

Dynamics of dispersal: Modeling the impact of dandelion proliferation on environmental and economic equilibria

Leyi Jiang^{1,5,†}, Mingjun Ouyang^{2,†}, Weiyu Qi^{3,†}, Yueyue Yu^{4,†}

¹High School Affiliated to Remin University of China

²Keystone Academy

³Beijing No.80 High School

⁴Cogdel Cranleigh School Changsha

⁵angele_susan@126.com

[†]All authors equally contributed to the research. In alphabetical order by last name.

Abstract. Dandelion, with its unique wind dispersal mechanism, can easily spread its seeds across vast land areas, encroaching on territories of other creatures. To address this issue, two models are considered: The first model considers wind patterns, temperature, and moisture in dandelion growth, using differential equations to analyze population changes over time. Sensitivity analysis reveals the model's responsiveness to these factors. The second model combines ecological and economic factors to assess the impact of invasive species, employing fuzzy mathematics to determine the severity of impact. The study identifies Canada Goldenrod as particularly destructive. These models offer insights for future invasive species management policies, showcasing innovations in incorporating partial differential functions and fuzzy mathematics for more accurate estimations.

Keywords: Partial Differential Equation, Numerical Simulation, Iterative Algorithm, Differential Equation, Fuzzy Mathematics, Analytic Hierarchy Process

1. Introduction

Dandelion is a common perennial herb, whose seeds can fly long distances with the airflow attached to a parachute-like structure. Therefore, it can spread to further areas and crowd out many local species. The spread mechanism of dandelions and the economic and environmental impacts of invasive species in new areas are studied [1].

Klemen et al studied the simulated average monthly air temperature to the flowering of the dandelion [2]. Seale et al researched the environmental morphing enables informed dispersal of dandelion diaspore [3]. The spread of dandelion may lead to loads of problems [4-6], so deeply understanding the proliferation of dandelion is extremely important.

Partial differential functions were used to study the propagation and growth mechanism of dandelion. Three parameters were set: temperature, humidity and wind pattern, and the spread of dandelion in several months of the year was simulated through the model. Also, determine how the arrival of invasive species may affect populations of other native species. Simulations were performed using differential equations considering the initial population of each species, the natural growth rate of the invasive species, and resource consumption. The model has two indicators: the time required to return to

equilibrium and the average percentage change in the population. This paper defines two metrics: implementation costs and profitability to determine economic impact. The analytic hierarchy process is used to determine the weights of the four indicators, and the fuzzy matrix is used to determine the result.

2. Model 1: Model of Dandelion Spread

Wind, temperature and moisture are considered in model 1 that will affect the population of dandelions. Partial differential equation is used to determine the change in the population of dandelions over time [7].

2.1. Assumptions

- Topography does not affect seed dispersal.
- The wind direction is uniform.
- Every dandelion successfully planted can spread its seeds the next month.
- All dandelion seeds will grow after first colonization.
- Mature dandelions can disperse all their seeds at once.
- Dandelions will grow 100% under ideal conditions and die 100% under extreme conditions.

2.2. Dandelion Spreading Model Functions

This section establishes a dandelion propagation model. The symbols used are in Table 1.

Table 1. Symbol Related to Dandelion Spread

Symbol	Definition	Unit
p	Amount (Population) of Dandelion	N/A
t	Time	Month
\varnothing	Wind Pattern	N/A
T	Temperature	Celsius
M	Moisture	%
r	Coefficient of natural spread	N/A
d	Coefficient of natural death	N/A
N_0	Capacity	Cluster / hector
T_p	Percentage of successful dandelion growth due to temperature	%
M_p	Percentage of successful dandelion growth due to moisture	%
T_c	Maximum dandelion land capacity percentage due to temperature	%
M_c	Maximum dandelion land capacity percentage due to moisture	%
T_d	Percentage of dandelion death due to temperature changes	%
M_d	Percentage of dandelion death due to moisture changes	%
T_0	Ideal temperature of dandelion growth (25 °C)	Celsius
M_0	Ideal moisture of dandelion growth (40 percent)	%
∇^2	Laplace Operator	N/A

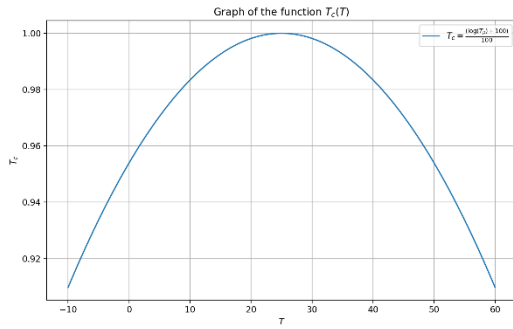
$$T_p = \frac{k \times e^{\frac{-(|T-T_0|-25)^2}{|T_0|^2}}}{(2\pi)^{\frac{1}{2}} \times 100} \quad (2.1)$$

$$M_p = \frac{k \times e^{\frac{-(|M-M_0|-40)^2}{|M_0|^2}}}{(2\pi)^{\frac{1}{2}} \times 100} \quad (2.2)$$

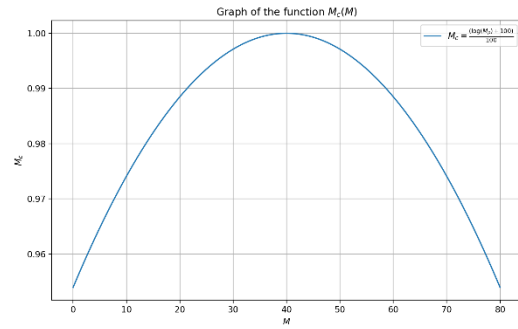
The percentage of successful growth of dandelions can be represented by a normal distribution curve, which means that when the temperature and humidity are relatively high or low, dandelions cannot grow successfully at a high percentage. The optimal temperature for dandelion growth is about 25 °C, and the optimal humidity is about 40%. In addition, for the convenience of calculation, set the constant k so that the y value of the two function vertices is 100, where k is approximately 250.6628. All dandelions can grow successfully when the environment is 25 °C and the moisture is 40%.

