Integrated study of dark matter and dark energy models

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Abstract. Dark matter and dark energy are used as two important concepts in cosmology to explain some of the observed phenomena in the universe. Dark matter is one of the most dominant constituents of the Universe, and it influences the structural formation of the Universe through gravity, including the formation and evolution of galaxies, clusters, and the large-scale structure of the Universe. Dark energy is believed to be one of the causes of the accelerated expansion of the Universe, and its presence explains the observed phenomenon of the accelerating rate of expansion of the Universe. Although their existence has not been directly observed, people understand through the study of the structure and evolution of the universe that they play an important role in the universe. This paper first introduces the background knowledge of dark matter and its related properties and explains the reasons why three types of models, namely WIMP, axion, and sterile neutrino, are candidates for dark matter in the light of existing observations. The paper then discusses the relevant properties of dark energy and analyses the mainstream dark energy models. For the cosmological constant Λ mode, the fine-tuning problem and cosmic coincidence problem it faces are analysed in detail. The evolution of the dark energy equation of state ω from the past $\omega > -1$ to the present $\omega < -1$ is then explained, and this is used to introduce the scalar field model involving dynamic, the Chaplygin gas model, the holographic dark energy model, and the interacting dark energy model.

Keywords: Dark matter, Dark energy, Model comparison, Non-standard models, Cosmological constant, Scalar fields

1. Introduction

Dark matter and dark energy have been called the "two dark clouds" in the clear sky of modern physics and astronomy in the 21st century. Understanding dark matter and dark energy is one of the great challenges of 21st century science, and their existence is crucial to our understanding of the structure and evolution of the universe. Professors Robert Caldwell and Marc Kamionkowski commented on this issue in Nature: According to observations, the universe is dominated by invisible dark matter and dark energy. The primary task of astronomers and physicists is to reveal the darkness of the universe [1].

Dark matter was first proposed in the first half of the 20th century. The existence of dark matter is a key factor in explaining the formation and movement of the structure of the universe. Although we cannot directly observe the particles of dark matter, some theoretical models (e.g. supersymmetric theories) have proposed some possible candidate particles of dark matter, such as WIMP, axions, and sterile neutrino.

Dark energy was first introduced as a theoretical concept in the late 1990s, when, by observing the luminosity curves of supernova outbursts, scientists discovered that the expansion of the Universe was accelerating, rather than decelerating. To explain this observation, scientists introduced the concept of dark energy. Dark energy is a theoretical form of energy that is thought to be responsible for the accelerating expansion of the universe. However, we have a limited understanding of the nature of dark energy, which may be the energy field that fills the universe or is related to the nature of space-time.

In this paper, we summarise the advantages and limitations of various dark matter models by comprehensively analysing the current research status of dark matter models. For each model, we make a comprehensive comparison in terms of theoretical foundations, observational support, and unresolved issues.

2. Dark matter

The history of dark matter can be traced back as far as 1933, when Swiss scientist F. Zwicky studied the Coma Cluster of Galaxies and found that the galaxies in it were moving much faster than the escape velocity of the cluster and that the cluster could only bind the galaxies when the cluster's mass was more than 100 times the mass of the material being observed. Through this, it was shown that there is a type of matter in the universe that cannot be directly observed, namely dark matter [2]. Subsequent studies of the rotation curves of galaxies in the Great Nebula in Andromeda [3] and observations of gravitational lensing [4] have provided the necessary evidence for this.

As modern observations become progressively more precise and the understanding of dark matter is further deepened, it should have the following basic characteristics: 1) it participates in gravitational interactions, does not participate in strong and electromagnetic interactions, and may be electrically neutral and colour-neutral; 2) it is stable on cosmic timescales; 3) it consists mostly of non-baryonic matter and non-relativistic particles; 4) it can account for the current observations of dark matter residual abundance [5]. Scientists have proposed several candidates based on these properties, with the currently favoured ones being weakly interacting massive particles, axions and sterile neutrinos. The properties, strengths and weaknesses of each of these three models will be discussed below.

