
| The cost of having a traffic warden survey at an intersection | C2 |
| Benefits brought to drivers by traveling on smooth roads 1 (time saved, etc.) | P1 |
| The cost incurred by drivers passing through congested roads 2 (wasted time, etc.) | C3 |
| Additional revenue to the government from drivers passing through Road 1 | P2 |

Table 4: (continued)

| | |
|--|-------|
| Additional revenue to the government from drivers passing through Road 2 | P_3 |
| The driver knows the probability that the road is clear | r |
| The driver knows the probability of road congestion | $1-r$ |
| Probability of driver investigation | p |
| Probability of driver not investigating | $1-p$ |
| Probability that a traffic controller directs road 1 | q |
| The probability that the traffic controller does not direct road 1 | $1-q$ |
| Probability that a traffic controller directs road 2 | s |
| Probability that traffic controller does not direct road 2 | $1-s$ |

2.2.2. Game Model Establishment

Through the above assumptions, it can get the profit matrix of the driver and traffic controller in the road 1 game, as shown in Table 5.

Table 5: Payoff matrix for driver and road 1

| | | Traffic controller | |
|--------|---------------------------|--------------------|-------------------------|
| | | Command(q) | Not commanding($1-q$) |
| Driver | Investigation(p) | P_1-C_1, P_2-C_2 | P_1-C_1, P_2 |
| | No investigation($1-p$) | P_1, P_2-C_2 | P_1, P_2 |

The profit matrix of the driver and traffic controller in the Road 2 game is shown in Table 6.

Table 6: Payoff matrix for driver and road 2

| | | Traffic controller | |
|--------|---------------------------|--------------------|-------------------------|
| | | Command(s) | Not commanding($1-s$) |
| Driver | Investigation(p) | $-C_1, -C_2$ | $-C_1, 0$ |
| | No investigation($1-p$) | $-C_3, P_3-C_2$ | $0, 0$ |

2.2.3. Game Model Solution

Suppose the probability that the driver investigates is p , and the probability that he does not investigate is $1-p$. The probability that the traffic commander directs road 1 is q , and the probability that he does not direct road 1 is $1-q$. The probability that the traffic commander directs road 2 is s , the probability of not directing road 2 is $1-s$.

(1) Solve the profit matrix of the road 1 game between the driver and the traffic controller.

In this article, using the underline method, this paper can get Table 7.

Table 7: Analysis results of income matrix using line drawing method

| | | Traffic controller | |
|--------|-----------------------|--------------------|---------------------|
| | | Command(s) | Not commanding(1-s) |
| Driver | Investigation(p) | P1-C1, P2-C2 | P1-C1, P2 |
| | No investigation(1-p) | P1, P2-C2 | P1, P2 |

(2) Solve the profit matrix of the road 2 game between the driver and the traffic controller.

Based on the above assumptions, the revenue expectation for road 2 is

$$\begin{aligned} EU(\text{Command}) &= p \cdot (-C2) + (1-p) \cdot (P3-C2) = P3-C2-pP3 \\ EU(\text{Not commanding}) &= p \cdot 0 + (1-p) \cdot 0 = 0 \end{aligned} \quad (1)$$

The profit expectation for the driver is

$$\begin{aligned} EU(\text{Investigation}) &= s \cdot (-C1) + (1-s) \cdot (-C1) = -C1 \\ EU(\text{No investigation}) &= s \cdot (-C3) + (1-s) \cdot 0 = -sC3 \end{aligned}$$

2.2.4. Game Result Analysis

In the game between the driver and road 1, this article can reach the Nash equilibrium through the line drawing method (no investigation, no command). That is to say, the driver adopts the non-investigation strategy, and the non-directing strategy for road 1 is the optimal solution.

In the game between the driver and road 2, assuming that $EU(\text{command}) > EU(\text{not command})$, that is related to the cost of a traffic warden directing the road, the probability of driver investigation, and the additional revenue to the government via Road 2. If the cost of the traffic commander directing the road and the additional revenue brought to the government through Road 2 is reduced, and the probability of driver investigation increases, it will be beneficial for the driver to choose a smoother road for travel. Assuming that $EU(\text{investigation}) > EU(\text{no investigation})$, that is. This is related to the cost of the driver's investigation of the road traffic situation, the cost of the investigation of a traffic conductor at the intersection, and the probability of the traffic conductor directing the road 2. If the driver's cost of investigating road traffic conditions decreases, the cost of having a traffic conductor investigate an intersection, and the probability of a traffic conductor directing Road 2 increases, it will be beneficial for the driver to conduct an investigation of the road in advance and save the driver's traffic time. At the same time, the probability of the traffic commander directing Road 2 increases, which can effectively reduce the occurrence of traffic accidents.

3. Conclusion

To sum up, in response to the morning peak road congestion problem, this article discusses the game between drivers and the game between drivers and roads from the perspective of game theory. In Model 1, drivers choose different roads and adopt cooperative strategies to achieve Nash equilibrium. In Model 2, for Road 1, a strategy should be adopted in which the driver does not investigate and Road 1 does not direct. This not only saves the cost of driver investigation and the government's deployment of traffic commanders but also saves the travel time of passers-by. For Road 2, drivers should investigate road conditions in advance, and traffic commanders should actively direct congested roads to achieve a win-win situation.

Research results show that gaming behavior among drivers is one of the important causes of morning peak road congestion. Since each driver pursues his or her own best interests, they often adopt uncooperative behaviors, such as ignoring traffic rules and changing lanes without authorization. These behaviors will lead to more traffic congestion and affect the efficiency of road traffic. Therefore, drivers should investigate the smoothness of the road and choose a travel road that

is beneficial to them, forming a cooperative and win-win traffic attitude. Drivers must abide by traffic rules and traffic commanders' instructions, cooperate with traffic management, and change lanes while ensuring safety to prevent traffic accidents.

On the other hand, the influence between drivers and traffic controllers is also crucial. The traffic management department should strengthen the training and management of traffic commanders and improve the command level and service quality of traffic commanders. Traffic commanders can divert traffic promptly and reduce congestion and traffic accidents.

Finally, the influence between drivers and the government also needs to be taken into account. The government can reduce competition among drivers by formulating relevant policies, such as promoting more flexible working hours and changing driver's prescribed travel time. The government can set toll policies based on vehicle travel time, allowing drivers to investigate road conditions in advance and thereby reduce the toll charged on the road. The government can set up traffic lights, which can help separate vehicles, ease road congestion, and reduce the occurrence of traffic accidents. In addition, the government can also use technological means, such as the use of intelligent transportation systems, to dynamically monitor and control traffic flow to more accurately predict the time of morning peak road congestion, so that drivers can reasonably arrange their travel and reduce the cost of setting up traffic controllers.

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