$$T_c = \frac{\log(T_p) + l}{100} \quad (2.3)$$

$$M_c = \frac{\log(M_p) + l}{100} \quad (2.4)$$



(a) T_c



(b) M_c

Figure 1. Maximum Dandelion Land Capacity Percentage

The maximum land capacity has the same vertex as the two percent successful growth functions above, meaning that at 25 degrees Celsius and 40% humidity, the land capacity percent is the theoretical maximum(as shown in Fig. 1). However, the capacity is not greatly affected by temperature and humidity, so we take the logarithm of T_p and M_p to obtain T_c and M_c . To keep the vertices at 100, a constant l of approximately 100 is added to the function.

$$T_d = \frac{8.3^{0.03\Delta T - 0.825}}{100} = \frac{8.3^{0.03|T - T_0| - 0.825}}{100} \quad (2.5)$$

$$M_d = \frac{8.3^{0.03\Delta M - 0.825}}{100} = \frac{8.3^{0.03|M - M_0| - 0.825}}{100} \quad (2.6)$$

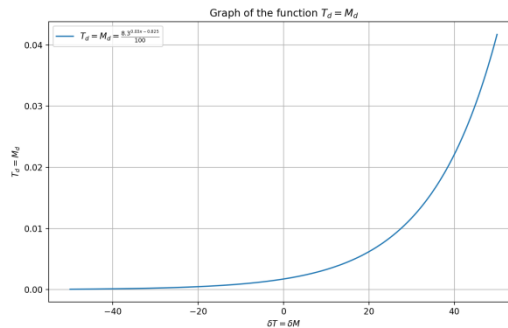


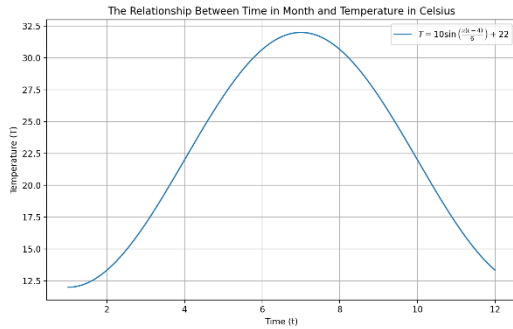
Figure 2. Percentage of Dandelion Death

The percentage of dandelions(see Fig. 2) that die due to temperature changes can be expressed as the same exponential function, which means that when the temperature and humidity changes are large relative to the optimal temperature and humidity, the mortality of dandelions will be high, and vice versa.

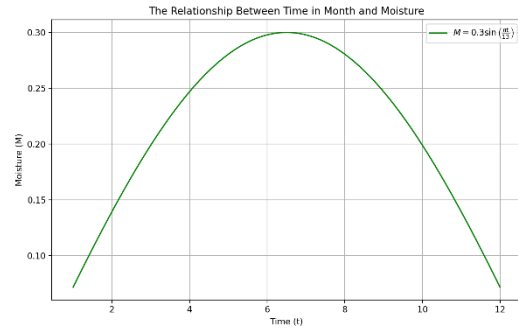
In order to find the relationship between the number of dandelions unfolding and time in a year, the variables T and M also need to be represented by t (see Fig. 3). Referring to the climate in northern my country, the formula is as follows:

$$T = 10 \sin\left(\frac{\pi(t-4)}{6}\right) + 22 \dots \{1 \leq t \leq 12\} \quad (2.7)$$

$$M = 0.3 \sin\left(\frac{\pi t}{13}\right) \dots \{1 \leq t \leq 12\} \quad (2.8)$$



(a) T_c



(b) M_c

Figure 3. Maximum Dandelion Land Capacity Percentage

The number of dandelions can be expressed using the following partial differential equation:

$$\frac{\partial p}{\partial t} = \left\{ D(\varnothing, T, M) \nabla^2 p \right\} + \left\{ rp \left(1 - \frac{P}{N_0 \times (T_c \times M_c)} \right) \times T_p \times M_p \right\} - \left\{ dp(T_d \times M_d) \right\} \quad (2.9)$$

Where $D(\varnothing, T, M)$ is the diffusion coefficient for seed dispersal that will be affected by wind, temperature, and moisture; the second term is the successful growth amount, and the third term is the successful death amount.

2.3. Numerical Simulation

A grid defined as 20 x 20 is constructed, with each square corresponding to a value in the matrix. In the initial state, some positions are randomly set so that they form clusters, and each cluster contains ten dandelions. The dispersal core of dandelion is then defined, taking into account the effects of different months, wind direction and seasonal characteristics, which will result in different dispersal patterns.

An iterative algorithm is used to simulate this partial differential equation, using convolution for two-dimensional operations. The spatial step (denoted as dx) is set to 1, and the time step (denoted as dt) is set to 0.01, which means 100 iterations per month. Additionally, since dandelions require two months of growth to flower and disperse seeds, a two-month lag was included in the simulation.

2.4. Result

The 12-month volume forecast is shown in Figure 4. Showing an exponential growth trend. This is because the set temperature difference is not very large, which avoids the death of a large number of dandelions in winter, while some dandelions can still germinate. Growth is generally slow from January to June due to cooler temperatures and smaller initial populations. As temperature and humidity changes increase, the number of dandelions gradually increases, resulting in faster growth. At the end of January, the number of dandelions was 4,865, in March it was 12,254, in June it was 51,889, and in December it was 839,265 (Fig. 4).

