DOI: 10.54254/2753-8818/43/20240849

Fundamentals of the flatness problem

Zican Zhang

Great Neck South High School, Great Neck, NY 11020, United States

zzhang2@student.gn.k12.ny.us

Abstract. The Flatness Problem is a fine-tuning problem (defined as when a physical constant appears to fall within an arbitrarily precise range such that the current status of the Universe was able to be developed) in physical cosmology, related to the density of matter and energy in the universe. In this paper, I provide an overview of the problem, describing the problem's history, beginning with its foundations by Albert Einstein and Alexander Friedmann in General Relativity and the discoveries of Edwin Hubble and Georges Lemaitre regarding the nature and expansion of the Universe. I also present the mathematical basis of the energy density of the Universe by completing a simple derivation from the first Friedmann Equation, before analyzing observational data regarding the apparent density of the Universe. This data comes from the Cosmic Microwave Background and distant Type 1a supernovae. Finally, I explore the possible solutions to the Flatness Problem, including Anthropism and Inflation, and recount each one's strengths and weaknesses.

Keywords: Flatness Problem, Inflation (Cosmology), Friedmann Equations.

1. Introduction

During the 20th century, observations and various insights into cosmology gave birth to the Big Bang Theory, the best working idea regarding the origin of the universe. However, despite its success, it was plagued by a variety of problems. One such problem was the Flatness Problem, which was raised in the 1970s. Along with other discrepancies, the Flatness problem compelled cosmologists, specifically Alan Guth, to form the Theory of inflation, a supplementary theory to the Big Bang and a key cosmological concept.

1.1. *Einstein and Friedmann*

In 1915, Albert Einstein published his Theory of General Relativity, which, through a set of equations known as the Einstein Field Equations, implied that Energy, both in and not in the form of matter, could curve spacetime. Naturally, the degree of curvature depends on the amount, or density, of matter/energy present.

General Relativity also serves as a mathematical theory from which the effects of gravity could be described, superseding Newton's old formulations of gravity. Attempting to verify this new mathematical framework, scientists put general relativity to the test by measuring its predictions against observational data, including the precession of mercury [1] and gravitational lensing [2]. Most of these predictions came out correct. However, when predicting the nature of the universe, the result was a

DOI: 10.54254/2753-8818/43/20240849

prediction that the universe was static and unchanging. This, as we know it today, is extraordinarily incorrect.

This mistake was the result of a mathematical error during Einstein's computation. It was eventually resolved by Alexander Friedmann [3], and the subsequent set of equations is now called the Friedmann Equations.

The theory that matters and energy bend spacetime and the first Friedmann Equation are the basis for the Flatness Problem.

1.2. Expansion of the Universe and the Big Bang

In 1912, Vesto Slipher discovered that various distant objects, now known to be other galaxies outside of the Milky Way, were moving away from Earth. However, Slipher did not recognize the importance of this discovery [4-5].

In 1927, Georges Lemaître suggested that the cause for these receding objects was the expansion of space itself [6]. This implied that the universe was, at one point in time, concentrated to a dot. Lemaître himself argued in favor of this interpretation, stating how this dot, which he called the "primeval atom", was the beginning of the universe [7]. This was the beginning of the Big Bang Theory.

Meanwhile, in 1929, Edwin Hubble discovered the relation between distance and recession, finding that objects more distant from Earth have faster recessional velocities [8]. This proportional relationship is called Hubble's law, and the constant by which the proportion is set is called the Hubble Parameter.

Following the confirmation of the universe's expansion, two rival hypotheses regarding the nature of expansion arose. One was called the "Steady State" theory, where, as the universe expands, matter spontaneously comes into existence, keeping the density of the universe constant. The other was what we consider now to be the Big Bang theory, where matter is diluted as the universe expands. It was eventually determined that the latter theory was correct. This is highly important, as the decreasing density of matter as time progresses is a major component of the geometry of space, as we shall see.

2. Formulation of the Flatness Problem

The discovery of the problem was a relatively recent endeavor. It was first done by Robert Dicke in 1970 [9], before being expanded upon by Dicke and Jim Peebles in 1979 [10]. The basic concept is as follows:

2.1. Conceptual and Mathematical Basis

As stated before, the geometry of the universe depends on the presence, or density, of matter/energy. Essentially, just like the Earth, the universe has a 3D geometry. This geometry could be hyperbolic, or negative (where the three angles of a triangle add up to less than 180 degrees), spherical, or positive (where the three angles of a triangle add up to more than 180 degrees), or Euclidean, or flat. Observations suggest that the geometry of the universe is of the latter type, differing from what theoretical physics suggests, as shown in the following derivation.

