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Python-enabled dynamic analysis of earth-moon tidal forces

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Abstract. This study investigates the origin of tidal forces within the Earth-Moon system, formulating the corresponding dynamic equations and solving them using Python for computational modeling. This allows for the prediction of tidal intensity at specific locations and times. The methodologies and Python algorithms developed quantify the impact of Earth's tidal forces on the Moon, analyzing the deformation and thermal effects caused by tidal force variations across different distances. This framework also elucidates the mechanisms behind Io's volcanic activity. The computational approach, characterized by accuracy and precision, enhances the interpretability of the results through data visualization. This study lays a foundational basis for extending the analysis to other celestial bodies under diverse conditions, contributing to the broader field of celestial mechanics.

Keywords: Scientific Computing, Tidal Force, Earth-Moon System, Volcanic Activity on Io, Uneven Gravity.

1. Introduction

Tidal forces exert a plethora of effects, ranging from their influence on virtual particles near black holes, resulting in Hawking radiation, to their role in the cosmic phenomena of galaxy engulfment [1].The increase in the Moon's internal temperature and the volcanic activities observed on Io [2] are attributed to their influence, with further implications for the mechanisms underlying supernova formation [3]. This manuscript presents a computational methodology for the examination and analysis of tidal forces, employing computational simulations [4].

2. Earth-Moon Motion Model

The Earth and Moon revolve around their common center of mass, situated at a point between them known as the barycenter of the Cislunar system. Given that Earth's mass is 81.367 times greater than the Moon's, the barycenter is located approximately 4800 kilometers from Earth's center, denoted as point O in Figure 1. In the depicted model, m_e represents the mass of the Earth with its center of mass at point O_1, and m_m signifies the mass of the Moon with its center of mass at point O_2. The distance between these two centers is designated as L. The Earth-Moon system completes a rotation around the z-axis at a uniform angular velocity, with a period of 27.32 days.

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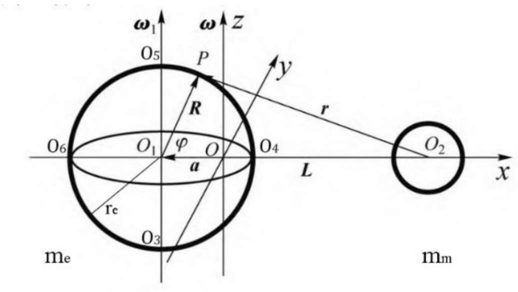

Figure 1. Coordinate Diagram of the Earth-Moon System

Consider an arbitrary mass m on the surface of the Earth. This mass undergoes rotation with an angular velocity ω around the Z-axis and simultaneously with an angular velocity ω_1 around the Earth's axis of rotation [5]. The dynamical equation for an object on the Earth's surface in an inertial coordinate system, derived from the product of mass and acceleration, is given by:

$$-\frac{Gm_e m_m \vec{R}}{R^3} - \frac{Gmm_m \vec{r}}{r^3} + F_N = m\vec{\omega}\times(\vec{\omega}\times\vec{a}) + m\vec{\omega}\times\left(\vec{\omega}\times\vec{R}\right) + 2m\vec{\omega}\times(\overrightarrow{\omega_1}\times\vec{a}) + m\overrightarrow{\omega_1}\times\left(\overrightarrow{\omega_1}\times\vec{R}\right) \quad (1)$$

In this study, the influence of the Coriolis force is not considered, and it is assumed that the rotation directions of ω and ω_1 are identical. Consequently, thetidal force F can be expressed as:

$$F = -\frac{Gmm_m\vec{r}}{r^3} - m\vec{\omega}\times(\vec{\omega}\times a) \quad (2)$$

According to Equation (2), the force condition of each particle on the Earth's surface varies in both direction and magnitude with the change in time and coordinates within the Cislunar system. The angular relationship at the intersection point P, where vectors $\mathbf{O_1P}$ and $\mathbf{O_2P}$ meet, can be expressed as:

$$\cos\theta = \frac{2r_e^2 + L^2 - 2r_e L\cos\varphi - L^2}{2r_e\sqrt{r_e^2 + L^2 - 2r_e L\cos\varphi}} \quad (3)$$