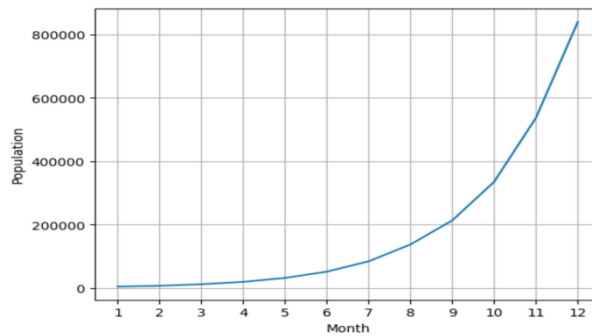


Figure 4. Predict quantity changes

Figure 5 shows the distribution of the number of dandelions in January, March, June, and December. The colored blocks represent the total amount of dandelions in a region. The spread was less obvious in January, March, and June, but there was obvious spread in December. This is due to a significant increase in dandelion numbers, causing the breeding areas of dandelion clusters to overlap.

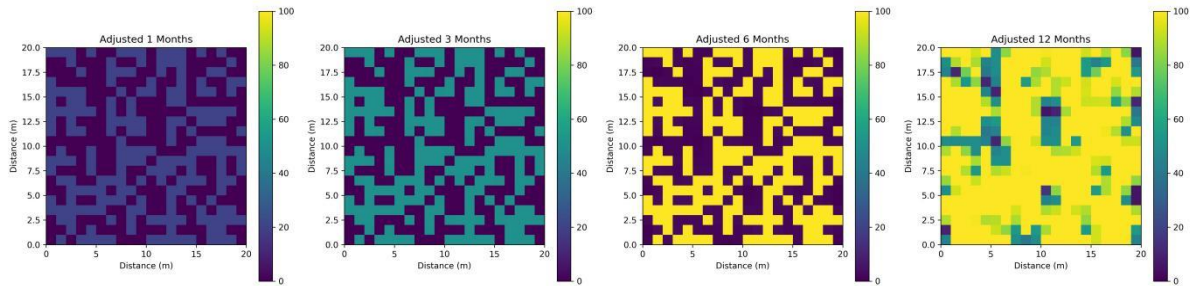


Figure 5. Dandelion's spread over month

Taking the number of dandelions in June as a benchmark, the impact of temperature and humidity span on the number of dandelions was studied. In Figure 6 and 7, as the temperature and humidity span increases, the number of dandelions first increases and then decreases. Therefore, for dandelions, the smaller the temperature span, the better. Temperature and humidity spans need to be within specific ranges for optimal results. In other words, growing dandelions requires specific climatic conditions.

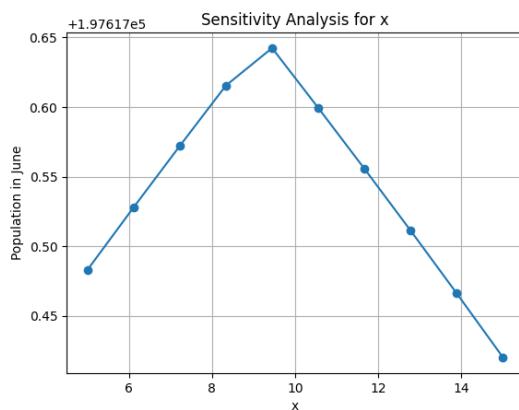


Figure 6. Temperature Span

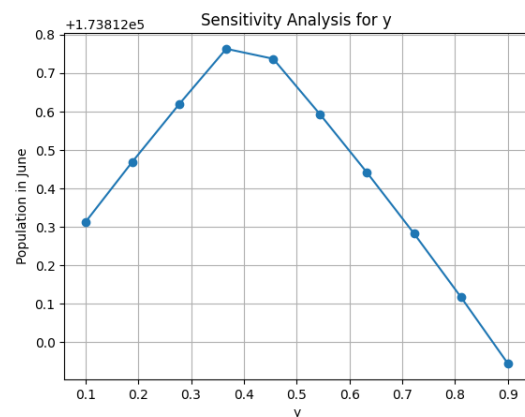


Figure 7. Humidity Span

3. Model 2: Determining the impact of an invasive species

This section provides an assessment from an ecological and economic perspective to determine the impacts of invasive species. Considering the relationship between invasiveness, two criteria are proposed, and then a fuzzy method is applied to combine them with economic factors.

3.1. Environmental impact

The arrival of invasive species can have varying impacts on local ecosystems. Consider three scenarios: harmonious symbiosis, resource competition, and directional competition. Equilibrium is the best state an ecosystem can achieve, in which the number of species remains relatively constant and the resources utilized are equal to the equilibrium resources, $R = R_e$. To quantify this impact, the time required for ecosystems to return to equilibrium after invasion was assessed. Additionally, the percentage change in native species should be considered.

3.1.1. Assumptions

- Environmental resources are limited, and R_e is the most stable amount of resources being used.
- Invasive species will bring two impacts to local ecosystems: resource competition and targeted competition. Resource competition is more common and refers to situations where invasive species take over general natural resources, such as soil and water, thereby “crowding out” some species that use the same resources [8,13]. Targeted competition means that invasive species have unique traits that are detrimental to certain native species, which is more harmful.
- All other native species are in equilibrium before the invasive species arrives. When invasive species arrive, native species cannot adapt quickly to the new changes. Therefore, native and invasive species cannot reach a complete balance in the short term.
- Changes in species abundance are sticky, meaning that such changes lack immediacy.

