

Influence of temperature and momentum on heavy quark energy loss

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Abstract. We investigated the correlation between temperature and momentum, as well as the impact of temperature and momentum on energy loss in quark-gluon plasma (QGP), along with the primary mechanisms of energy dissipation. Our findings reveal that the threshold momentum increases with rising temperatures, indicating a greater possibility to energy loss at higher temperatures and momenta. Numerical estimates of the losses show that the energy loss in the hot matter is greater than the energy loss in the cold matter. Upon considering energy loss due to polarization, radiation, and collision, we observe that under low transverse momentum conditions, polarization dominates the energy loss, whereas under high transverse momentum conditions, radiation takes precedence. In addition, we found that heavy quarks with lower mass are more prone to energy loss because of the faster thermalization rate.

Keywords: heavy quark, quark-gluon plasma (QGP), quark energy loss.

1. Introduction

Quantum chromodynamics (QCD) is a non-abelian gauge field theory that describes strong interactions. It has two basic properties: progressive freedom and quark confinement. The nature of progressive freedom allows us to use perturbation QCD for calculations during large momentum or very short-range interactions. The nature of quark confinement prevents us from observing the existence of free quarks. However, quantum chromodynamics predicts that quark gluons can be in the unconfined phase at high temperatures and high densities, and exist freely in a space-time range larger than the hadron scale. High temperature and high density can achieve the conditions required for phase transition, so that the shell of hadron ceases to exist, and quarks and gluons form a new material form. This new state of matter came to be called quark gluon plasma (QGP). Theoretically, this high-energy-density material form can be achieved by high-energy heavy ion collisions (i.e., nuclear-nuclear magnetic collisions). Because the relativistic heavy ion magnetic collision with a sufficiently high center of mass energy can provide the high temperature and high density required for QGP formation [1].

2. Correlation between Temperature and Transverse Momentum

In the context of soft hadron-hadron systems, we acknowledge that two-body dynamics and geometry are crucial factors contributing to energy loss and fluctuations in multiple distributions.

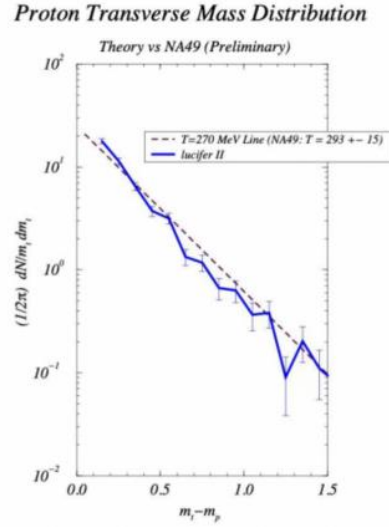


Figure 1. A calculated of protons transverse in momonmentum spectrum in Pa+Pb using a rapidly $-1.0 \leq y \leq +1.0$. The comparison exhibits an exponential behavior with an inverse slope or ‘temperature’ T , approximately equal to 270 MeV.

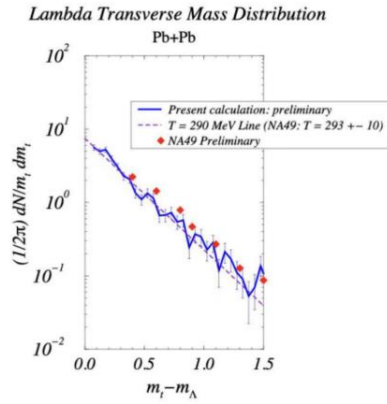


Figure 2. A calculated of protons transverse in momonmentum spectrum in Pa+Pb. Comparison is made to NA49 data.

In Fig. 1 and Fig. 2, the transverse momentum spectrum of the proton in Pb+Pb collisions is calculated within the velocity interval $[-1.0 \leq y \leq +1.0]$. We compare the index of inverse slope or temperature T approaching 270 MeV, and compare our calculations of central collisions of Au+Au nuclei at RHIC with the partial cascade results obtained by K. Geiger [2]. Mesons may appear very similar spectra when the binding molecules are hard scattered and the splitting process occurs. This observation indicates that the two-body processes and the conservation of energy play a dominant role in production, rather than thermodynamics.