The first Friedmann Equation can be expressed as:

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} \tag{1}$$

Where H is the Hubble Constant, i.e. the current value of the Hubble Parameter, which changes with time. It is currently about 70 km/s/Mpc. Furthermore, G is Newton's Gravitational constant, about 6.67×10^{-11} $m^3/kg/s^2$, c is the speed of light, 3×10^8 m/s. ρ is the density of matter and energy of the universe, the measure that determines by how much spacetime is curved. Speaking of which, k is the parameter for spacetime curvature. Finally, *a* is the scale factor of the Universe.

The value k is the central theme of the flatness problem. Its value determines the geometry of the Universe. A negative k would mean a closed universe, a k of 0 would mean a flat universe, and a positive k would mean an open universe. Importantly, k does not change as the universe evolves. For k to be 0, the density ρ must be of the following value:

DOI: 10.54254/2753-8818/43/20240849

$$\rho_c = \frac{3H^2}{8\pi G} \tag{2}$$

Notably, this value, called the critical density, is not dependent on the scale factor. Since H and G are known, ρ_c can be calculated, and its value returns as 10^{-26} kg/m^3. The ratio between the actual density of the universe and this critical density is Ω. The value of Ω as less than, equal to, or greater than 1 is the key to the universe's geometry.

By multiplying both sides of (1) by $3a^2/8\pi G$, we arrive at the following equation.

$$\frac{3a^2}{8\pi G}H^2 = \rho a^2 - \frac{3kc^2}{8\pi G} \tag{3}$$

Further manipulation can be done by plugging in the formula from (2) into the left-hand side, and subtracting both sides by ρa^2, resulting in the following:

$$\rho_c a^2 - \rho a^2 = -\frac{3kc^2}{8\pi G} \tag{4}$$

Finally, by applying $\Omega = \rho/\rho_c$, the equation is reduced to:

$$\left(\Omega^{-1} - 1\right)\rho a^2 = \frac{-3kc^2}{8\pi G} \tag{5}$$

Because the right-hand side contains nothing but constant values, meaning that even though the individual values of ρ and a change throughout time, the left-hand side always remains the same. As the scale factor is, by definition, related to the size of the universe, it increases as time progresses. ρ, meanwhile, decreases as space expands, which makes sense. However, ρ falls faster than a rises, and ρa^2 has fallen by about 10^{60} since the Big Bang [11], meaning that (1/Ω - 1), or the difference between Ω and its critical value if the universe is flat, must have increased by 10^{60} to balance this. Unless Ω is exactly 1, the 10^{60} would exponentially inflate any difference between the ideal and actual values of Ω.

2.2. *Observational measurement of Ω*

We can measure Ω in multiple ways. The two most prominent are through observation of the Cosmic Microwave Background Radiation and observation of Type Ia Supernovae.

CMB Radiation is radiation caused by photons during an early stage of the universe as this universe was small and hot enough that the plasma could absorb photons. As the universe expanded and cooled, the plasma was eventually unable to absorb these photons. However, leftover photons continue to propagate, spreading throughout the universe.

This radiation is extremely balanced, but there are minute variations in its temperature. These variations, called Anistrosopies, have power spectrums that can be measured. The angular scale of these peaks corresponds to the curvature of the universe.

Another method is to observe Ia supernovae—supernovae caused by the accretion of matter onto a white dwarf such that its mass exceeds the Chandrasekhar Mass of 1.44 Solar Masses, reigniting fusion and causing an explosion—at different distances, as all Ia Supernovae have a set absolute magnitude of -19.3. We can use this as well as the apparent magnitude when we observe them to determine their distance from us. Using the Distance Modulus,

$$m - M = 5\log_{10}(d) - 5 \tag{6}$$

Where m is the apparent magnitude, M is the Absolute magnitude, and d is the distance to the supernova. The apparent magnitude can be determined by how faint the supernova is when observed. Thus, d to any particular Ia supernova can be calculated [12].