3.1.2. Variables

Table 2. Variables for the ecological model

Symbol	Definition	Unit
N	the total number of different indigenous species	N/A
R	the real total resources utilized	N/A
R_e	the total resources utilized at the equilibrium state	N/A
R_{\max}	the maximum resources that the environment can offer the number of indigenous species in a certain area	N/A
n	the number of indigenous species in a certain area	N/A
$P_{n,t}$	the relative population of the n th indigenous species at period t	N/A
$P_{i,t}$	the relative population of the invasive species	N/A
r_i	the natural growth rate of the invasive species	%
$index_n$	the resource consumption index of the n th indigenous species	N/A
$index_i$	the resource consumption index of the invasive species	N/A

3.1.3. Relative population. Rank native species from most to least abundant. Then the populations are normalized, and 100 represent the largest population. For the following species, we multiply the proportion of its population to the largest population by 100.

3.1.4. Three different scenarios

(1) Harmonious symbiosis means that the arrival of invasive species such as dandelions will not significantly affect the survival of other native species. This is because invasive species do not take up any of the resources that the original native species consumed. For example, while some mosses and ferns are not native to the environment, they have little impact on other plant species: they simply take root in rocks where normal plants cannot grow, and therefore extract few resources from native species. In this case, the numbers of existing native species will not be affected [9].

(2) Resource competition: The arrival of dandelions has had large-scale impacts on every native species with which it shares natural resources. To determine the function of population change of the n th native species following invasion, we should find:

$$\frac{\Delta p_n}{\Delta t} = \frac{\Delta p_n}{\Delta p_i} \times \frac{\Delta p_i}{\Delta t} \quad (3.1)$$

The relative population size of native species tends to be more sensitive when there are large differences in the ability of native and invasive species to consume the same resources. It is believed that the rate of change of p_i is related to the proportion of resources it occupies. When it has access to relatively more resources than other native species, its population will grow faster. Therefore, we get:

$$\begin{aligned} \frac{\Delta p_n}{\Delta t} &= \frac{index_i}{index_n} \times \frac{p_{i,t} \times (1 + r_i) \times index_i}{p_{1,t} \times index_1 + p_{2,t} \times index_2 + \dots + p_{N,t} \times index_3 + p_{i,t} \times index_i} \\ \frac{\Delta p_i}{\Delta t} &= \frac{p_{i,t} \times (1 + r_i) \times index_i}{p_{1,t} \times index_1 + p_{2,t} \times index_2 + \dots + p_{N,t} \times index_3 + p_{i,t} \times index_i} \end{aligned} \quad (3.2)$$

When the invasive species enter the environment, it will cause a sharp decrease in the population of other related indigenous species. Thus, at the first stage, based on (3.1) and (3.2), we can get:

$$\begin{aligned} p_{n,t+1} &= p_{n,t} - \frac{\Delta p_n}{\Delta p_i} \times \frac{\Delta p_i}{\Delta t} \\ p_{i,t+1} &= p_{i,t} + \frac{\Delta p_i}{\Delta t} \end{aligned} \quad (3.3)$$

In the first stage of Expansion, the invasive species will consume as much as they can, intensifying resource competition within species. There is a possibility that the real resources consumed, R , are higher than R_e . When that competition cannot be more intensified, which is the condition where $R = R_{\max}$, the population of both the invasive species and the indigenous species shrinks. We call the second stage Compression, where

$$\begin{aligned} p_{n,t+1} &= p_{n,t} - \frac{\Delta p_n}{\Delta p_i} \times \frac{\Delta p_i}{\Delta t} \\ p_{i,t+1} &= p_{i,t} - \frac{\Delta p_i}{\Delta t} \end{aligned} \quad (3.4)$$

As the growth in population has an “inertia”, which means that they cannot stop exactly at the point of R_e , but really close. This is what we define as “quasi-equilibrium”. After R becomes close to R_e , the population of indigenous species and the invasive species will increase/decrease alternatively, making their populations and R to fluctuate around the real equilibrium level.

After stage Compression, there are always some excess resources left, which may give rise to the third stage: Rehabilitation I. Here, we assume that after shrinking for such a long time, the indigenous species will be more sensitive to the new resources available thus having a short-term bounce back first.

$$\begin{aligned} p_{n,t+1} &= p_{n,t} + \frac{\Delta p_n}{\Delta t} \\ p_{i,t+1} &= p_{i,t} - \frac{\Delta p_i}{\Delta t} \end{aligned} \quad (3.5)$$

As the indigenous species grow, the resources utilized will again surpass the equilibrium state, entering the fourth stage Compression II, where the invasive species have a lagged bounce back, and the indigenous species have a short-term decrease.

$$\begin{aligned} p_{n,t+1} &= p_{n,t} - \frac{\Delta p_n}{\Delta t} \\ p_{i,t+1} &= p_{i,t} + \frac{\Delta p_i}{\Delta t} \end{aligned} \quad (3.6)$$

Then for the following stages, Rehabilitation and Compression alternate, making the total resources utilized fluctuate subtly around the equilibrium state, R_e .

(3) Targeted competition: on the basis of resource competition, the invasive species may cause some further damage to certain indigenous species. In this case, we introduce the new variable $d_{i,n}$, which measures the targeted damage caused by the invasive species to the n th indigenous species.

$$\frac{\Delta p_n}{\Delta t} = \frac{\Delta p_n}{\Delta p_i} \times \frac{\Delta p_i}{\Delta t} \times (1 + d_{i,n}) \quad (3.7)$$

3.2. Economic impact

The economic impact of invasive species depends largely on how people respond. The use of fuzzy mathematics gives a final model for calculating plant influence factors, which can be used for situations where the background and relationship of the model are unclear, such as there is no precise data to show the relationship between dandelions and other organisms in the ecological environment. First, when faced with invasive non-native species, governments have two options: inaction or action. For each option, two criteria were used to determine the negative impact of the plant: implementation costs and unprofitability, where implementation costs refer to the costs required for governments to implement measures and policies against invasive species, and species-defined unprofitability. Profitability does not lie within the confines of the market. For example, dandelions are less profitable than tulips because people are more likely to buy tulips for decoration, but the market for dandelions won't be as large. The other two factors are date and percentage in determining the environmental impact of plant species in the model.