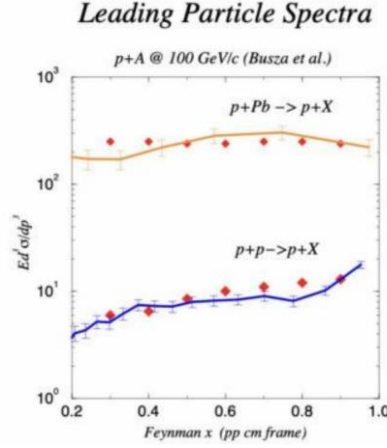


Figure 3. The inclusive proton is produced in pp and p+Pb. Simulations at 100 GeV/c are compared as a function of Feynman x to FNAL data from Busza et al.

We also considered the behavior of leading particles in pp and p+Pb collisions, and Fig.3 presents a comparative analysis of proton and pion spectras in Feynman x, showcasing the measured data at 100 GeV/c from FNAL. A narrower distribution of transverse momenta is evident in both the elastic and inelastic factors within the model when considering Feynman x values approaching. Although these findings may be associated with parton structures of the proton. A straightforward inference is that the nucleon-nucleus datas can be derived from nucleon-nucleon interactions [2].

To investigate the correlation between temperature and momentum, the study titled ‘Radiative energy loss of heavy quarks in a quark-gluon plasma’ presents a graphical representation with temperature on the x-axis and threshold momentum on the y-axis. Fig. 4 demonstrates that when $(q_{\perp}^2 < \mu^2 D)$, there is an absence of energy loss within the shaded region depicted in Fig. 2 (it should be noted that this behavior may differ for massless quarks). It is worth noting that the threshold of heavy quarks evidently increases with rising temperature, and when the transverse momentum falls below this threshold, the emission of radiation by the heavy quark no longer exists [3].

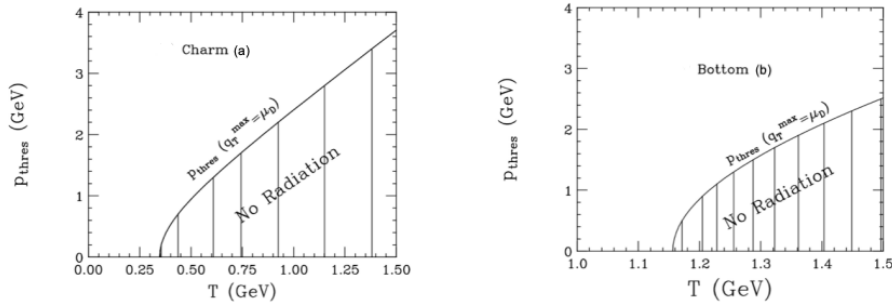


Figure 4. The threshold momenta of Charm (left) and Bottom (right) as a function of T.

3. Energy loss of finite conical hard quark jets in hot and cold matter

In the model (GW model) proposed by Gyulassy and X.N.Wang, it is postulated that the energy dissipation resulting from particle traversal through a partially thermalized submedium is predominantly carried away by gluons generated via multiple scatterings. A similar phenomenon of energy loss occurs when particles traverse cold nuclear matter. Therefore, in the subsequent analysis, we will quantitatively assess and compare the energy dissipation of QCD particles in two distinct environments, namely cold and hot matter, with the aim of investigating the impact of temperature on QCD energy loss [1,4].

Next, we will consider classical calorimetric calculations of hard jets resulting from heavy-ion collisions. A large amount of energy loss results in a spectral decay, usually expressed as a jet burst. In

order to quantitatively predict the energy lost as the jet traverses' hot matter, The study of the angular distribution of radiative gluons is essential. Only gluons radiating out of the cone that defines the jet will lead to energy loss [4].