2.1. WIMP

To address issues such as the hierarchical specification in the Standard Model, scientists have proposed new physics models in which supersymmetry offers the possibility of dark matter candidates. Supersymmetry is a symmetry between fermions and bosons, as having different R-parity results in non-transformability between particles and their sparticles. WIMP (Weakly Interacting Massive Particles) is the lightest of these sparticles [6], and is a linear superposition of quantum states of the sparticle of the Higgs boson, the sparticle of the photon, and the sparticle of the Z intermediate boson.

WIMP does not participate in electromagnetic and strong interactions and therefore belongs to the category of dark matter. Its high mass, relatively small velocity and easy aggregation are consistent with large-scale observations of the Universe, and WIMP can explain the formation and evolution of the observed large-scale structure of the Universe in a way that is consistent with the understanding of the formation and early evolution of the Universe. For example, in the ΛCDM cosmological model, the residual abundance of WIMP dark matter is consistent with the dark matter density observed today. Standard calculations of the residual abundance of dark matter show that WIMP particles naturally satisfy the astronomical observation that $\Omega_{DM}h^2 \approx 0.11$ [7].

It should be noted that this model describes the 1Mpc to 1Gpc scale structure well, but at scales below 10Kpc its predictions differ significantly from experimental observations, which is a drawback of the model.

2.2. Axion

Axions were created to explain the problem of strong CP violation problem. CP describes a composite symmetry, namely the parity and charge conjugation symmetry. Some terms allow the CP to be broken under strong interactions, but the fact that the CP is conserved in strong interactions leads to formulas that are not natural, and to make them natural in a theoretical framework, R. D. Peccei and H. R. Quinn

Peccei introduced the new Goldstone boson to make the term strictly zero, and this boson is the axion, whose mass can be given as QCD given by the instanton effect:

$$V(a) = \Lambda_{QCD}^{4}[1 - cos(a/f_a)] \tag{1}$$

Where a refers to the axion field and f_a is the PQ symmetry breaking scale.
For axion interactions, one can couple them to the axial vector flow of fermions in derivative form:

$$\partial_\mu (a/f_a)\bar{\Psi}\gamma^\mu\gamma^5\Psi \tag{2}$$

It follows that the mass of the axion is very small, with a scale of $1\mu eV$, and that its interaction with other matter is extremely weak. This makes it a candidate for dark matter. Theoretical calculations show that an axion with a mass of $10^{-5}eV$ can give the dark matter density required by the universe when f_a is 10^11 GeV:

$$\Omega_a h^2 \approx (0.04 - 0.3)\left(\frac{f_a}{10^{11}Gev}\right)^{\frac{7}{6}} \tag{3}$$

However, the shortcoming is that the interactions are too small to be in thermal equilibrium and do not explain the abundance well. More discussion on axions can be found in the dissertation [8-10].

2.3. Sterile Neutrino

Once neutrinos were taken as candidates for dark matter because they are uncharged, stable, and have no strong interactions; however, the upper limit of their total mass is $\sum m_v \lesssim 0.23eV$ from observations such as WMAP-9, 6df LSS, etc., and their contribution to the cosmic density parameter is only$\Omega_v <$ 0.005. Therefore, neutrinos have become the main component of dark matter for the loser.

In the new physics model, Dodelson and Widrow proposed sterile neutrinos, which are single neutral fermions often produced by neutrino oscillations [11,12]. The hotspot of dark matter modelling is the KeV sterile neutrino, which can decay to three ordinary neutrinos with a corresponding lifetime [13] of:

$$\tau \simeq 2.88 \times 10^{27}\left(\frac{M_s}{1KeV}\right)^{-5}\left(\frac{\theta^2}{10^{-8}}\right)^{-1} sec \tag{4}$$

Where M_s is the mass of the KeV sterile neutrino and θ is its mixing angle with ordinary neutrinos. It can be found that the lifetime of the KeV sterile neutrino is much larger than the age of the universe($\sim 10^{27}sec$). Therefore, it is relatively stable on the time scale of the age of the universe, and because it does not participate in all interactions except gravity, KeV sterile neutrino are taken as one of the candidates for dark matter.

The current detection of sterile neutrino is not ideal, and although current neutrino oscillations may occur, the probability that a neutrino will normally become a sterile neutrino through standard oscillations in space is considered to be very small, $(10^{-5} \leq p \leq 10^{-10})$. More knowledge about sterile neutrino can be found in dissertation [14-16].