Now determine a fuzzy set A_1 on x , in which A_1 represents “invasive” and x shows the discrete data of the plants on this criteria. Furthermore, $\mu_{A_1}(x)$ is called the membership degree of x to fuzzy set A_1 .

$$\mu_{i1} = \begin{cases} 1 \dots 0 < x < 40 \\ \frac{60-x}{60-40} \dots 40 \leq x \leq 60 \\ 0 \dots x > 60 \end{cases} \quad (3.8)$$

$$\mu_{i2} = \begin{cases} 0 \dots 0 < x < 25 \\ \frac{x-20}{40-20} \dots 20 \leq x \leq 40 \\ 1 \dots 2 \leq x \leq 60 \\ \frac{80-x}{80-60} \dots 60 \leq x \leq 80 \\ 0 \dots x > 80 \end{cases} \quad (3.9)$$

$$\mu_{i3} = \begin{cases} 0 \dots 0 < x < 40 \\ \frac{x-20}{40} \dots 40 \leq x \leq 60 \\ 1 \dots x > 60 \end{cases} \quad (3.10)$$

$$A_1 = \sum_{i=0}^n \frac{\mu_A(i)}{x_i} = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots + \frac{\mu_A(x_n)}{x_n} \quad (3.11)$$

Hence the same method is used to find the model of the other fuzzy set A_2 , which indicates “noninvasive”. Additionally, we define a factor set U and an appraisal set V :

$$\begin{aligned} U &= \{u_1, u_2, u_3, u_4\} \\ V &= \{v_1, v_2, v_3\} \end{aligned} \quad (3.12)$$

In order to determine the weight of each factor, we used AHP (analytic hierarchy process), which is shown below:

Table 3. Table of AHP in order to find the weights of factors

Criteria	Economic Loss	Unprofitability	Date	Percentage
Cost implementation	1	2	1/3	1/4
Unprofitability	1/2	1	1/4	1/5
Date	3	4	1	1/2
Percentage	4	5	2	1

The table can get the weight of the following factors: 1. Cost implementation: about 12.6%; 2. Profitability: about 7.9%; 3. Date: about 30.5%; 4. Percentage: about 49.0%. Additionally, for each factor u_i , create a single-factor evaluation. That is, r_{ij} ($0 \leq r_{ij} \leq 1$) represents v_j 's evaluation of factor u_i , and the single factor evaluation matrix $R = (r_{ij})_{n \times m}$ is obtained.

$$(r_{i1}, r_{i2}, r_{i3}) \quad (3.13)$$

Where u_1 and u_2 represent the two factors “cost implementation” and “unprofitability”; v_1, v_2 and v_3 are the order of evaluation “invasive”, “indifferent” and “noninvasive”. Among them, “indifferent” means it is indifferent whether people decide that the plant species is an invasive species. Now we set a weight of 0.5 for both factors, hence the three rows and two columns fuzzy relation matrix of them is $R = [R_1, R_2, R_3]^T$ Therefore, for a particular plant species, the judgment matrix is:

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \\ R_{41} & R_{42} & R_{43} \end{bmatrix} \quad (3.14)$$

Where the four rows represent the four indicators and three columns represent three comments. Therefore, we can calculate the degree of membership B by multiplying the weight of each factor W and the fuzzy vector R of each plant:

$$B_{1 \times 3} = W_{1 \times 3} \cdot R_{4 \times 3} \quad (3.15)$$

The total economic loss is obtained by multiplying the percentage calculated in the environmental impact determination model for dandelions replacing other plants (percent change in the environmental impact model) by the unit price of the replaced plants. Use the import volume of the plant to represent the macro market demand for that plant species and divide the import volume by 1 to obtain unprofitability.

3.3. Result

3.3.1. Variable value of Common Dandelion, Pineapple and Canada Goldenrod

Table 4. Values for the variable

Variable	Value of Common Dandelion	Value of Pineapple	Value of Canada Goldenrod
R_e	the total resources utilized at the equilibrium state		
R_{\max}		1.2 R_e	
n		3	
$P_{1,1}$		100	
$P_{2,1}$	40	30	30
$P_{3,1}$	95	65	65
$P_{i,1}$	10	10	10
r_i	0.1	0.03	0.2

Table 4. (continued)

<i>index₁</i>	1	1	1
<i>Index₂</i>	1	1	1
<i>Index₃</i>	1	1	1
<i>Index_i</i>	2	5	4

3.3.2. Common Dandelion. Take the common dandelion in Katmai National Park, Alaska. In Katmai National Park, the yellow-flowered common dandelion is an invasive species. There are three other common native herbaceous species: northern goldenrod (NG), little black tip (SBR), and western rattlesnake root (WR). These three herbs belong to the same family as dandelions, which means they are highly similar and thus compete for the same resources. Of the three native tree species, NG and WR are more common, while SBR is less common. Species 1 refers to NG. Species 2 is SBR. Species 3 is WR. Furthermore, assume that a single common dandelion takes up more resources than three native species.

$$R_e = p_{1,1} \times index_1 + p_{2,1} \times index_2 + p_{3,1} \times index_3 \quad (3.16)$$

Using the model and value shown above, we can get the following results, where the periods are 100, 200, 1000, and 10000 respectively. Then, we calculate the periods taken before Rehabilitation I, which in this case is 43 periods.