There we would consider HQs jets of high energy, which are generated by hard scattering in a dense QCDs medium and spread over a distance L in it. We are focusing on the combination of losses out of the opening angle θ_{cone} angle cone

$$\int_0^\infty d\omega \int_{\theta_{\text{cone}}}^\pi \frac{\omega d^3 l}{d\omega dz d\theta} d\theta \quad (1)$$

We have thought about the normalized loss by defined ratios

$$R(\theta_{\text{cone}}) = \frac{\Delta E(\theta_{\text{cone}})}{\Delta E} \quad (2)$$

This ratio $R(\theta_{\text{cone}})$ is only affected by a single variable

$$R = R(c(L)\theta_{\text{cone}}) \quad (3)$$

and

$$c^2(L) = \frac{N_c}{2C_F} \hat{q}(L/2)^3 \quad (4)$$

The “scaling behavior” of R claims that the medium case and size dependencies are usually represented in the function of $c(L)$., where $c(L)$ is the transport coefficient function \hat{q} , and the length L depend on Eq. Fig. 6 shows transformations of R with the θ_{cone} [4].

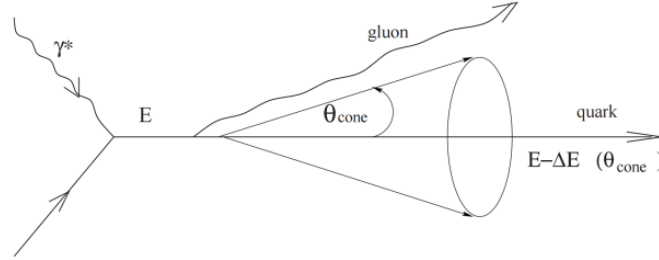


Figure 5. The hard process which produces a quark jet. The gluon in this jet is emitted outside the cone with angle θ_{cone} .

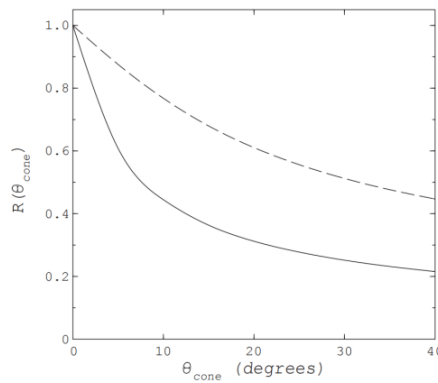


Figure 6. Medium-induced (normalized) energy loss distribution as a function of cone angle θ_{cone} for hot matter ($T = 250$ MeV) (solid curve) [4] and cold matter (dashed curve) [4] at decided length $L = 10$ fm.

We now calculate the data of energy loss caused by radiation. We need to ensure the value q of the transfer coefficient that affects the amount of energy lost to perform the following calculation. We get the following data from quark jets in the matter [4]:

For hot matters at $T = 250$ MeV, with $\frac{\mu^2}{\lambda} \sim 1$ GeV/fm² (based on perturbative estimates) [4].

and the typical value for $\hat{v} \approx 2.5$, we can find $\hat{q} \approx 0.1$ GeV³. When $\alpha_s = 13$, leading to the determination of the total energy loss:

$$-\Delta E \approx 60 \text{ GeV} \left(\frac{L}{10 \text{ fm}}\right)^2 \quad (5)$$

For cold matters, it is possible to establish a connection between the \hat{q} and the gluon structure function G evaluated at an average scale $\mu^2 \frac{\lambda}{L}$.

$$\hat{q} \cong \frac{2\pi^2 \alpha_s}{3} \rho [xG(x)] \quad (6)$$

Selecting the nuclear density $\rho \sim 0.16 \text{ fm}^{-3}$, $\alpha_s = \frac{1}{2}$, and $xG \sim 1$ when $x < 0.1$, it is found that

$$-\Delta E \approx 4 \text{ GeV} \left(\frac{L}{10 \text{ fm}}\right)^2 \quad (7)$$

As these Fig.s demonstrate, matter can effectively stimulate energetic particles to produce significant losses of radiant energy. The loss of energy in a thermal medium that expands is greater than in the related static medium. Then we consider the energy loss $-\Delta E(\theta_{\text{cone}})$ -induced by the medium of the energetic jet. We can use the above estimates to give the magnitude of $c(L)$ in the situations of hot and cold media [4].