3. Dark energy

Before the discovery of dark energy, according to general relativity, the universe should be in a state of decelerated expansion when only matter and radiation are present. However, in 1998, two supernova observation teams, HZT (High z Search Team) and SCP (Supernovae Cosmology Project), discovered that supernovae are fainter than expected [17], i.e., they are farther away than originally predicted by the standard cold dark matter, suggesting that the universe is in an accelerated state of expansion. This power, which can transcend the gravitational pull between matter, is known as dark energy. Subsequently, through observations of the Cosmic Microwave Background Radiation (CMBR), scientists have found that dark matter, baryonic matter, and other known constituents make up only about 27% of the universe, leaving a 73% gap that offers the possibility of dark energy.

Based on the Friedmann equations [18]:

$$\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=\frac{8\pi G}{3}\rho \tag{5}$$

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3P) \tag{6}$$

From this it follows that:

$$P<-\frac{1}{3}\rho \tag{7}$$

Where a is the cosmic scale factor, G is the gravitational constant, P and ρ are respectively the pressure and density, and the $\frac{\dot{a}}{a}$ term is essentially the Hubble constant $H_0(2.13h\times 10^{-42}\mathrm{Gev})$. Because of the accelerated expansion of the universe, the acceleration is positive, and $\ddot{a}>0$. Friedmann equations' results suggest that dark energy has a negative pressure and exists in a different form than before.

It has been found that it is characterised by 1) Its spatial distribution is approximately uniform at large scale structure of the Universe, and it does not accumulate into observable matter at the scale of galaxy clusters; 2) It does not participate in electromagnetic interactions, and it does not emit nor absorb photons; 3) It has a considerable negative pressure; 4) Its density does not change with the evolution of the universe. For these properties, the mainstream ones are the cosmological constant Λ model, the scalar field model, the Chaplygin gas model, the holographic dark energy model, and the interacting dark energy model. Each of them will be discussed below.

3.1. Cosmological Constant Λ Model

The cosmological constant Λ was first introduced in the 1910s, when the existence of dark energy was not recognised. According to the original Einstein field equations, the universe collapses or decelerates its expansion under the influence of gravity, which contradicts the prevailing view of the universe as being static, so to obtain a static solution to Einstein's field equations Einstein introduced cosmological constant Λ, which has the action of a repulsive force, whereupon the Einstein field equations were supplemented with:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}=8\pi GT_{\mu\nu} \tag{8}$$

Using equation (8), solving in the context of the Friedmann-Robertson-Walker metric spacetime allows equations (5) and (6) to be improved as:

$$\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=\frac{8\pi G}{3}\rho+\frac{\Lambda}{3} \tag{9}$$

$$\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3P)+\frac{\Lambda}{3} \tag{10}$$

It can be seen that a large negative pressure can be obtained when the cosmological constant Λ is large enough, and thus cosmological constant Λ model has become one of the mainstream models of dark energy.

It should be pointed out that although the cosmological constant Λ is in good agreement with the current observational data, there are still some problems with it, such as the fine-tuning [19] problem and cosmic coincidence problem [20]. These two issues are discussed below:

After WMAP observations the value of the cosmological constant Λ is H_0^2,so from the Einstein field equation (8) the vacuum energy density is:

$$\rho_\Lambda=\frac{\Lambda}{8\pi G}\approx 10^{-47}Gev^4 \tag{11}$$

However, a zero-point energy with mass m in a quantum field has a vacuum energy density estimated at the Planck scale of:

$$\rho_\Lambda\approx 10^{74}Gev^4 \tag{12}$$

It can be seen that the theoretical value differs from the observed value by 121 orders of magnitude, and so the cosmological constant Λ needs to be fine-tuned to 121 bits of accuracy, which is the fine-tuning problem.

