Applications of fourier transforms in engineering

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Abstract. The Fourier transform was proposed by Fourier in 1807. Fourier transform is a method of analyzing signals, which can analyze the components of the signal or synthesize the signal using these components. Many waveforms can be used as signal components, such as sine waves, square waves, sawtooth waves, etc. The Fourier transform uses sine waves as components of the signal. Due to its excellent properties, the Fourier transform has a wide range of applications in physics, number theory, combinatorial mathematics, signal processing, probability, statistics, cryptography, acoustics, optics, and other fields. This paper focuses on the study of fractional-order Fourier transform in the engineering field, for the equipment of the tiny fault diagnosis method, and according to some existing diagnostic methods, put forward the idea of diagnostic method enhancement.

Keywords: fractional-order Fourier transform, tiny fault diagnosis

1. Introduction

The traditional Fourier transform is suitable for smooth signals. However, it still has limitations and is not so effective in solving all problems in practical applications. For example, the analysis of systems that consist of non-smooth signals is very insufficient, which is due to the Fourier transform using the global basis function decided. Therefore, the fractional order Fourier is proposed, and the fractional order Fourier can show good characteristics in analyzing certain non-smooth signal transforms.

The mathematical definition of the Fourier series states that a non-sinusoidal function (signal) can be stated by an infinite number of sums of sinusoidal with frequencies that are integer (including zero) multiples of its fundamental frequency. The Fourier transform expands the period of integration to the infinity of the formation.

Fractional Fourier transform can be defined from the perspective of integration, from the perspective of eigenvalues and feature functions, and from the perspective of time-frequency surface rotation, and the three definitions are equivalent to each other. The following will introduce the definition and properties of FRFT from the above three:

From the perspective of integration, the Fourier transform of function $\mathrm{f}(x)$is represented by $\{Ff\}(x)$ and the integral power F^j of operator $\mathrm{F} \equiv \mathrm{F}^1$ can be defined as its continuous application. Then, there are $\{F^2 f\}(x) = f(-x)$ and $\{F^4 f\}(x) = f(x)$. The a-order fractional Fourier transform of function $\{F^a f\}(x)(0 < |a| < 2)$ can be defined as:

$$F^a[f(x)] \equiv \{F^a f\}(x) \equiv \int_{-\infty}^{+\infty} B_a(x, x')f(x')dx'$$

$$B_a(x,x') \equiv A_\varphi e^{i\pi(x^2 cot\varphi - 2xx' csc\varphi + x'^2 cot\varphi)}$$

$$A_\varphi = \frac{e^{-i\pi\, sgn(sin\varphi)/4 + i\varphi/2}}{|sin\varphi|^{\frac{1}{2}}}$$

Among them, φ is the rotation angle of the fractional Fourier transform, $\varphi = \frac{a\pi}{2}$, i is the imaginary unit, x is a variable in the time domain, x' is a variable in the a-order fractional domain, $B_a(x,x')$ is the kernel function of the fractional Fourier transform, A_φ is the amplitude of the fractional Fourier transform. For $a=0$ and $a = \pm 2$, there is a kernel function

$$B_0(x,x') \equiv \delta(x - x')$$

$$B_{\pm 2}(x,x') \equiv \delta(x + x')$$

Where $\delta(x)$ is the Dirac function.

2. Application of Fractional Order Fourier Transform to Detection of Minor Faults in Industrial Equipment

2.1. Related Research

The fractional order Fourier transform can be regarded as a generalized form of the ordinary Fourier transform. Compared with the ordinary Fourier transform, it is more capable of reflecting the joint characteristics of time and frequency of the signal. And it has the advantages of stronger noise suppression, high time-frequency resolution, and absence of cross terms [6]. Because most of the data signals in industrial processes are non-stationary signals, ordinary Fourier transform is not enough to analyze their significant features. Fractional order Fourier transform can be used to analyze the problem from the perspective of the most concentrated information, that is, to select the result with the largest amplitude among the results obtained by different fractional orders, and then the fractional order of this result is the optimal order. Moreover, the fractional order Fourier transform can reduce the estimation distortion and the mean square deviation of the noise signal, which is more advantageous for dealing with multi-component industrial process fault data. Wiener [10] firstly proposed the concept of Fractional Order Fourier Transform (FRFT) in 1929. He proposed a new transform kernel, whose eigenfunction is a Hilbert-Gaussian function but whose eigenvalue form is more complete than that of the common. In 1937, Condon independently studied some basic definitions of the FRFT, and in 1939, Kober defined the FRFT as a fractional power of the Fourier transform. 1980, Namias reintroduced the concept of the FRFT in a purely mathematical way and applied it to the solution of partial differential equations. In 1992, Mendlovic and Ozaktas introduced the FRFT from Wigner's work. Ozaktas redefined the FRFT in terms of the Wigner distribution, interpreted its physical meaning as the rotation of the signal's representation axis in the time-frequency plane, and successfully applied it to the field of optics. In 1996, Ozaktas et al. [11] proposed the discrete fractional Fourier transform (DFRFT), which is a method for solving partial differential equations. It decomposes FRFT into convolutional form and realizes DFRFT by Fast Fourier Transform. 2007, Xinghao Zhao [12] proposed a new fast algorithm for fractional order Fourier transform. Fractional order Fourier transform has been gradually utilized in feature extraction and fault detection because it can transform from a time domain to a frequency domain step by step, decompose the conversion process in detail, and show all the features of the signal from the time domain to the frequency domain completely.