As more distant supernovae occurred further back in time, we can use redshift to determine the rate of universal expansion, giving us an estimate of Ω from the rate of expansion throughout history.

The WMAP space probe and the Sloan Digital Sky Survey have used the former technique, and various other observations have used the latter. Both suggest that Ω must be within 1% of 1 [13]. This is further supported by results from the Planck spacecraft [14]. This 1%, combined with the 10^{60} shift in ρa^2 since the Big Bang, means that Ω must have been within 10-62 at the Big Bang, all but confirming the Universe is flat. This is an exceptionally unlikely scenario since ρ must be exactly the same as ρc. Furthermore, if ρ is larger than ρc, the overdense universe would quickly stop expanding and collapse. If ρ is smaller than ρc, the universe would continue to expand on such a scale that gravity would not be strong enough to form galaxies and other structures. Neither universe would allow for life to exist [15]. The absurdity of such a situation is the theme of the Flatness Problem, which, along with the Horizon Problem and the Problem of Magnetic Monopoles, is one of the key issues with the Big Bang Theory.

3. Solutions to the Flatness Problem

3.1. Anthropism

There are several solutions to the issue. The simplest is to invoke Anthropism and the existence of multiple universes, which argues that any deviation from flatness due to a difference in density would cause galaxies and stars to fail to form, resulting in a universe lacking life. Thus, the fact that we can observe the apparent flatness of the universe is a testament to a flat universe's ability to support life. This mentality has been supported by Christopher Collins and Stephen Hawking [16].

A similar solution that also uses Anthropism is to state that the universe is infinite in size but with varying densities across large portions of space. Life would only evolve in the regions which have the right density [17].

However, as these thought processes do not use the scientific process, they are far from perfect.

3.2. Inflation

A much more usable explanation is the Theory of Inflation, proposed by Alan Guth in 1980 [18]. In fundamental cosmology, the History of the Universe is divided into various periods, known as epochs, which vary in characteristics, most notably, temperature. The Epoch of Inflation was a period of rapid exponential growth in the size of the universe, ending about 10^{-32} seconds after the Big Bang.

Inflation had several effects on the early universe. Firstly, it removed any large inhomogeneities across space, forming what we see as a homogeneous universe today (however, quantum fluctuations did cause some inconsistencies). Secondly, it massively decreased the density of magnetic monopoles, among other particles, in the universe, making their existence unlikely.

During the early universe, during this period of rapid expansion, the energy density did not decrease but rather remained constant. ρa^2 must have thus increased dramatically, forcing Ω down to a value that was very close to one, such as 10^{-62}, which evolved into the 0.01 we see today. This is the explanation most favored by cosmologists today, especially as it also solves the Horizon and Magnetic Monopole problems. However, a drawback is that the cosmological field driving inflation is currently still unknown. As such, various conjectures have been proposed [19]. Scientists are still working on the issue. Currently, one of the most promising approaches is the Inflaton, proposed by Alan Guth himself.

There is also the possibility that the flatness problem isn't a problem at all, but rather a misunderstanding of cosmology [20]. However, it does not appear that this hypothesis is widely accepted in the scientific community.

4. Conclusion

In summary, the Flatness Problem is a problem regarding the density of matter and energy in the universe, where the expansion of space causes said density to decrease, forcing it to move farther and farther away from the critical density. This is shown by the first Friedmann Equation. However, observations have revealed that even after such movement, the density remains close to or is at the critical density. This forces the hand that the Universe is indeed flat, which, given the range of possible initial densities, feels highly unlikely. Possible solutions to this problem include the possibility that we only observe this

DOI: 10.54254/2753-8818/43/20240849

problem *because* the universe is flat or that cosmological inflation forced the density to be near critical. There remain various problems with the cause of Inflation, however, and a key component of cosmological research is investigating this problem.