After reaching a quasi-equilibrium state, the population undergoes short-term fluctuations, alternating with recovery and compression(as shown in Fig. 8). There will also be a long-term plan for these groups, separated by two consecutive compression phases. The scheme cycles every 149 cycles. The final fluctuation range of the population of species 1 is [89.9, 90.2]. Therefore, the average percentage change is 9.95%. Similarly, for species 2 and 3, the rates are 24.88% and 10.47% respectively. The total average percentage change of the three is 15.1%.Then, we incorporate date and percentage in the appraisal set, which will be further used to calculate the final index.

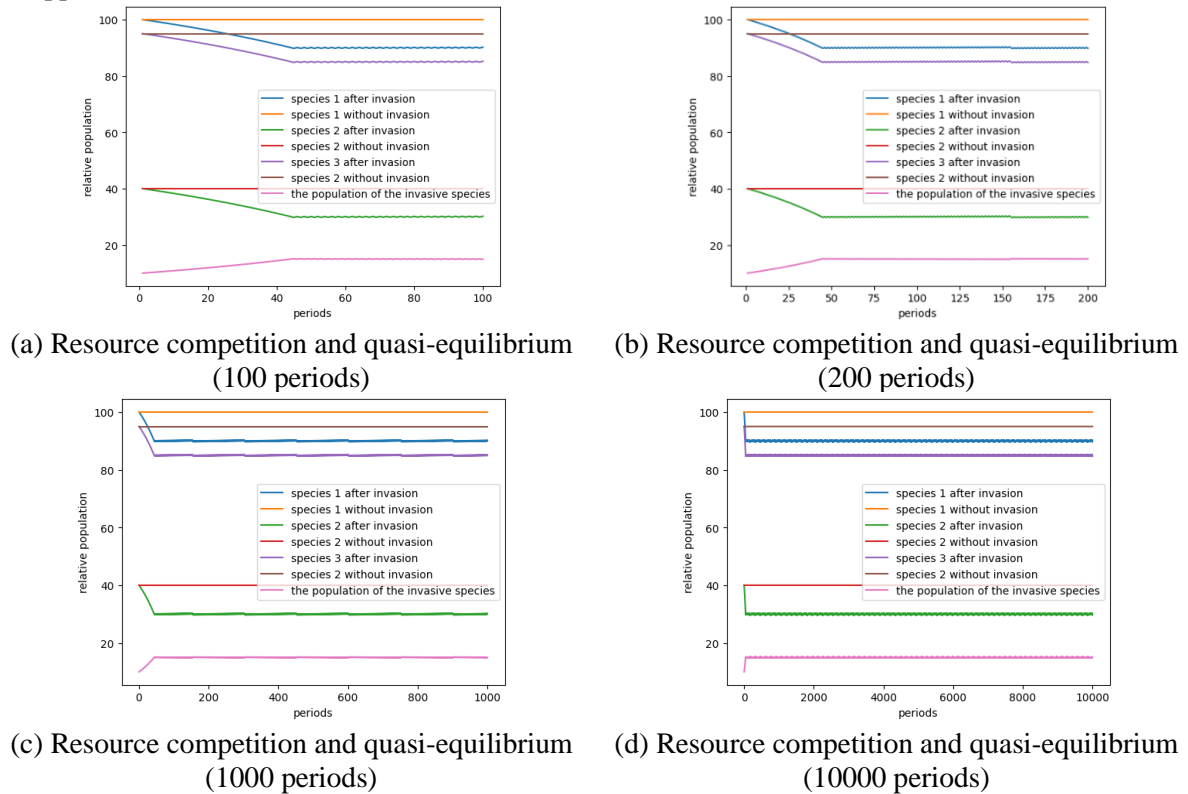


Figure 8. Impact of common dandelions

3.3.3. Pineapple. Pineapple is native to South America and is an invasive species in my country. Suppose there are three species with the same resource consumption index, namely common, uncommon and rare. Unlike most herbs, pineapple requires more resources to produce juicy fruit [5,6,8]. Therefore, we hypothesize that its resource consumption index is much greater than that of the other three native species. Moreover, the fruiting cycle of pineapples usually takes 2-3 years, which is about three times that of ordinary herbs. Since we set the natural growth rate of an herb that blooms once a year (such as a dandelion) to 0.1, we approximate the natural growth rate of a pineapple to 0.03. The date is 10 and the percentage is 42.45%(see Fig. 9).

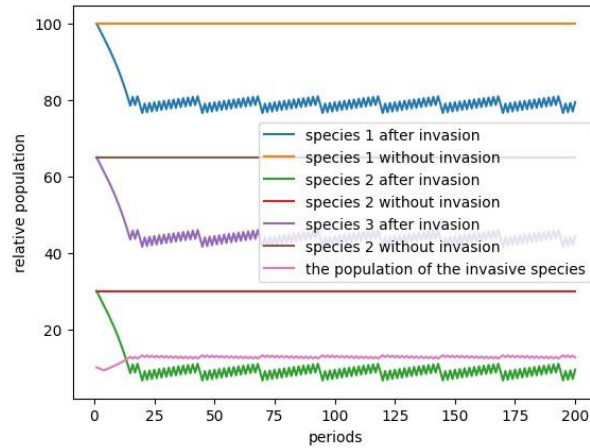


Figure 9. The impact of pineapple

3.3.4. Canada Goldenrod. Canada goldenrod (CG) is native to North America and is an invasive species in China. The reproductive capacity of CG is very strong, making it aggressive in competition for resources. Moreover, the metabolites of CG contain large amounts of limonene, pinene, camphor, etc., which can inhibit the growth of other plants. Therefore, in this case, we have to consider not only resource competition, but also targeted competition. Likewise, we assume that there are three species with the same resource consumption index, namely common, uncommon, and rare. Canada goldenrod has an additional negative impact on them due to its secretions. We set the natural growth rate of CG to 0.2 and its resource competition index to 4. Furthermore, we introduce $d_{i,n}$ and set it to 0.1 for these three species. The date is 13 and the percentage is 36.97%(see Fig. 10).