With the increasing complexity of modern industrial production systems, large industrial systems are more prone to major safety accidents. To ensure that the industrial production process can operate stably for a long period of time, the timely detection of early failures or minor failures has become a problem. When monitoring industrial processes, if the occurrence of faults is caused by very small changes, it is difficult to be noticed in the early stage of occurrence, and often causes a series of reactions in the later stage, and eventually leads to accidents, which poses a challenge for detecting these tiny faults. Because of the small deviation from normal values and low amplitude of minor faults at the early stage of occurrence, and the slow process of change compared to obvious faults, the impact on the system is almost imperceptible. However, the accumulation of these glitches over a long period of time can have serious consequences for the system. If the fault information can be detected in the early stage of the failure, timely intervention and adjustment, a series of unnecessary losses can be avoided. Therefore, the detection of minor faults is particularly important in industrial systems.

Most of the micro-fault data signals in modern industrial processes are non-stationary signals, and the ordinary Fourier transform is not enough to analyze their significant features. Using the fractional-order Fourier transform, the time domain amplitude characteristics of the micro-fault signal are not obvious to transform the fractional-order domain, in the amplitude characteristics of the obvious fractional-order domain to analyze the information of the micro-fault and detection can effectively improve the detection rate of the micro-fault. Moreover, the fractional-order Fourier transform can reduce the estimation distortion and the mean square deviation of the noise signal, which is more advantageous for processing multi-component industrial process fault data.

Let the original data matrix be $X, X \in R^{m*N}$. Among them, *m* is the number of control variables and N is the number of observation samples. The covariance matrix corresponding to $C = \frac{1}{N-1}XX^T$. Because covariance matrix *C* is an *m*-dimensional real symmetric matrix, *m* unit orthogonal vectors can be found to make covariance matrix *C* similar to diagonalization. Let *m* feature vectors be a_1, a_2, a_3,.. form a matrix $D = (a_1, a_2, a_3)$ in columns, then $\mathrm{C} = \frac{1}{\mathrm{N}-1}XX^T = D\Lambda D$. Among them, Λ is a diagonal matrix, and its diagonal elements are the corresponding eigenvalues of each eigenvector λ, the variance of each variable.

$$\Lambda = D^T C D = diag(\lambda_1, \lambda_2, \ldots \lambda_m)$$

Enlarge the variance of all variables λ times which means amplifying the eigenvector of the covariance matrix C λ Times. Namely,

$$\Lambda^{1/2} D^T C D \Lambda^{1/2} = \Lambda^2 = diag(\lambda_1{}^2, \lambda_2{}^2, \ldots \lambda_m{}^2)$$

Perform a base transformation on data *X*, where *Y=PX*, *P* is the base matrix, and *Y* is the data obtained after *X* performs the base transformation on *P*, so that the fault data after the base transformation is independent of each other and the eigenvectors are amplified λ times. Let the covariance matrix of *Y* be *D:*

$$D = \frac{1}{n} YY^T = \frac{1}{n} PXX^T P^T = P(\frac{1}{n} XX^T) P^T = PCP^T$$

And from equation, it can be seen that $\mathrm{D} = \Lambda^2 = \Lambda^{1/2} D^T C D \Lambda^{1/2}$. From this, the base matrix $P = \Lambda^{1/2} Q^T$. Multiplying the base matrix *P* with the original data matrix *X* yields the data matrix Y after scaling the feature vector by λ times.

At the same time, the main reason why it is difficult to detect small faults is that the amplitude characteristics are not obvious, the transformation rate is slow, and it is easily masked by interference signals. Fractional order Fourier transform can transform the digital signal with insignificant amplitude characteristics in the time domain to the fractional order domain with more obvious amplitude characteristics and filter part of the high-frequency interference signals and noise signals. Moreover, most of the modern industrial systems are nonlinear, which increases the difficulty of detecting small faults. Kernel Principal Component Analysis (KPCA) can map the nonlinear data into a linearly divisible high-dimensional space, extract the principal elements in the high-dimensional feature space, and complete the fault detection.

