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# Interdisciplinary applications of finite group theory—From epidemic modeling to financial market analysis

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**Abstract.** Finite group theory has a long history. Many mathematicians have done researches on it. As time goes on, it becomes a useful tool for human as it has a lot of applications. Finite group theory becomes more than just mathematical. As a result, its applications on epidemic modeling and financial market analysis are introduced respectively. These focus on SIR (Susceptible-Infective-Recovered) model and stock markets. What's more, there are some other applications in different areas. What we came to the conclusion is that wired groups have developed in many fields, not only in financial markets and infectious disease models, but also in many other fundamental disciplines, chemistry, physics, computer science. They are going to be showed in the following essay.

**Keywords:** Finite group. Epidemic model. Financial market.

## 1. Introduction

The definition of finite group is a group with finite many elements, which is one of the important contents of group theory. The number of elements it contains is called the order of a finite group. Finite groups can be divided into two main categories: solvable groups and non-solvable groups. Finite groups have composite group columns or principal group columns, and any two composite group columns or principal group columns are equivalent. Finite group has a long and complex history. The classification of finite groups is an important mathematical problem. Many mathematicians have worked hard to find a perfect answer to this question, such as finite groups of prime order are cyclic groups. In history, many abstract theories and concepts have their origins in finite group theory. There are many important events. Galois found about the concept of a normal subgroup in 1832. Then, Camille Jordan flagged Calois' distinction between two groups [1]. Also, in Trait, Jordan set out to build a database of finite simple groups with at least 5 alternating groups and most classical projective linear groups on the prime cardinal field. Finally, in 1872, Ludwig Sylow published his theorem on power subgroups of prime numbers. However, Holder's paper is a milestone. The holder laid down a challenge and was quickly decided by Frank Cole to order all the simple group 500 in 1892 [2] (except for some uncertainties with the 360 and 432 years), and in 1893 [3] extended this 660, discovering a new simple group SL (2, 8). Finally, the first paper mentioning the family of infinitely finite simple groups, starting with a hypothesis on the structure of these subgroups, was published by Burnside in 1899 [4].

In recent years, since the last paper of the first proof of the classification theorem of finite single groups (hence shortened to classification) was likely to be published in 2001 or 2002 (Aschbacher and S. D. Smith's classification of characteristic quasi-thin single groups), classification efforts came close to accurately spanning the 20th century. With the rapid development and increasing application of finite group theory, finite group theory has become one of the mathematical foundations of modern science and technology, and it is a mathematical tool that most scientific and technological workers are willing to master. In addition, finite group theory occupies a prominent position both from the theory itself and from the practical application. Its permutation groups, solvable and non-solvable groups, nil potent groups, and group representation theory are all important research objects.

According to many papers and researches about finite group theory and its applications, this paper summarizes some of them in order to help make finding information more convenient. The main objective of this paper is to introduce the basics of finite group theory and discuss the application and relationship of finite group theory in epidemiological models and financial markets. Moreover, it summarizes the functions of finite group theory in other fields.

## 2. Basics of Finite Group Theory

### *2.1. Definition and main concepts*

Even though one may be inclined to think that finite groups are easier to understand, this is not necessarily true. However, finite groups are absolutely fundamental to the development of algebra, and more generally, mathematics. They are a rich source of examples that have led to deep and beautiful mathematical theories. They have also given us insight into infinite groups. But their intrinsic structure could be incredibly complex and quite fascinating.

We have chosen to present H. Wieland's proof of the existence of Sylow subgroups, even though the theorem itself dates back to 1872. This proof is concise and efficient but also has a few limitations. It was based on a technique that didn't seem to be applied, and it turned out not to be really constructive; It gives no guidance on how to find Sylow subgroups in practice. But Wieland's proof is beautiful, and that is our main motivation for presenting it here. Moreover, Wieland's proof provides us with an excuse to quickly review the theory of group actions, which are almost as ubiquitous in the study of finite groups as Sylow's theorem itself.

The set G, with its finite collection of elements, is referred to as finite if there are only a finite amount. If it is infinite, then the group is known as infinite.