The distance from the center of mass **O** of the Earth-Moon system to the Earth's center of mass $\mathbf{O_1}$ is denoted by $\boldsymbol{\alpha}$. The force exerted on a point mass **dm** at point P on the ground due to the rotation of the Earth-Moon system is given by:

$$df_1 = dm\vec{\omega}\times\left(\vec{\omega}\times\overrightarrow{OP}\right) = dm\omega^2(r_m - a)\frac{2r_m^2 - 2r_m L\cos\varphi}{2r_m\sqrt{r_m^2 + L^2 - 2r_m L\cos\varphi}} \quad (4)$$

The gravitational force exerted by the Moon on a point mass **dm** located at point **P** on the Earth's surface is given by Newton's law of universal gravitation:

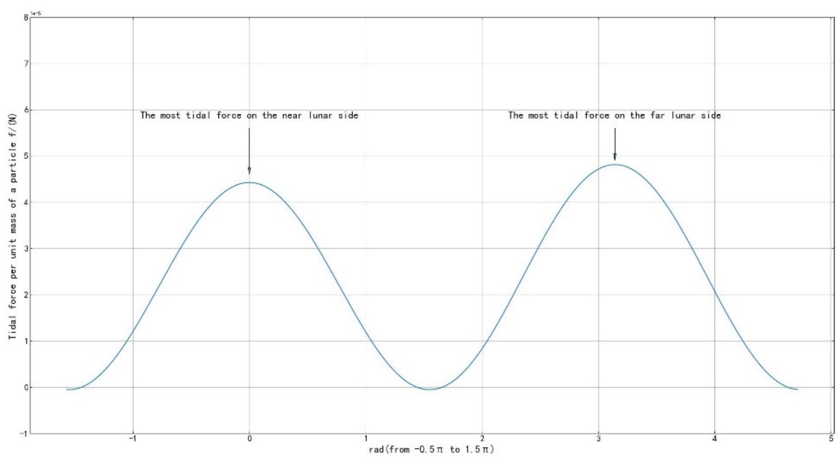

Figure 2. Distribution and Variation Map of Surface Tidal Forces

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$$df_2 = \frac{dmGm_m\vec{r}}{r^3} = dmGm_m \frac{2r_e^2 - 2r_eL\cos\varphi}{2r^2 r_e\sqrt{r_e^2 + L^2 - 2r_eL\cos\varphi}} \tag{5}$$

Based on the aforementioned mathematical principles, Python is utilized to calculate the force values experienced by particles at various points around the Earth and to visualize these data graphically.

3. Algorithms and Analysis

Algorithm 1: Strength calculation and variation trend of surface tidal force(Focus solely on presenting the core concepts and implementation methods)

Input: Data sets of physical quantities of the Earth-moon system

Output: Tidal force intensity change map

```
t=np.linspace(-np.pi/2,np.pi/2,500)
L=r_earth+r_earthtolunar+r_lunar
r_square=L**2+r_earth**2-2*L*r_earth*np.cos(t)
r=np.sqrt(r_square)
cos_angle=(r_earth**2+r_square-L**2)/(2*r_earth*r)
angle=np.arccos(cosangle)
f_nearside_gravitation=m_lunar*G/r_square*cos_angle
f_nearside_centrifugal=(r_earth*np.cos(t)-
r_earthcoretobarycentre)*np.cos(t)*w_spinoflunarencircleearth**2
f_nearside_vector=f_nearside_gravitation+f_nearside_centrifugal
t1=np.linspace(np.pi/2,np.pi*1.5,500)
r1_square=L**2+r_earth**2-2*L*r_earth*np.cos(t1)
r1=np.sqrt(r1_square)
cos_angle1=(r_earth**2+r1_square-L**2)/(2*r_earth*r1)
f_farside_gravitation=m_lunar*G/r1_square*cos_angle1
f_farside_centrifugal=-(r_earth*np.cos(t1)-
r_earthcoretobarycentre)*np.cos(np.pi-t1)*w_spinoflunarencircleearth**2
f_farside_vector=f_farside_gravitation+f_farside_centrifugal
t_total=np.linspace(-np.pi/2,np.pi*1.5,1000)
f_vector=np.append(f_nearside_vector,f_farside_vector)
plt.rcParams['font.sans-serif']=['SimHei']
plt.rcParams['axes.unicode_minus']=False
plt.plot(t_total,f_vector)
```

The variation in tidal forces, as influenced by the Earth's rotational angle data, has been calculated using the aforementioned code and is depicted in Figure 2.