Applying Hubble's law, substituting H_0 instead of $\frac{\dot{a}}{a}$ and deforming equation (9) leads to:

$$\frac{8\pi G}{3{H_0}^2}\rho_0 + \frac{\Lambda}{3{H_0}^2} - \frac{k}{{H_0}^2{R_0}^2} = 1 \tag{13}$$

The individual meanings and observations are:

$$Material\ density\ parameter: \Omega_m = \frac{8\pi G\rho_0}{3{H_0}^2} = 0.27 \pm 0.02 \tag{14}$$

$$Vacuum\ density\ parameter: \Omega_\Lambda = \frac{\Lambda}{3{H_0}^2} = 0.73 \pm 0.02 \tag{15}$$

$$Spatial\ curvature\ density\ parameter: \Omega_k = \frac{k}{{H_0}^2{R_0}^2} = 0 \tag{16}$$

It can be seen that Ω_Λ is a constant, while Ω_m is proportional to the density of matter $\Omega_m \propto p_0 \propto a^{-3}$, and as the Universe skyrockets the value of Ω_m decreases. It can be surmised that in the early universe, the scale factor a was very small, when $\Omega_\Lambda \ll \Omega_m$, but nowadays it happens that $\Omega_m = 0.27$ has the same magnitude as $\Omega_\Lambda = 0.73$. To ensure today's coincidence, it is required that the ratio of the two in the early universe should be set to a very small value precisely, and the small deviation will lead to the universe to develop into a completely different way from the present one. This is the cosmic coincidence problem.

3.2. *Scalar Field Model*

The scalar field model is derived from the kinetic observation of the dark energy equation of stateω. According to the Einstein field equation (8), the vacuum energy pressure can be obtained as:

$$p_\Lambda = -\frac{\Lambda}{8\pi G} \tag{17}$$

According to the equation of state $p = \omega\rho$, combining equation (17) with equation (11) leads to $\omega = -1$, this is obtained in the cosmological constant Λ model. However, according to observations, ω is allowed to vary with time, and current data observations show that ω evolves in a small interval around -1. In order to achieve such a dynamical evolution, scientists have proposed that there exists a scalar field model ϕ that varies with time in the energy-momentum tensor model, and with this, many models of dark energy have been born, such as the Quintessence model, the Phantom model, the K- essence model, Quintom model, and so on.

The Quintessence model [21,22] is represented by a scalar field that slowly rolls down to its base state at the bottom of the potential, which is a regular scalar field, coupled only to the gravitational field, with a dark energy equation of state $-1 \le \omega \le 1$.

The Phantom model [23] has a negative kinetic energy term for the scalar field and achieves $\omega \le -1$.

The K-essence model [24] is a scalar field with a non-regular kinetic energy term and has a dark energy equation of state of $\omega \ge -1$ or $\omega \le -1$.

One defect of the above models is that ω cannot cross -1.And according to the observation data of Type Ia supernovae by Huterer [25] and Riess [26] and so on, it can be concluded that ω evolved from $\omega > -1$ in the past to $\omega < -1$ at present with the time advancement of the Universe.To be able to describe this phenomenon better, it was firstly by the scientists, such as Feng Bo, combined the Quintessence Model with the Phantom model to give birth to the Quintom model [27], which is discussed below:

Quintom has a Lagrangian of:

$$L_\phi = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 - \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - V(\phi_1, \phi_2) \tag{18}$$

Where the ϕ_1 field representing the positive kinetic energy $\frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1$ term is the Quintessence field and the ϕ_2 field representing the negative kinetic energy term $-\frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2$ is the Phantom field, and $V(\phi_1,\phi_2)$ is the interaction potential of the two fields, from which one can obtain the energies and densities [28] in the flat FRW universe as:

$$\rho = \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 + V(\phi_1,\phi_2) \tag{19}$$

$$p = \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 - V(\phi_1,\phi_2) \tag{20}$$

Therefore, its dark energy equation of state can be written as:

$$\omega = \frac{p}{\rho} = \frac{\dot{\phi}_1^2 - \dot{\phi}_2^2 - 2V(\phi_1,\phi_2)}{\dot{\phi}_1^2 - \dot{\phi}_2^2 + 2V(\phi_1,\phi_2)} \tag{21}$$

From this equation, it can be seen that $\omega > -1$ when $\dot{\phi}_1^2 > \dot{\phi}_2^2$ and $\omega < -1$ when $\dot{\phi}_1^2 < \dot{\phi}_2^2$. The dark energy equation of state is realised from past $\omega > -1$ to the present $\omega < -1$. by the reciprocal evolution of the two fields.

It should be mentioned that although the scalar field model can explain the cosmic coincidence problem [29] by its dynamical evolution, there is still a fine-tuning problem. In addition to the Quintom model, the Hessence model and the Hantom model can also accomplish the -1 crossing, which is not discussed here, see dissertation [30,31].