### *2.2. Theorems and results*

Lagrange's theorem, named after Joseph Louis Lagrange, is a theorem in the field of group-theoretic mathematics that states that the order of any finite group G is divisible by the order of every subgroup of G. A further modification of the theorem states that for any subgroup H of a finite group G, the value of the index [G:H] is not only an integer, but also a left cosine of H in G.

Proof: A left coset of H in G is viewed as an equivalence class of an equivalence relation on G. If there is h in H such that x = yh, then x and y are equivalent in G. The left cosets form a partition of G. Thus, the left cosets constitute a partition of G. Every left coset aH has the same cardinality as H, since x maps to ax, which defines a bijection from H to aH (with the inverse being y maps to a-1y). The number of left cosets equals the index [G: Therefore, applying the three previously mentioned sentences leads to the equation |G|=[G: H]·|H|.

The order of any element a in a finite group, also known as the smallest positive integer number k where ak=e and e represents the identity element of the group, is a divisor of the order of the group.

This is a theorem that is induced by the fact that the order of the cyclic subgroup generated by a is equivalent to a's order. If the group has n elements, it follows an=e. Fermat's Little Theorem and its generalisation, Euler's Theorem, can be demonstrated using this method. It is worth noting that these specific cases were already known before the general theorem was proved. Furthermore, it is demonstrated that any prime-ordered group is both cyclic and straightforward, as the subgroup created by any non-identity element must be the whole group. Lagrange's theorem can demonstrate that there is an infinite number of primes if there was a prime p with the greatest value. The prime divisor q of the Mersenne number 2p-1 satisfies 2p≡1 (mod q) (see modular arithmetic). This implies that the order of 2 in the multiplicative group (Z/qZ)* is p. According to Lagrange's theorem, the order of 2 has to divide the order of (Z/qZ)*, which is q-1. Thus, p divides q-1 and gives p.

In his article "Reflections on the Algebraic Solving of Equations", Lagrange stated that if a polynomial has n variables and these variables are permuted in all possible ways, then the resulting distinct polynomials will always be a factor of n! (For instance, if x, y and z are permuted in all 6 possible ways in the polynomial x+y-z, then the total number of polynomials obtained is 3 - x+y-z, x+z-y, and y+x-z). However, the theorem was not proven in its broadest form. The index of the symmetric group Sn for the subgroup H, which preserves the polynomial 3 multiplied by 6, is known as the size of H. This size divides n!, and Lagrange's result on polynomials, now referred to as the general theorem about finite groups, was later extended to abstract groups. For instance, the subgroup H in S3 comprises the identity and the transposition (x,y).

### *2.3. Tools and techniques used in applications*

G.A. Miller (2012) proposed that normality is not a transitive relation between the subgroups of a group. In other words, if K and H are both equal to G, it is not usually possible to conclude that K is equal to G. In accordance with Wielandt, who was the originator of much of the theory discussed in this chapter, we attempt to rectify this lack of transitivity by establishing a new relation. The notion of normality is extended by the transitive relation of subnormality, which states that a subgroup S in G is said to be subnormal if there exist subgroups Hi of G such that S=H0 ◃ H1 ◃ H2 ◃ …◃ Hr=G. This is expressed as S≪G. We emphasize that the chain of subgroups Hi, extending from S up to G, must be finite, even if G is an infinite group. It is not necessary for the subgroups Hi to be distinct, yet it is always feasible to eliminate repeated groups and to assume H0. The subnormal depth of S is denoted by the length of the shortest chain, or the least possible integer r.G's entirety has a depth of 0 within it; subgroups that are normal have a depth of 1, and those that are not normal have a depth greater than 1 in subnormal subgroups. Although we are concerned almost exclusively with finite groups in this book, it should be noted that part of the subnormality theory that we present also works for infinite groups. Some of the theorems that we prove by induction on the group order can also be proved by induction on the subnormal depth, and most of those results hold for infinite groups too. But people should be cautioned that many of the more interesting theorems about subnormality are simply false for general infinite groups.