In summary, compared to obvious faults, minor faults are characterized by low rate, low frequency, smaller amplitude, and easy to be overwhelmed by disturbances and noise [1]. One of them is a potential fault with a small deviation from the normal operating condition, but after a long-time accumulation, it can lead to a safety problem in the system. For example, in electric drive systems, failures are caused by the presence of ring defects within the bearing runners [2-3]. The other type of faults is early faults that have less impact on the system. Early faults have low amplitude in the pre-fault period. However, during the evolution of the fault, it develops into an obvious fault once it exceeds a defined maximum value,

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such as faults in industrial systems due to wear and tear and normal aging of the devices [4], and faults in transformers of power supply system's due to arc discharges [5]. Therefore, the study of detection of minor faults is of theoretical significance and practical value. Generally, a fault in an industrial process is defined as a deviation of at least one system indicator or variable beyond the tolerance range. The general process of fault detection and diagnosis is to analyze the production state of an industrial system, then determine whether an abnormality occurs in the system, and if a fault occurs, further determine the cause of the fault, and finally draw a conclusion. When a fault occurs in the production process, the task of fault detection is to be able to detect the fault in a timely and accurate manner. However, due to the limitations of the existing fault detection methods due to the level of technological development, there are usually false alarms and omissions, and we cannot make a completely accurate judgment on whether the fault occurs. Moreover, the process data of modern complex industrial systems are characterized by diversity, nonlinearity and sea quantization, and traditional fault detection methods are difficult to apply in modern industrial processes. Therefore, how to mine effective information based on these data and construct an efficient intelligent fault detection model, so as to realize the rapid and accurate judgment of the tiny faults occurring in the industrial process is the biggest challenge in the field of industrial fault detection.

Wiener first proposed the concept of Fractional Fourier Transform (FRFT) in 1929. The fractional Fourier Transform is the same time-frequency transform, reflecting the signal's information in both time and frequency domains. As the order grows continuously from 0 to 1, the fractional-order Fourier transform shows all the characteristics of the signal that change gradually from the time domain to the frequency domain, which can provide a greater choice for the time-frequency analysis of the signal. Moreover, because it can transform from the time domain to the frequency domain step by step, decompose the conversion process in detail, and show all the features of the signal from the time domain to the frequency domain, it has gradually started to be utilized in feature extraction and fault detection.

The main reason why it is difficult to detect micro faults is that the differentiation between micro fault data and normal data is not obvious. Suppose the eigenvectors of the glitch data are scaled, and the variance of the data is enlarged. In that case, the main features of the glitches can be emphasized, making the differentiation between the glitch data and the normal data obvious and thus improving the detection rate of glitches. The eigenvalues of the data, i.e., the variance, portray how much the data fluctuates with respect to its mathematical expectation. The data has different values projected in different directions in space, and the projection direction can be determined by a set of bases. So, the ultimate goal of eigenvector scaling of data is to find a set of bases such that the variance value of the data becomes large after the data is subjected to base transformation. So, the sampled values need to be computed by the discrete FRFT algorithm. Currently, the main ways to implement fast algorithms for DFRFT are weighting methods, decomposition methods, eigenvalue and eigenvector methods, and direct discretization of the FRFT [13]. a discrete algorithm introduced by Haldun M. Ozaktas and Orhan Arikan [11] et al. The decomposition method proposed by Ozaktas decomposes the expression of the FRFT into the signal's convolutional the decomposition method proposed by Ozaktas decomposes expression of FRFT into the convolutional form of the signal, and utilizes the FFT to compute the FRFT, the computational speed is comparable to that of the FFT, and it is one of the faster methods. Zhao Huimin et al [6] proposed a fault diagnosis model based on fractional-order Fourier transform and long and short-term memory network, which solved the problem of weak fault features that are difficult to be extracted and recognized in the early fault diagnosis of transmission gears. Wu Xin [7] applied fractional-order Fourier transform to preprocess and feature extraction of the collected fault signals to obtain the feature vector suitable for subsequent fault classification, and effectively extracted the fault features in the inverter circuit. Peng Zhang [8] applied the fractional order Fourier transform to the fault diagnosis of rotating machines and achieved good innovative results. Luo Hui et al [9] proposed a fault feature extraction method for analog circuits based on optimal fractional-order Fourier transform (FRFT), which enhances the differentiability of different fault modes in analog circuits and improves the accuracy of diagnostic results. However, most of the above methods are researched for fault diagnosis. Moreover, most of the application scenarios are small industrial systems, and there are few studies on

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the detection of tiny faults in large industrial systems. Therefore, this paper applies the fractional order Fourier transform to studying tiny fault detection in industrial processes, which has certain theoretical significance and practical value.