3.3. Chaplygin Gas Model

The Chaplygin gas model [32,33] was originally derived from the realm of aerodynamics, it was proposed by the scientist Chaplygin in 1904 to study the lift of aircraft wings in the air. It was later used as one of the candidates for dark energy because its properties in a cosmological setting were more consistent with dark energy.

As an ideal gas its equation of state is:

$$p = -\frac{A}{\rho} \tag{22}$$

Bringing this into the continuity equation $\dot{\rho} + 3H(p + \rho) = 0$ leads to:

$$\rho = \sqrt{A + \frac{B}{a^6}} \tag{23}$$

Where B is the constant of integration.

When a^6≪B/A equation (23) can be approximated as:

$$\rho \sim \frac{\sqrt{B}}{a^3} \tag{24}$$

This is consistent with the dominance of matter in the early Universe when the a scale factor was small.

When a^6≫B/A equation (23) can be approximated as:

$$\rho \sim \sqrt{A}, \rho \sim -\sqrt{A} \tag{25}$$

This suggests that it has a negative pressure that provides the impetus for the accelerated expansion of the universe, which is similar to the cosmological constant Λ.

Thus, in the early universe, Chaplygin Gas behaves like non-relativistic matter due to the small cosmic scale factor a, while in the late universe it behaves like the cosmological constant Λ, leading to an accelerated expansion of the universe, and to a certain extent providing the possibility of an interaction between dark matter and dark energy. It is regarded as a unified model of dark matter and dark energy [34].

It should be noted that the original Chaplygin model did not match the experimental data, so scientists improved it to obtain the GCG (generalised Chaplygin gas) model [35] and the NGCG (new generalised

Chaplygin gas) model [36]. Their interaction terms can be regarded as interacting dark energy models. Its interaction term is:

$$Q \propto \frac{\rho_{de}\rho_c}{\rho_{de}+\rho_c} \quad (26)$$

Where ρ_{de} and ρ_c respectively represent the energy densities of dark energy and dark matter [37].

3.4. Holographic Dark Energy Model

The holographic principle [38] is one of the basic principles of quantum gravity, which is expressed as "the full information of a gravitational system that is in a region Ω can somehow be stored on its boundary $\partial\Omega$, and the theory on the boundary can be gravitationally free". To compensate for the lack of quantum gravity theory in dark energy, Fischler and Susskind first attempted to apply this principle to cosmolog [39], followed by Cohen et al. who proposed the holographic dark energy model [40], stating that the establishment of a local gravitational field theory requires that gravitational effects can be neglected and in particular black hole formation should be avoided, and thus the energy within a region of scale L should be no more than the equivalent scale black hole energy, which then requires that the zero-point energy in the field theory can have:

$$L^3 \rho \Lambda \leq L M_P{}^2 \quad (27)$$

Where L refers to the size of the entire physical system, also known as the IR cutoff scale. In cosmology it refers to a certain horizon. M_P refers to the Planck scale.

If the zero-point energy is determined by the UV cutoff Λ, there is then a UV-IR relation:

$$\Lambda^4 \approx m_P{}^2 L^{-2} \quad (28)$$

When the holographic principle is fulfilled, the dark energy density [41] is:

$$\rho_\Lambda = 3c^2 M_P{}^2 L^{-2} \quad (29)$$

Different holographic dark energy models can be obtained by selecting different L. The popular ones are HDE (holographic dark energy) model [42], NADE (new agegraphic dark energy) model [43], RDE (Ricci dark energy) model [44] and so on. The HDE model is introduced below:

Let L be taken as:

$$L = R_h = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2} \quad (30)$$

Bringing this into the dark energy density equation (29) gives:

$$\rho_\Lambda = 3c^2 M_P{}^2 R_h{}^{-2} = 3a^2 M_P{}^2 a^{-2\left(1+\frac{1}{c}\right)} \quad (31)$$

This can further lead to the dark energy equation of state:

$$\omega = -\frac{1}{3} - \frac{2}{3c} < -\sqrt{\Omega_\Lambda} \quad (32)$$

It follows that there is only one covariate in the dark energy equation of state, c. The value of c can be determined from observational data, thus determining the evolution of the equation and the final fate of the universe. If $c = 1$,the dark energy behaves similarly to the cosmological constant Λ; if $c > 1$,the dark energy has $\omega > -1$, like the quintessence model; if $c < 1$,the dark energy exhibits a traversing behaviour from $\omega > -1$ to $\omega < -1$, similar to the quintom model.