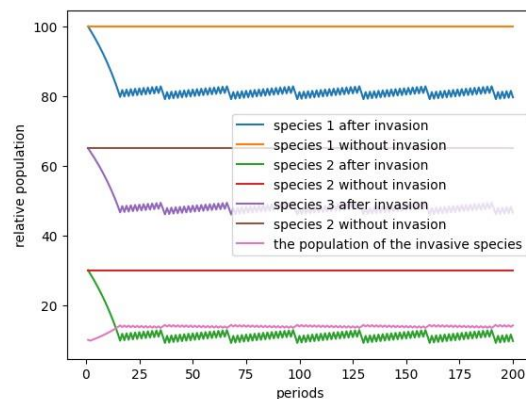


Figure 10. The impact of Canada Goldenrod

3.3.5. Determine the Impact Factor of Invasive Species. The final direct economic damage is calculated by multiplying the change in percentage of other competitors and their price, we plug the model into dandelion first and get 39.55. And its demand in 2022 is 12.35k tones, so the unprofitability of dandelion

is $(1/12.35) \times 1000 = 81.97$. The range of x is 0 to 100 in our membership function so that is why there is 2/3, 100 and 1000 in calculating economic loss and unprofitability. Therefore the fuzzy matrix for dandelion is:

$$R = \begin{bmatrix} 0.68 & 1 & 0.98 \\ 0 & 0 & 1 \\ 0.85 & 1 & 0.58 \\ 1 & 0 & 0 \end{bmatrix} \quad (3.17)$$

So the final result for dandelion is:

$$B = [0.126, 0.0079, 0.305, 0.49] \times R = [0.576, 0.431, 0.308] \quad (3.18)$$

And we take the greatest value in vector B as the final result, which is 0.576. Because 0.576 lies in our second grade—"indifferent", so dandelion would not make a huge negative impact overall environment. Our second species is pineapple. The cost implementation is: $(21.69 \times 10.39 + 72.30 \times 10.75 + 33.37 \times 21.24) \times 2/3 \times 0.1 = 94.09$ and the quantity import of pineapple in 2022 in China is 208.2k tones (China's Pineapple Production), so its cost implementation is $1/208.2 \times 1000 = 4.8$. Repeat the calculation process above, we can get the fuzzy vector for pineapple is $[0.126, 0.126, 0.126, 0.490]$ and its maximum value is 0.490, lies in our second grade, so considered an invasive species, pineapple's impact is limited [10].

The last species is the Canada Goldenrod. Its cost implementation is: $(19.35 \times 10.39 + 64.51 \times 10.75 + 27.07 \times 21.24) \times 2/3 \times 0.1 = 97.93$ according to Research report on China's Kirin vegetable import volume and Investment strategy planning, the import of Canada Goldenrod is 109.1 thousand tones, so its unprofitability is $1/109.1 \times 9.2$. Repeat the calculation process above, we can get the fuzzy vector for Canada Goldenrod is $[0.126, 0.016, 0.305, 0.926]$.

The maximum value 0.926 in this vector lies on our third grade, so Canada Goldenrod is an invasive species with severe impact.

3.4. Sensitivity analysis

This section analyzes the sensitivity of the model to the resource consumption index(see Fig. 11). The resource consumption index of invasive species changes by 40%, while the average percentage changes in the time required to restore equilibrium and population change are approximately 21% and 42% respectively, indicating that the results of the ecological model have a certain impact on the resource consumption index (invasive species consumption). The degree of aggressiveness of resources) is more sensitive to changes.

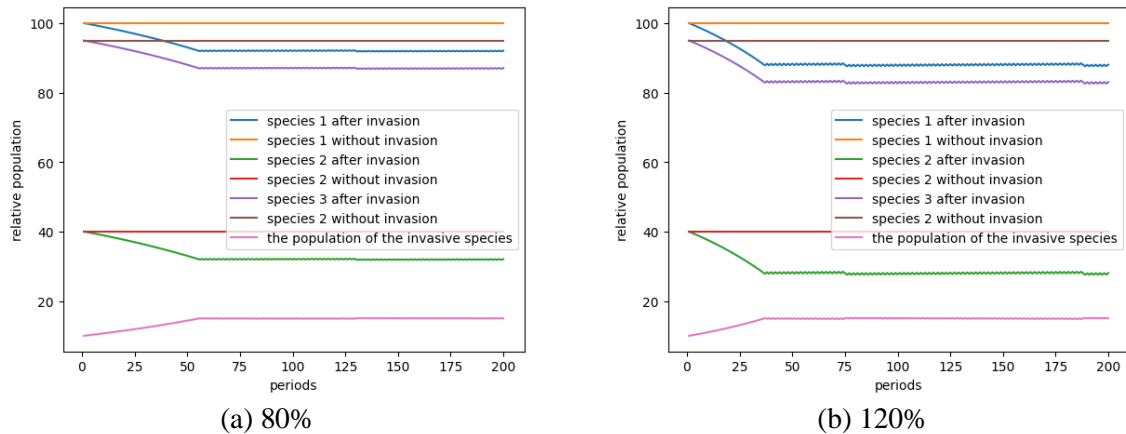


Figure 11. Indigenous species

Table 5. Sensitivity analysis of resource consumption index

Index	date	percentage at period 200
1.8* $index_i$	54	12.1832%
1.9* $index_i$	48	13.7717%
1.1* $index_i$	43	15.1000%
1.2* $index_i$	39	16.7695%
1.3* $index_i$	35	18.4646%

4. Model evaluation

For model 1, the strengths are firstly, we considered all the main factors of dandelion spreading and growing, like the wind pattern, temperature, and moisture. The consider of various factors makes the model fit more to reality, which makes the stimulation more useful in an analytical way. Secondly, the model uses references to real-world climate. We mainly used the northern China climate to simulate our model. The real life data also improve on the reasonableness of the model. Last but not least, the use of partial differential equation as the model creates numerous benefits to the model. It not only could simulate the spread of dandelion on a two-dimensional plain with population density as a very visual way to express the spread of dandelion, but also could it be very flexible in the factors added to the model to affect the spread. Not every term is influencing one another, and every term can be expressed in terms of time. This makes adding new terms or changing the value of constants very easy because they are not bonded together. Therefore, this model could be easily used on a more complex condition where more factors are considered, and it could be used to the spread of other objects that have similar mechanisms.