$$c(L)_{\text{hot}} \cong 40(L/10 \text{ fm})^{3/2} \quad (8)$$

Compared to thermonuclear matter, the value of $c(L)$ is much smaller in cold nuclear matter

$$c(L)_{\text{cold}} \approx 10(L/10 \text{ fm})^{3/2} \quad (9)$$

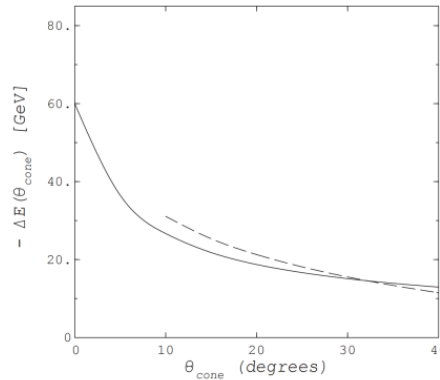


Figure 7. The dependence of energy loss on θ_{cone} in a hot medium ($T=250$ MeV), with the dashed curve representing the medium-independent component of energy loss for $E=250$ GeV. The length scale is $L=10$ fm [4].

Fig. 7 shows that the quark jet is more collimated in a hot medium than in a cold medium since the normalised loss $R(\theta_{\text{cone}})$ normally depends on $c(L)\theta_{\text{cone}}$. However, the magnitude of the loss remains significantly large even for cone size approximately equals to $\theta_{\text{cone}} \approx 30^\circ$. Note that our estimates are calculated using the leading logarithmic approximation [4].

In conclusion, we calculate the energy lost by a quark jet with a given aperture in order to predict the suppression (quenching) produced by a hard jet. We explore the data differences of energy

loss between hot matter and cold matter. The numerical estimation of the loss indicates that the loss of hot matter may be significantly greater than that of cold matter. This result claims that we can judge the formation of quark-gluon plasma according to the radiation energy loss

4. Dynamical Evolution of Heavy Quarks

The dynamic evolution of Heavy-quark can be described as the Brownian motion, governed by the classical Langevin equation. In order to calculate the energy loss caused by radiation and collision, the modified Langevin equation is applied as follow:

$$\frac{d\vec{p}}{dt} = -\eta_D(p)\vec{p} + \vec{\xi} + \vec{f}_g. \quad (10)$$

The first two terms on the right of this equation describe the drag and thermal random forces, and a third recoils term $\vec{f}_g = -d\vec{p}/dt$ is used to describe the effect of gluon radiation on the Brownian motion of the heavy quark. The possibility of occur radiation energy loss at each time interval and the momenta of the radiated gluons are simulated by Monte Carlo method as follow:

$$\frac{dN_g}{dxdk_{\perp}^2 dt} = \frac{2\alpha_s(k_{\perp})}{\pi} P(x) \frac{\hat{q}}{k_{\perp}^4} \sin^2\left(\frac{t-t_i}{2\tau_f}\right) \left(\frac{k_{\perp}^2}{k_{\perp}^2 + x^2 M^2}\right)^4 \quad (11)$$

By using this new framework to calculate the RAA (nuclear modification factor), FIG. 8 compared the R_{AA} of gluon radiation and collision at different momenta and compared with the test results, we find that the heavy nuclear modification is dominated by collision energy loss in the low p^T region, but the gluon radiation dominates the energy loss in the high p^T region.

By assuming that the HQs achieve thermal equilibrium after a long period of time, we can establish the fluctuation-dissipation relationship between drag and heat $-\eta D(p) = \kappa/(2TE)$, which κ is consider as the momentum space diffusion coefficient. To account for the equilibrium between emission and absorption, we impose a lower limit of gluon energy $\omega 0 = \pi T$ in the radiation process. Different transport coefficients are related via $D = 2T^2/\kappa$ and $\hat{q} = 2\kappa C_A/C_F$.