## 3. Application in Epidemic Modeling

### *3.1. Definition of epidemic modeling*

Epidemic models don't have a long history. Daniel Bernoulli modeled the spread of smallpox in 1760 [3]. He was the first to come up with a mathematical model to explain infectious diseases. Epidemic models only began to develop in the 20th century. In 1906, Hamer constructed and analyzed a discrete-time model to research the repeated epidemics of measles. In 1926 Kermack and McKendrick constructed the famous SIR(Susceptible-Infective-Recovered) model [4]. Afterword, various epidemic models was built and used. Such as agent-based model [5] and Bayesian model [6].

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*3.2. How finite group theory is applied to model the spread of diseases*

Epidemic modelling is mainly based on algebra, statistics, etc. Among then, finite group theory can be used to describe population heterogeneity [7]. Finite group also can find changes in the composition and behavior patterns of crowds, it is useful for epidemiological modeling [8].

In a lot of time, epidemiological models are based on the situation at the time of the outbreak. This model works fine at small scales, but becomes much less useful as the population size increases [9]. The combination of finite group theory and some other theories can improve the usability of many epidemiological models in the face of large samples.

*3.3. Case studies and examples*

The use of finite group theory in epidemic modelling is not mainstream. But, there are also have some studies in this area. Finite group analysis can analyze the structure and symmetry of epidemic spread.

The transition matrix $T$ for SIS model considering $N$ two-state agents is [10]:

$$\hat{T} = \mathbb{1} - \beta \sum kj[Ajk(1 - \hat{n}j - \hat{\sigma} + j) + \Gamma\delta kj(1 - \hat{\sigma} - j)]\hat{n}k \quad (1)$$

Using finite group theory in practical applications can simplify the difficulty of application. In questions about transfer matrices, according to the given symmetry, use a suitable basis to create some small matrix, represented by some small matrix. Blocking can greatly reduce the computational pressure in computing such problems. Each block can be calculated independently, which reduces memory storage and retrieval. In particular, if only individual blocks need to be analyzed, the rest can be ignored. In this regard, calculations can also be greatly simplified [7]. This is a good attempt to apply finite group phenotypes to the transition matrix of epidemiological models.

## 4. Applications in financial market analysis

*4.1. Introduction to financial market analysis*

An introduction to financial market analysis is necessary before describing the uses of finite group theory in this field. Due to their support of economic growth, financial markets are crucial to economies all over the world. According to the type of financial instrument, one of the most common methods to separate the financial markets is into the stock market, corporate bond market, Treasury bill market, and commercial paper market. The Central European financial and capital markets are relatively new, having emerged after 1990 [11]. “A market for financial claims can be viewed as the process or mechanism that connects the buyers and sellers of claims regardless of where they happen to be physically located.” [12] The goal of investors is to correctly predict price changes in the financial and commodity markets. They believe that past and present events and facts will determine what will happen in the future [13]. The options market is another thing. In this options market, volatility is always priced and traded. For instance, these investors will purchase puts and calls if they anticipate more market volatility.

*4.2. Finite group theory in financial market analysis*

The fractional Black-Scholes model with a -order temporal fractional derivative can be explained by the price volatility of the associated fractal transmission system. “The fractional Black-Scholes model with a temporal fractional derivative can be explained by the price volatility of the associated fractal transmission system. The fractional Black–Scholes model is used to price up and down options on stocks that do not pay dividends in the United States or Europe. When encountering fractional differential equations, because fractional derivatives are limited, an efficient and reliable numerical format must be obtained. [14] The solution of the Time fractional black scholes model is obtained using the boundary conditions.[15]

*4.3. Real world examples and success stories*

An interesting example about Dow-Jones provided by the referee is the following:

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$$x_{t+\mu} = \sum_{i=1}^{k} \alpha_i \, x_{t-i} + \sum_{i=1}^{n} \beta_i \, D_{t-i} + \epsilon_{t+\mu} \; \mu > 0 \; n < k \tag{2}$$

How many periods in the future one is anticipating are determined by the parameter μ. It encapsulates the argument that the moving average method only detects changes in long-term trends and is not always appropriate for forecasts one period out. Therefore, a value of μ that is bigger than 1 or 2 months should be chosen. In the empirical research described below, μ is chosen as a year. The results continue to be qualitatively comparable for μ greater than 12. The equation provided appears to make inference simple at first. But if μ>1, the errors of equation will be serially correlated. In fact, the error structure in these equations will always be given by a (μ -1) th-order MA process:

$$\epsilon_t = v_t + a_1 v_{t-1} + a_2 v_{t-2} + \cdots + a_{\mu-1} v_{t-\mu-1} \tag{3}$$