For model 2, we consider both the ecological impact and the economic impacts brought by the invasive species, which is more comprehensive and closer to reality. Moreover, the application of Fuzzy Mathematics can deal with uncertain information in practical problems, making the result more scientific and accurate.

For model 1, the weaknesses are that firstly, we made a lot of assumptions that make the scenario too idealized. For example, the land is assumed flat and identical in every place. It is too ideal to happen in real life, which makes the model off the real-life situation to some extent. Secondly, we did not find sufficient data to verify that our model is correct, but according to our pre-knowledge gain before and some pre-research, the trend is reasonable to some extent.

For model 2, since we have limited access to data, the data we use to determine our impact factor may be inaccurate, which may affect the accuracy of our result. In addition, in the application of Fuzzy Mathematics, the determination of membership function is not very rigorous.

5. Conclusion

Partial differential equations were used to simulate the dispersal of dandelion seeds and the growth of dandelions on a hectare of land during a year by differentiating the dandelion population at different times.

The impact factors of invasive species were considered from both environmental and economic perspectives. The result is that although pineapples have a smaller impact on society as a whole than dandelions, they still do not cause huge damage. , but Canada goldenrod will [13].

Furthermore, when we consider the many different factors that contribute to the impact of non-native plant species, humans may discover a method or pattern that allows plants to exploit their strengths, avoid their weaknesses during an “invasion”, and render invasive species detrimental to the original species. There is less damage to ecosystems or economic systems and even benefits to the environment. For example, in the results from our second model, Solidago Canada would be very dangerous to its surroundings when it becomes an invasive species [12]. But it still has some advantages, such as it can be used as medicine to treat snake bites (Efficacy and functions of Canadian goldenrod - pictures - taboos - country doctor) [13]. Therefore, we should plant invasive species on certain areas of land and improve

measures to control their fertility elsewhere. So this model is actually valuable for further research, taking into account more factors, including more real-life conditions, simulating more complex calculations to remind humans how harmful some invasive species can be, but equally, it can also provide insights into how invasive species can contribute positively to the environment and human society.

References

- [1] "Invasive Plant Species", National Park Service, 2016, <https://www.nps.gov/katm/learn/nature/invasive-species.htm>. Accessed 10 Nov. 2023.
- [2] Bergant, K., Kajfež-Bogataj, L., & Črepinšek, Z. (2002). Statistical downscaling of general-circulation-model-simulated average monthly air temperature to the beginning of flowering of the dandelion (*Taraxacum officinale*) in Slovenia. *International journal of biometeorology*, 46(1), 22-32.
- [3] Seale, M., Zhdanov, O., Soons, M. B., Cummins, C., Kroll, E., Blatt, M. R., ... & Nakayama, N. (2022). Environmental morphing enables informed dispersal of the dandelion diaspore. *Elife*, 11, e81962.
- [4] Lis, B., & Olas, B. (2019). Pro-health activity of dandelion (*Taraxacum officinale* L.) and its food products—history and present. *Journal of Functional Foods*, 59, 40-48.
- [5] Grauso, L., Emrick, S., de Falco, B., Lanzotti, V., & Bonanomi, G. (2019). Common dandelion: A review of its botanical, phytochemical and pharmacological profiles. *Phytochemistry Reviews*, 18, 1115-1132.
- [6] Froese, N. T., & Van Acker, R. C. (2003). Distribution and interference of dandelion (*Taraxacum officinale*) in spring canola. *Weed science*, 51(3), 435-442.
- [7] "What is the optimal temperature for Red-seed dandelion?" www.picturethisai.com/question/Taraxacumerythrospermum-temperature0.html. Accessed 12 Nov. 2023.
- [8] "Common Dandelion Watering Instructions." www.picturethisai.com/care/water/Taraxacumofficinale.html. Accessed 12 Nov. 2023.
- [9] "Grind fine BiZhi information consultation. "dandelion extract market insight report 2017- 2027." zhihu column,zhuanlan.zhihu.com/p/385667753.
- [10] "China's pineapple production, price, import and export and province distribution in 2022, the competitiveness of domestic pineapple continues to improve 'Figure'", China Economic Information Network, China Economic Industry Research Institute." www.huaon.com/channel/trend/870917.html. Accessed 12 Nov. 2023.
- [11] "China Economic Industry Research - Action Research Report. "Research Report on China's Kirin Vegetable import volume and Investment Strategy Planning in 2022." Zhihu column," zhuanlan.zhihu.com/p/519575773.
- [12] Foster, Joe. "Canadian Goldenrod – a Complete Guide to *Solidago Canadensis*." Growit Buildit, 25 June 2023, growitbuildit.com/canadian-goldenrod-solidago-canadensis/. Accessed 12 Nov. 2023.
- [13] Grant, Amy, "Pineapple Plant Fruiting: Do Pineapple Plants Fruit More Than Once", 2021, <https://www.gardeningknowhow.com/edible/fruits/pineapples/pineapple-plant-fruiting.htm>