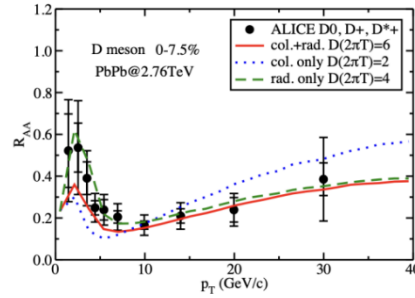


Figure 8. The R_{AA} of gluon radiation and collision at different momenta

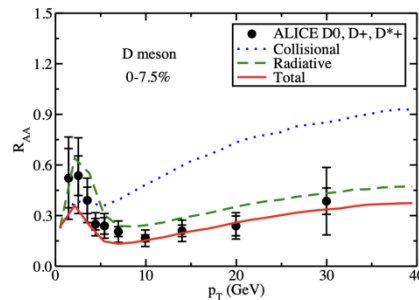


Figure 9. Tuning the diffusion coefficient to fit D meson RAA with various energy loss mechanisms.

The diffusion coefficient in this section is set as $D = 6/(2\pi T)$ for the presented results. By adjusting the transport coefficient, it can be observed that either collisional or radiative energy loss alone can provides a reasonable excuse of the data of ALICE experimental, although not as accurate as when both radiative and collisional are considered [5].

5. Polarization effect on energy loss

In order to increase the accuracy of energy loss calculation, we consider the effect of dielectric polarization on QGP as a mechanism of energy loss. The polarization energy can be described as an HQ passing through the QGP medium, generating a color electric field against its motion, and generating a reaction force against its propagation. In that case, HQs are subject to resistance from the induced field, resulting in energy dissipation. So now we're going to consider three parts of the energy loss, the polarization energy loss, the gluon radiation energy loss, and the collision energy loss.

Named lossing energy as “ $-dE/dx$ ”, coefficient of drag as “ γ ” We compare the temperature and momentum dependence of the polarization effect (simplify as PE), elastic collision (simplify as EC) and radiation (simplify as IC) of two heavy quarks with different masses, the charm quark and the bottom quark.

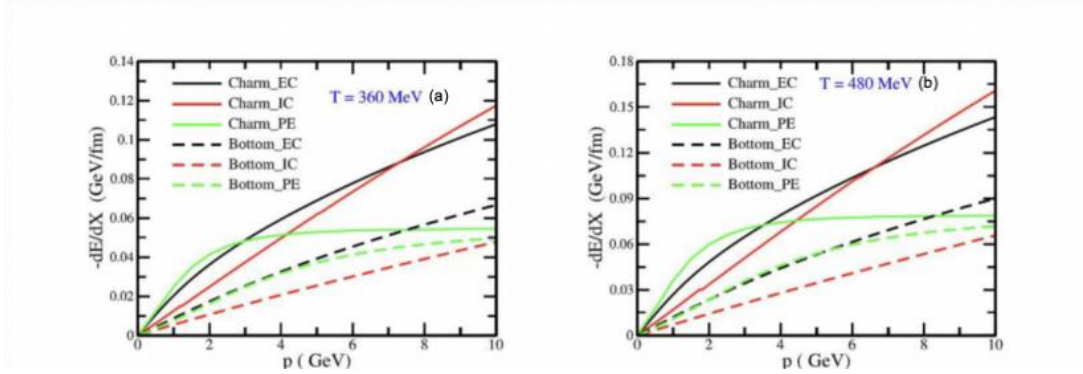


Figure 10. Comparison of momentum-dependent energy loss between charm and bottom quarks considering the polarizability effect (PE), elastic collisions (EC), and radiation (IC) at temperatures $T = 360$ MeV (left) and $T = 480$ MeV (right)

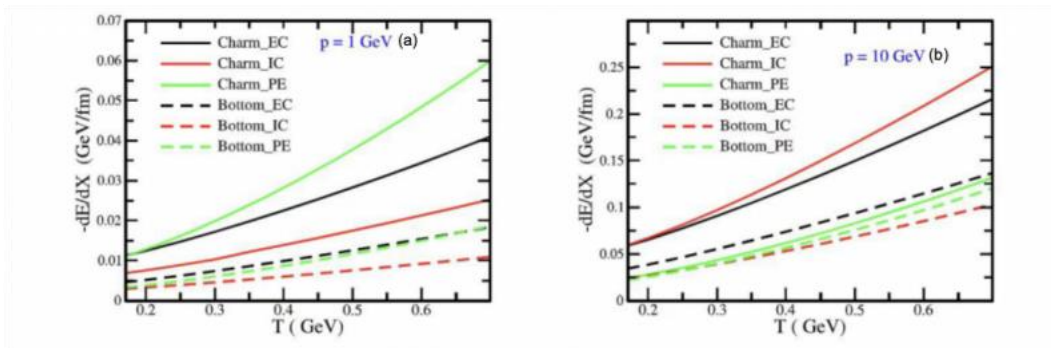


Figure 11. Comparing the polarization effects (PE) of charm quarks and bottom quarks, as well as the impact of elastic collisions (EC) and radiation (IC), on the temperature-dependent energy loss for hq at $p = 1.1$ GeV (left) and $p = 10$ GeV (right).