The $\{v_t\}$ are the innovations in the $X_t$process. Hannan's efficient method is used to correct them for the serial correlation. In reality, using common least squares , the$\{\epsilon_t\}$ can be approximated. Following that, the periodogram of these (first-stage) residuals is determined. The relevant entries of the square root are then divided by the Fourier transform of X. The time domain is then restored to this series. These modified data are used to estimate an equation. The three tables below provide empirical results. The outcomes are intriguing. The sub periods of 1795–1851 and 1852–1910 for the F-tests on the D are not significant (Tables 1 and 2). However, they have a considerable impact over the years 1911 to 1976 (Table 3). As a result, the specific moving average rule of 150 days appears to have a strong ability to anticipate the future. A rule like this is really useful. In the last period, The dummy variable's lags that represent the buy(+1), sell(-1), and no action(0) signals are all statistically significant. The signals are also all favorable, indicating that things are moving in the right direction. Intriguingly, the dummy variable's coefficients exhibit a pleasing reverse V shape, with the peak arriving at lag 23. (Table 1).

Therefore, the approach does appear to have some predictive validity beyond the delays of the Dow-Jones industrial. With the exception of μ=1, the results shown in these tables really remained the same when alternative values for were employed. In this case, the 150-day moving average proved to be irrelevant in all equations. [16].

**Table 1.** Dow-Jones industrials, 1792:1-1851:12

| Label and Lag | Coefficient | t-statistic |
|---|---|---|
| Dow-Jones industrials | | |
| 18 | .83 | 3.8 |
| 19 | -.14 | -.45 |
| 20 | .03 | .05 |
| 21 | .02 | .03 |
| 22 | .03 | .05 |
| 23 | -.03 | -.09 |
| 24 | -.05 | -.15 |
| 25 | -.01 | -.04 |
| 26 | .01 | .04 |
| 27 | -.05 | -.15 |
| 28 | .01 | .05 |
| 29 | .01 | .05 |
| 30 | -.05 | -.15 |

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**Table 1.** (continued)

| | | |
|---|---|---|
| 31 | .01 | .04 |
| 32 | .26 | 1.34 |
| Dummy: | | |
| 18 | .45 | 1.16 |
| 19 | .63 | 1.39 |
| 20 | .67 | 1.35 |
| 21 | .58 | .92 |
| 22 | .61 | 1.06 |
| 23 | .50 | .71 |
| 24 | .32 | .69 |
| Constant | 1.51 | 7.12 |

Note: R^2=.73; sum of square of residuals =3592; F-statistic=.46. 1792:1 =January1792

**Table 2.** Dow-Jones industrials, 1852:12-1910:12

| Label and Lag | Coefficient | t-statistic |
|---|---|---|
| Dow-Jones industrials | | |
| 18 | .86 | 4.60 |
| 19 | .00 | .01 |
| 20 | -.08 | -.31 |
| 21 | .04 | .17 |
| 22 | -.21 | -.80 |
| 23 | -.09 | -.34 |
| 24 | .08 | .29 |
| 25 | -.06 | -.20 |
| 26 | -.04 | -.16 |
| 27 | .07 | .27 |
| 28 | -.16 | -.66 |
| 29 | -.06 | -.24 |
| 30 | .06 | .23 |
| 31 | -.01 | -.02 |
| 32 | .55 | 3.15 |
| Dummy: | | |
| 18 | .53 | .67 |
| 19 | .85 | 1.0 |
| 20 | 1.1 | 1.3 |
| 21 | 1.5 | 1.9 |
| 22 | 1.9 | 2.4 |
| 23 | 1.8 | 2.3 |
| 24 | 1.2 | 1.8 |
| Constant | 4.1 | 5.4 |

Note: $R^2$=.72; sum of square of residuals =36120.202; F-statistic=.050330. 1852:12 =December 1852