References

[1] Einstein, A. (1916). Die grundlage der allgemeinen relativitätstheorie [The Foundation of the General Theory of Relativity]. Annalen Der Physik, 354(7), 769-822. https://doi.org/10.1002/andp.19163540702
[2] IX. A determination of the deflection of light by the sun’s gravitational field, from observations made at the total eclipse of May 29, 1919. (1920). Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character, 220(571-581), 291-333. https://doi.org/10.1098/rsta.1920.0009
[3] Friedmann, A. A. (1922, December 6). From Alexander Friedmann [Letter to Albert Einstein]. https://einsteinpapers.press.princeton.edu/vol13-trans/363
[4] Slipher, Vesto M. (1913). “The Radial Velocity of the Andromeda Nebula”. Lowell Observatory Bulletin. 1 (8): 56–57. Bibcode:1913LowOB...2...56S.
[5] Slipher, Vesto M. (January 1915). “Spectrographic Observations of Nebulae”. Popular Astronomy. 23: 21–24. Bibcode:1915PA.....23...21S.
[6] Lematre, A. G. (1931). A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae. Monthly Notices of the Royal Astronomical Society, 91(5), 483-490. https://doi.org/10.1093/mnras/91.5.483
[7] LemaÎtre, A. G. (1931). Contributions to a British association discussion on the evolution of the universe. Nature, 128(3234), 704-706. https://doi.org/10.1038/128704a0
[8] Hubble, E. (1929). A relation between distance and radial velocity among extra-galactic nebulae. Proceedings of the National Academy of Sciences, 15(3), 168-173. https://doi.org/10.1073/pnas.15.3.168
[9] Dicke, R. H. (1970). Gravitation and the Universe. American Philosophical Society.
[10] Dicke, R.H., & Peebles, P.J.E. Israel, W. (Ed.). (1979). The big bang cosmology - enigmas and nostrums. United Kingdom: University Press.
[11] Coles, P., & Luccin, F. (1995). Cosmology. Wiley.
[12] Stompor, R., Abroe, M., Ade, P., Balbi, A., Barbosa, D., Bock, J., Borrill, J., Boscaleri, A., de Bernardis, P., Ferreira, P. G., Hanany, S., Hristov, V., Jaffe, A. H., Lee, A. T., Pascale, E., Rabii, B., Richards, P. L., Smoot, G. F., Winant, C. D., & Wu, J. H. P. (2001). Cosmological implications of the maxima-1 high-resolution cosmic microwave background anisotropy measurement. The Astrophysical Journal, 561(1), L7-L10. https://doi.org/10.1086/324438
[13] Spergel, D. N., Bean, R., Dore, O., Nolta, M. R., Bennett, C. L., Dunkley, J., Hinshaw, G., Jarosik, N., Komatsu, E., Page, L., Peiris, H. V., Verde, L., Halpern, M., Hill, R. S., Kogut, A., Limon, M., Meyer, S. S., Odegard, N., Tucker, G. S., . . . Wright, E. L. (2007). Three-YearWilkinson microwave anisotropy probe</i>(wmap</i>) observations: Implications for cosmology. The Astrophysical Journal Supplement Series, 170(2), 377-408. https://doi.org/10.1086/513700
[14] Aghanim, N., Akrami, Y., Ashdown, M., Aumont, J., Baccigalupi, C., Ballardini, M., Banday, A. J., Barreiro, R. B., Bartolo, N., Basak, S., Battye, R., Benabed, K., Bernard, J.-P., Bersanelli, M., Bielewicz, P., Bock, J. J., Bond, J. R., Borrill, J., Bouchet, F. R., . . . Chluba, J. (2020). Planck</i>2018 results. Astronomy & Astrophysics, 641, A6. https://doi.org/10.1051/0004-6361/201833910
[15] Ryden, B. S. (2003). Introduction to cosmology. Addison-Wesley.
[16] Collins, C. B., & Hawking, S. W. (1973). Why is the universe isotropic? The Astrophysical Journal, 180, 317. https://doi.org/10.1086/151965
[17] Barrow, J. D., & Tipler, F. J. (1986). The anthropic cosmological principle. Oxford University Press.

DOI: 10.54254/2753-8818/43/20240849

[18] Guth, A. H. (1981). Inflationary universe: A possible solution to the horizon and flatness problems. Physical Review D, 23(2), 347-356. https://doi.org/10.1103/physrevd.23.347
[19] Bird, S., Peiris, H. V., & Easther, R. (2008). Fine-tuning criteria for inflation and the search for primordial gravitational waves. Physical Review D, 78(8). https://doi.org/10.1103/physrevd.78.083518
[20] Helbig, Phillip (2021). Arguments against the flatness problem in classical cosmology: A review. The European Physical Journal H, 46(1). https://doi.org/10.1140/epjh/s13129-021-00006-9