Although the holographic dark energy model can solve the cosmic coincidence problem similarly like the scalar field model, because its event horizon exists only in the accelerating expanding universe, its existence requires the universe to continue to expand at an accelerating rate in the future, which may ultimately lead to the Big Rip [45,46].

3.5. Interacting Dark Energy Model

To solve the cosmic coincidence problem and to explain the evolution of the dark energy equation of state from the past $\omega > -1$ to the current $\omega < -1$ behaviour. Unlike the previous models describing only the interaction between dark energy and gravity, scientists have tried to introduce an interacting dark energy model, firstly the interaction between scalar fields and matter was proposed [47], where Amendola's group proposed the Quintessence model coupled with the interaction model of dark matter

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[48]. Subsequently Szydlowskl [49] assumed the existence of energy exchange between dark matter and dark energy and proposed a test [50], in the following, we discuss this model.

An ideal fluid with energy density ρ fulfils the conservation equation:

$$\dot{\rho} + 3(1 + w)H\rho = 0 \tag{33}$$

In the absence of energy exchange, dark matter and dark energy are respectively denoted as:

$$\dot{\rho_m} + 3H\rho_m = 0 \tag{34}$$

$$\dot{\rho_x} + 3(1 + w_x)H\rho_x = 0 \tag{35}$$

In interacting dark energy systems, equations (34) and (35) can be improved by the introduction of the energy exchange term Q leading to:

$$\dot{\rho_m} + 3H\rho_m = Q \tag{36}$$

$$\dot{\rho_x} + 3(1 + w_x)H\rho_x = -Q \tag{37}$$

Where Q can take different forms, in which case, as the universe evolves beyond a certain stage, dark matter and dark energy have a fixed ratio, which also implies the existence of attractor solutions for this system. It is due to the introduction of the exchange energy Q. When a specific relation is taken, the dark energy equation of state can traverse -1 during the evolution of the universe, in addition to this, some interacting dark energy models can also describe the nature of the dark energy equation of state parameter that traverses -1, as can be seen in the dissertation [51,52].

4. Conclusion

This paper starts with the nature of dark matter and dark energy and analyses their hotspot models comprehensively. Each model agrees with experimental data to different degrees, but at the same time it has defects.

The non-participation in strong and electromagnetic interactions is the most fundamental condition for being dark matter, and at the same time a constraint for its detection and confirmation. This has led to the fact that particles such as sterile neutrino, which are a good description of dark matter, have not been experimentally confirmed so far. Another example is the axion, whose interactions are too small to explain the abundance problem well, and the WIMP model, although one of the most popular models, suffers from large discrepancies with experimental results at small scales.

The dark energy equation of state ω is an important parameter for calibration to distinguish between different dark energy models, and the value of ω affects the evolution of the universe and the nature of the dark energy, the cosmological constant Λ can be rigorously defined at -1, but there is a cosmic coincidence problem. Scalar field models, although they can simulate the evolution of ω, are unstable at -1, like the Quintessence and Phantom models, and even the Quintom model, although it can span -1, still suffers from fine-tuning problem. To alleviate this problem scientists have proposed the Chaplygin gas model, the holographic dark energy model, the interacting dark energy model and so on.

Therefore, future research could further explore different theoretical models, improve experimental methods, and enhance the precision of observations to gain deeper insights into dark matter and dark energy. In addition, the development and refinement of the fundamental theories of the universe should be strengthened, and if the nature of dark matter and dark energy is understood, it will certainly trigger a revolution in cosmology and physics.

References

[1] Caldwell, R., & Kamionkowski, M. (2009). Dark matter and dark energy. Nature, 458(7238), 587-589.

[2] Zwicky, F. (1933). Die rotverschiebung von extragalaktischen nebeln. Helvetica Physica Acta, Vol. 6, p. 110-127, 6, 110-127.

[3] Rubin, V. C., & Ford Jr, W. K. (1970). Rotation of the Andromeda nebula from a spectroscopic survey of emission regions. Astrophysical Journal, vol. 159, p. 379, 159, 379.