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**Table 3.** Dow-Jones industrials, 1911:1-1976:12

| Label and Lag | Coefficient | t-statistic |
|---|---|---|
| Dow-Jones industrials | | |
| 18 | .19 | 2.34 |
| 19 | -.04 | -.39 |
| 20 | .01 | .05 |
| 21 | .18 | 1.36 |
| 22 | .05 | .34 |
| 23 | .04 | .25 |
| 24 | .19 | 1.31 |
| 25 | .12 | .78 |
| 26 | .01 | .04 |
| 27 | .09 | .61 |
| 28 | .02 | .13 |
| 29 | .07 | .48 |
| 30 | .02 | .14 |
| 31 | -.01 | -.05 |
| 32 | .09 | 1.11 |
| Dummy: | | |
| 18 | 26.9 | 2.9 |
| 19 | 28.3 | 2.9 |
| 20 | 28.4 | 2.9 |
| 21 | 28.2 | 2.9 |
| 22 | 30.1 | 3.1 |
| 23 | 30.8 | 3.2 |
| 24 | 21.8 | 3.4 |
| Constant | 19.6 | 4.6 |

Note: R^2=.91; sum of square of residuals =6521516; F-statistic=3.71. 1911:1 =January 1911[15]

## 5. Other Emerging Applications

Finite group theory is not only applied in the field of mathematics, but also plays an instrumental role in many other disciplines. In this chapter, the application of finite group theory in different disciplines and daily life will be described in detail.

### *5.1. Physics*

The first is the application of finite group theory to physics. Among them, the concept of quantum theory is considered in terms of “finite”. Introduce a continuous or actually infinite destructive structure into physics without the need for them to describe their observations. The results show that quantum behavior is a natural result of the symmetry of dynamical systems. The main reason is that people cannot find out the identity of indistinguishable objects during their evolution - only the information about the words and data of these phenomena is available. Mathematics shows that all quantum dynamics can be reduced to a series of permutations. Quantum phenomena, such as interference, produce dynamic systems of symmetric groups that arrange invariant sub spaces.

Observable can be expressed in terms of permutation in variants. The results show that nonconstructive number systems, such as complex numbers, do not describe quantum phenomena, but only use partial ring numbers, a minimal extension of the natural numbers applicable to quantum mechanics. The use of finite groups in physics is currently the basis of the method, but there is an additional motivation. Many experiments in particle physics have shown that relatively small finite groups are important in some fundamental processes [17, 18].

### *5.2. Computers*

The second is the application of finite group theory in computers. This is mainly reflected in three aspects. First, we need to prove that revertible cyclic cellular automat with periodic boundary conditions is virtual. These virtual machines have two characteristics, one of which is that there are active and inactive units at a specific time, and there is a certain periodicity. These properties relate to the finite group theory of the cellular machine, and it is proved that the inverse transition two polynomials are invariant. Then we use REN algorithm to generate examples of virtual cyclic cellular automat to generalize cellular automat in collective control and traffic modes [19, 20].

### *5.3. Future potential of finite group theory*

The theory of finite groups will be developed continuously in various fields in the future. For example, it is found that the application of finite groups in signal processing has not been fully developed. Due to the current signal processing algorithm, each different signal processing transformation has a unique ability, simply put, different types of signals have different combinations. So there is still room for improvement in the signal space of finite groups, and with the current background knowledge, the implementation of using multiple transformations to process a signal. The purpose is achieved by using a group generator as an action on the signal to generate an output signal for each input signal. Therefore, in the future, it is possible to establish a transformation through finite group theory, but to represent a different signal processing method [21].

## 6. Conclusion

Nowadays, with the development of science and technology, finite groups are becoming more and more important in the development of various fields, and it is a very useful tool. And we looked at a lot of literature and existing papers and integrated them, hoping to make it easier for people to look up. We also learned from the literature about finite groups and the basis of epidemiological models. We also show the performance of finite groups in the financial market through the form of tables and data. This article mainly describes the basic definition of finite groups and the development of finite groups in various fields, mainly in epidemiological models and financial markets. This article also explores the use of finite groups in other fundamental disciplines such as physics and computer science, and finally we discuss our vision for the future and the role of finite groups in the future.

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