The energy loss of QGP with respect to the HQs momenta, p , is depicted in Fig. 10 for fixed temperatures across all three probable cases: polarization-induce energy loss, elastic collision-induced energy loss, and inelastic collision-induced energy loss. The energy loss through all three processes is observed to increase with momentum. At low momentum, the polarization effect dominates and then

saturates, as the momentum approaches to 10GeV, the polarization is inhibited, and the energy loss is dominated by elastic collision and gluon radiation. Compared with elastic collision, the energy loss caused gluon radiation is larger.

When plotting the energy loss of temperature, as shown in Fig. 11, a similar conclusion can be drawn. For charm quarks, the polarization effect prevails in energy loss at $p = 1 \text{ GeV}$ (left), while at $p = 10 \text{ GeV}$ (right) the polarization is subdued while the collisions and gluon radiation effects play a leading role in energy loss, and the radiation loss here is the highest of these three cases.

The charm quark is observed to exhibit a greater energy loss compared to the bottom quark at the same parameters, which supports the idea that light particles heat up relatively quickly.

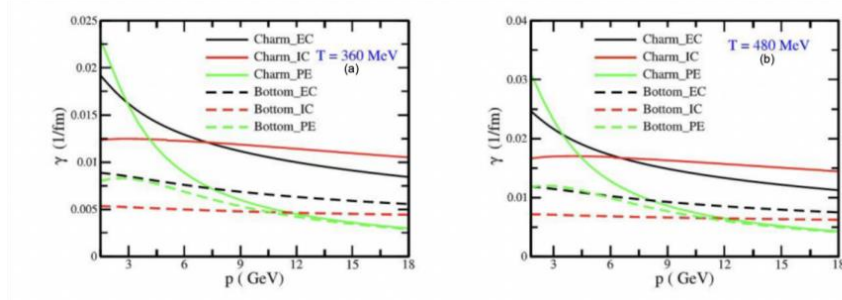


Figure 12. Comparative momentum dependence of drag coefficients for charm and bottom quarks considering the influence of polarization (PE), elastic collisions (EC), and radiation (IC) at temperatures $T = 360 \text{ V}$ (left) and $T = 480 \text{ V}$ (right).

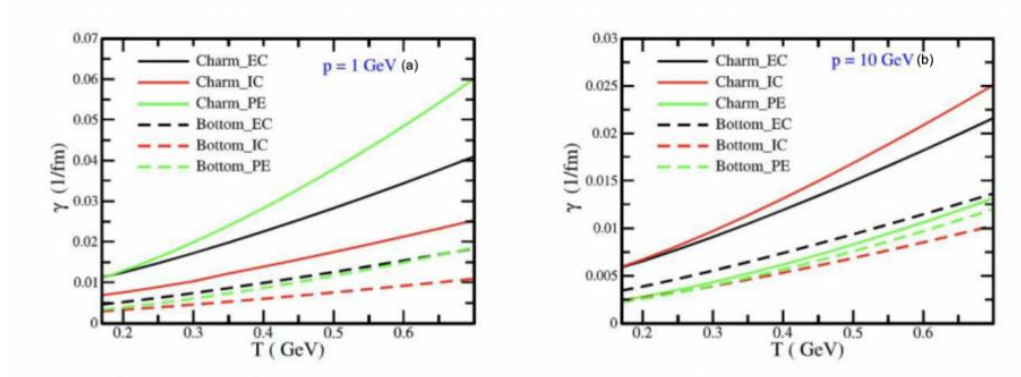


Figure 13. Comparative temperature dependence of drag coefficients for charm and bottom quarks considering the influence of polarization (PE), elastic collisions (EC), and radiation (IC) at temperatures $p=1\text{GeV}$ (left) and $p=10\text{GeV}$ (right).