[4] He, C. (1994). Gravitational lensing measurement of dark matter. Modern Physics Knowledge, (5), 26-29.

[5] Arcadi, G., Dutra, M., Ghosh, P., Lindner, M., Mambrini, Y., Pierre, M., ... & Queiroz, F. S. (2018). The waning of the WIMP? A review of models, searches, and constraints. The European Physical Journal C, 78, 1-57.
[6] Kolb, E. W., & Turner, M. S. (1990). The early universe addison-wesley. Redwood City, 69, 1-547.
[7] He, Y., & Lin, W. B. (2016). Research progress of WIMP dark matter model. Astronomy Progress, 34(3), 287-311.
[8] Kim, J. E., & Carosi, G. (2010). Axions and the strong C P problem. Reviews of Modern Physics, 82(1), 557.
[9] Hook, A. (2019). TASI lectures on the strong CP problem and axions, PoS. arXiv preprint arXiv:1812.02669, 4.
[10] Adams, C. B., Aggarwal, N., Agrawal, A., Balafendiev, R., Bartram, C., Baryakhtar, M., ... & Wooten, M. (2022). Axion dark matter. arXiv preprint arXiv:2203.14923.
[11] Dodelson, S., & Widrow, L. M. (1994). Sterile neutrinos as dark matter. Physical Review Letters, 72(1), 17.
[12] Shi, X., & Fuller, G. M. (1999). New dark matter candidate: Nonthermal sterile neutrinos. Physical Review Letters, 82(14), 2832.
[13] Li, Y. F. (2015). Sterile neutrino. Modern Physics Knowledge, 6.
[14] Adhikari, R., Agostini, M., Ky, N. A., Araki, T., Archidiacono, M., Bahr, M., ... & Zuber, K. (2017). A white paper on keV sterile neutrino dark matter. Journal of cosmology and astroparticle physics, 2017(01), 025-025.
[15] Kusenko, A. (2009). Sterile neutrinos: the dark side of the light fermions. Physics Reports, 481(1-2), 1-28.
[16] Boyarsky, A., Drewes, M., Lasserre, T., Mertens, S., & Ruchayskiy, O. (2019). Sterile neutrino dark matter. Progress in Particle and Nuclear Physics, 104, 1-45.
[17] Riess, A. G., Filippenko, A. V., Challis, P., Clocchiatti, A., Diercks, A., Garnavich, P. M., ... & Tonry, J. (1998). Observational evidence from supernovae for an accelerating universe and a cosmological constant. The astronomical journal, 116(3), 1009.
[18] Kolb, E. W., & Turner, M. S. (1990). The early universe addison-wesley. Redwood City, 69, 1-547.
[19] Weinberg, S. (1989). The cosmological constant problem. Reviews of modern physics, 61(1), 1.
[20] Carroll, S. M. (2001). The cosmological constant. Living reviews in relativity, 4(1), 1-56.
[21] Ratra, B., & Peebles, P. J. (1988). Cosmological consequences of a rolling homogeneous scalar field. Physical Review D, 37(12), 3406.
[22] Carroll, S. M. (1998). Quintessence and the rest of the world: suppressing long-range interactions. Physical Review Letters, 81(15), 3067.
[23] Caldwell, R. R. (2002). A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of state. Physics Letters B, 545(1-2), 23-29.
[24] Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. (2001). Essentials of k-essence. Physical Review D, 63(10), 103510.
[25] Huterer, D., & Cooray, A. (2005). Uncorrelated estimates of dark energy evolution. Physical Review D, 71(2), 023506.
[26] Riess, A. G., Strolger, L. G., Casertano, S., Ferguson, H. C., Mobasher, B., Gold, B., ... & Stern, D. (2007). New Hubble space telescope discoveries of type Ia supernovae at $z \geq 1$: narrowing constraints on the early behavior of dark energy. The Astrophysical Journal, 659(1), 98.
[27] Feng, B., Wang, X., & Zhang, X. (2005). Dark energy constraints from the cosmic age and supernova. Physics Letters B, 607(1-2), 35-41.
[28] Guo, Z. K., Piao, Y. S., Zhang, X., & Zhang, Y. Z. (2005). Cosmological evolution of a quintom model of dark energy. Physics Letters B, 608(3-4), 177-182.
[29] Copeland, E. J., Sami, M., & Tsujikawa, S. (2006). Dynamics of dark energy. International Journal of Modern Physics D, 15(11), 1753-1935.