In addition, fractional order Fourier transforms must be analyzed simultaneously in conjunction with PCA analysis methods. In the last decades, the need for effective quality monitoring and safe operation in the chemical industry has driven the research of statistically based methods for fault detection and diagnosis. Multivariate statistical methods such as Principal Component Analysis (PCA) [13-15], Partial Least Squares (PLS) [16-18], and Independent Component Analysis (ICA) [19-21] have been developed and applied to this process. Among them, PCA is the most popular one. However, these methods assume that the relationship between the variables is linear, and they are no longer applicable to nonlinear systems. To address the nonlinear challenges in industrial processes, Mika S [22] et al. proposed a new kernel principal component analysis (KPCA) method, which has been rapidly developed since then. KPCA can efficiently compute principal elements in high-dimensional feature spaces by means of dot product operations and nonlinear kernel functions. The main advantage of KPCA over other nonlinear methods is that it does not involve nonlinear optimization and essentially requires only algebraic operations, making it as simple as the standard PCA process. The method only needs to solve the eigenvalue problem, and using different kernel functions allows it to handle many different nonlinear processes. In addition, KPCA does not require the number of principal elements to be specified prior to extracting features from the data. Due to these advantages, KPCA has shown better performance than linear PCA in feature extraction and classification of nonlinear systems. The basic idea of kernel principal element analysis (KPCA) is to first map the data space to the high-dimensional feature space through a nonlinear mapping. Then, principal elements are computed in the high-dimensional feature space. In general, principal element analysis can effectively handle observations that vary linearly. However, when the variation is nonlinear, the PCA method is no longer applicable. According to the Cover theorem, nonlinear data in a low-dimensional space can always be mapped to a higher-dimensional space with linear variation. That is, the nonlinear data structure in the input space is more likely to be linear after a high-dimensional nonlinear mapping, and this high-dimensional linear space is called the feature space. kPCA essentially constructs a nonlinear mapping from the low-dimensional input space to the high-dimensional feature space, and by introducing the kernel function, the agnostic nonlinear mapping function can be computed explicitly by the kernel function. kPCA can be regarded as a general-purpose nonlinear PCA extension that can utilize integral operators and nonlinear kernel functions to compute principal elements in high-dimensional feature spaces efficiently. Moreover, the fact that KPCA can contain different kernel functions allows KPCA to handle a wide range of nonlinear problems. In addition, KPCA does not need to predict the number of preserved principal elements (PCs), making it more efficient for nonlinear data processing.

2.2. Major problems

(1) In industrial systems with complex backgrounds, micro-faults are difficult to extract due to their weak fault characteristics, and they are highly susceptible to being overwhelmed by noise signals and interference signals, which increases the difficulty of detecting micro-faults.

(2) Currently, most fault detection methods have good results for fault detection of linear processes. However, for the detection of minor faults in nonlinear processes, most linear methods are not good at extracting the main features of minor faults.

(3) Currently, most industrial systems are closed-loop systems. Closed-loop systems will have a certain degree of fault-tolerance due to the compensating effect of their feedback control, resulting in tiny faults that are more difficult to detect.

3. Conclusion

For the problem that the amplitude of the glitches is small, the features are not obvious, and they are easily masked by interference and noise signals, the fractional-order Fourier transform glitch detection method based on the scaling of data feature vectors can be considered. Firstly, the feature vector of the

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glitch data is scaled to amplify the variance of the glitch data, highlight the main features of the glitch, and increase the separability of the glitch data from the normal data. Then, fractional Fourier Transform (FRFT) is performed on the scaled data to transform the data signals with inconspicuous characteristics in the time domain to the fractional order domain and analyze the information of the glitches in the fractional order domain where the glitch amplitude changes are most obvious. Not only can it overcome the shortcomings of the inconspicuous characteristics of the amplitude of the glitches, but it also filters part of the perturbation signal and noise signal. Finally, the principal component analysis (PCA) is used in the fractional order domain to detect micro faults.

In view of the nonlinearity, real-time problems, and small deviations generated by micro faults in industrial processes, the kernel principal element analysis method based on the sliding window technique fractional-order Fourier transform can be considered to detect micro faults. First, using the sliding window technique, the deviation generated by the tiny fault data within the window is accumulated by choosing the appropriate window width so as to realize the amplification of the tiny faults and achieve the degree of easy detection. Then, the processed data are transformed to the fractional order domain to amplify the amplitude of the tiny faults and make them easier to be detected. Finally, for the chemical process monitoring data characterized by high dimensionality and nonlinearity, the KPCA is used to complete the detection of tiny faults.

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