The drag coefficients in Figs.12 and 13 are presented using the expression $\gamma = -\frac{1}{p} \frac{dE}{dx}$ for all three processes considered. The findings reveal that, in contrast to energy loss, the drag coefficient exhibits a decrease as HQs momenta rise across all three processes. At low momentum, polarization effects dominate for charm quarks, while elastic collision effects dominate for bottom quarks at both temperatures. The drag coefficient which due to the radiation exhibits the least magnitude at first, but it gradually becomes dominant as momentum increases. Furthermore, Fig. 13 illustrates a similar pattern of drag effect as energy loss, albeit with slightly suppressed magnitude at higher temperatures.

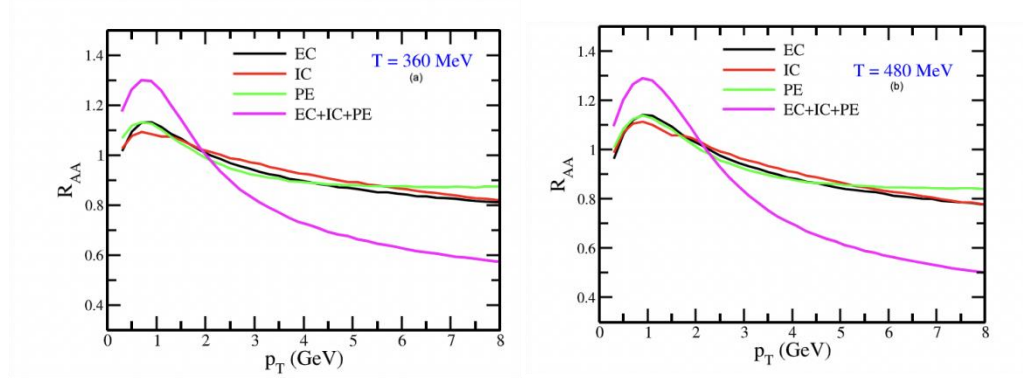


Figure 14. present the R_{AA} plots for charm quarks considering all three processes at two distinct temperatures

In Fig.14 we present the R_{AA} plots for charm quarks considering all three processes at two distinct temperatures, namely $T = 0.360 \text{ GeV}$ (left) and $T = 0.480 \text{ GeV}$ (right). At low p_T values, there is an overlap in the RAA of the three processes; however, at high p_T values, the polarization process exhibits a lesser degree of suppression compared to the other two processes. Furthermore, as anticipated, an increase in temperature leads to a stronger level of suppression.

This result well explains the error of the experimental and calculated results in the third part [6,7].

6. Conclusion

From the above experiments, we can learn: (a) Compared to a cold matter, the QGP exhibits a higher susceptibility to energy dissipation in a heated environment; (b) The threshold momentum value of heavy quarks obviously increases with the increase of temperature, but the radiation below heavy quarks does not exist; (c) Energy loss is primarily governed by two-body processes and the principle of energy conservation, rather than being solely dictated by thermodynamics; (d) The change in energy loss of HQ relative to HQ momentum is observed to increase with increasing momentum. Notably, the polarization effect exhibits a significant impact at low momenta. Furthermore, under identical parameters, charm quarks experience greater energy loss compared to bottom quarks. Additionally, at higher temperatures, the magnitude of energy loss escalates; (e) For charm quarks, due to mass reduction, the polarization effect predominates when momentum $p=1 \text{ GeV}$, whereas at very high momentum $p=10 \text{ GeV}$, the effect of polarization is smaller than elastic collision and gluon radiation, which predominate. However, for bottom quarks, due to their high mass, the polarization effect, elastic collision and gluon radiation are almost led to a same calculate result; (f) After analyzing all three processes, it was observed that the HQ drag coefficient cut down with increasing HQ momentum in all polarization effect, elastic collision and gluon radiation cases, unlike energy loss at a fixed temperature. The Fig depicting the drag effect demonstrates an energy loss pattern with a slight decrease in amplitude as the temperature rises.

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