DOI: 10.54254/2753-8818/34/20241173

[30] Wei, H., Cai, R. G., & Zeng, D. F. (2005). Hessence: a new view of quintom dark energy. Classical and Quantum Gravity, 22(16), 3189.
[31] Zhao, W., & Zhang, Y. (2006). Quintom models with an equation of state crossing− 1. Physical Review D, 73(12), 123509.
[32] Kamenshchik, A. Y., Moschella, U., & Pasquier, V. (2002). it Phys. Lett. B 511, 265 (2001); MC Bento, O. Bertolami, AA Sen. Phys. Rev. D, 66, 043507.
[33] Bilić, N., Tupper, G. B., & Viollier, R. D. (2002). Unification of dark matter and dark energy: the inhomogeneous Chaplygin gas. Physics Letters B, 535(1-4), 17-21.
[34] Bento, M. C., Bertolami, O., & Sen, A. A. (2002). Generalized Chaplygin gas as a scheme for Unification of Dark Energy and Dark Matter. arXiv preprint astro-ph/0210375.
[35] Bento, M. C., Bertolami, O., & Sen, A. A. (2002). Generalized Chaplygin gas, accelerated expansion, and dark-energy-matter unification. Physical Review D, 66(4), 043507.
[36] Zhang, X., Wu, F. Q., & Zhang, J. (2006). New generalized Chaplygin gas as a scheme for unification of dark energy and dark matter. Journal of Cosmology and Astroparticle Physics, 2006(01), 003.
[37] Li, Y. H., & Zhang, X. (2014). Large-scale stable interacting dark energy model: Cosmological perturbations and observational constraints. Physical Review D, 89(8), 083009.
[38] Bousso, R. (2002). The holographic principle. Reviews of Modern Physics, 74(3), 825.
[39] Fischler, W., & Susskind, L. (1998). Holography and cosmology. arXiv preprint hep-th/9806039.
[40] Cohen, A. G., Kaplan, D. B., & Nelson, A. E. (1999). Effective field theory, black holes, and the cosmological constant. Physical Review Letters, 82(25), 4971.
[41] Cohen, A. G., Kaplan, D. B., & Nelson, A. E. (1999). Effective field theory, black holes, and the cosmological constant. Physical Review Letters, 82(25), 4971.
[42] Li, M. (2004). A model of holographic dark energy. Physics Letters B, 603(1-2), 1-5.
[43] Wei, H., & Cai, R. G. (2008). A new model of agegraphic dark energy. Physics Letters B, 660(3), 113-117.
[44] Gao, C., Wu, F., Chen, X., & Shen, Y. G. (2009). Holographic dark energy model from Ricci scalar curvature. Physical Review D, 79(4), 043511.
[45] Simpson, F. (2007). An alternative approach to holographic dark energy. Journal of Cosmology and Astroparticle Physics, 2007(03), 016.
[46] Zhang, J., Zhang, X., & Liu, H. (2007). Holographic dark energy in a cyclic universe. The European Physical Journal C, 52, 693-699.
[47] Ellis, J., Kalara, S., Olive, K. A., & Wetterich, C. (1989). Density-dependent couplings and astrophysical bounds on light scalar particles. Physics Letters B, 228(2), 264-272.
[48] Amendola, L. (2000). Coupled quintessence. Physical Review D, 62(4), 043511.
[49] Szydłowski, M. (2006). Cosmological model with energy transfer. Physics Letters B, 632(1), 1-5.
[50] Szydłowski, M., Stachowiak, T., & Wojtak, R. (2006). Towards testing interacting cosmology by distant supernovae. Physical Review D, 73(6), 063516.
[51] Damour, T., & Polyakov, A. M. (1994). The string dilation and a least coupling principle. Nuclear Physics B, 423(2-3), 532-558.
[52] Zhang, X., Wu, F. Q., & Zhang, J. (2006). New generalized Chaplygin gas as a scheme for unification of dark energy and dark matter. Journal of Cosmology and Astroparticle Physics, 2006(01), 003.