3.1. The Contribution of Tidal Fluctuations in the Global Tide Cycle

Observations from Figure 2 reveal that there is one tidal event on both the near and far sides of the Moon, with two ebb tides occurring in between. The peak tidal force experienced by a unit mass particle on the near side of the Moon reaches $4.42\text{x}10^{-5}$ Newtons, while on the far side, it attains $4.81\text{x}10^{-5}$ Newtons. At the collocation points, the tidal force reaches its minimum value of $-5.16\text{x}10^{-7}$ Newtons, indicating a negative force, which suggests that at points O_3 and O_5 in Figure 1, the tide is at its lowest ebb.

The height of ocean tides is directly proportional to the intensity of the tidal forces in the region. The transition between high tide and low tide at the same location alternates approximately every 6 hours. When considering both the Earth's rotation and the rotation of the Earth-Moon system, the interval between two successive high tides is 12.5 hours, which is consistent with real-world observations [5].

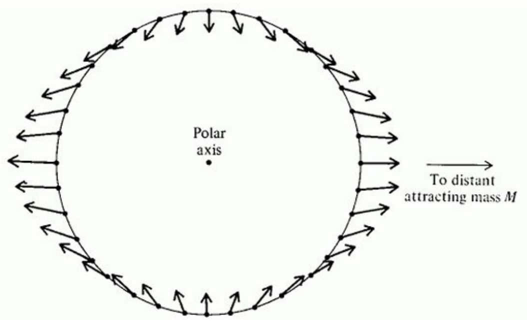

Figure 3. Vector illustration of tidal forces on the surface

3.2. The Contribution of Tidal Fluctuations in the Hawking Radiation Theory

This computational model can be extended to analyze the Hawking radiation, similar to the Earth-Moon system, calculating it requires considering the combined effects of gravitational forces and the centrifugal forces resulting from the rotational dynamics of the system. In the vicinity of a black hole, tidal forces may lead to the separation of particle pairs. The calculation of tidal forces exerted by a rotating black hole necessitates an in-depth examination of the Kerr metric, which accounts for the black hole‘s angular momentum and the frame-dragging effect on the surrounding spacetime. The magnitude of tidal forces is associated with the rate of change of spacetime curvature, which can be described by the partial derivatives of the metric tensor:

$$ds^2 = -\left(1-\frac{2GMr}{\rho^2}\right)dv^2 + \frac{\Sigma}{\rho^2}\sin^2\theta d\chi^2 + p^2 d\theta^2 + 2dvdr$$
$$-2a\sin^2\theta d\chi dr - \frac{4GMar}{\rho^2}\sin^2\theta dv d\chi \quad (6)$$

Despite the complexity, numerical solutions can be obtained through computational modeling approaches. As the virtual particle pair is separated by the tidal forces of the black hole to a certain distance, the negatively energized virtual particle falls into the black hole, while the positively energized virtual particle remains outside [6]. With no partner for annihilation, the positively energized virtual particle transitions into a real particle [7]. The positively energized virtual particle may gain energy from the tidal forces and escape the black hole, resulting in Hawking radiation [8].

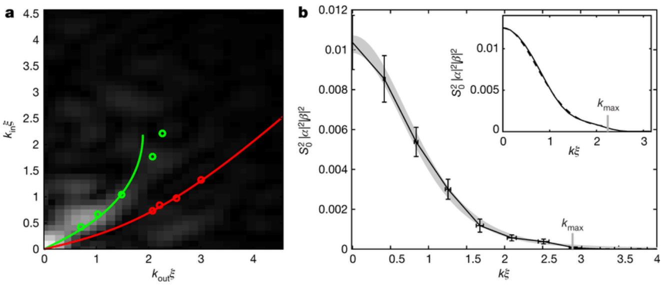

Figure 4. Spectrum of the Hawking radiation

4. The dynamic equation of tidal force exerted by the Earth on the Moon

The gravitational attraction from the moon causes tides on Earth, while Earth‘s tidal effect on the moon is faster and stronger due to its significantly larger mass. The near-earth side of the moon experiences a net force towards the earth, while the far-earth side experiences a net force away from the earth, with ‘d‘ representing the distance from Earth‘s center to the lunar surface. The dynamic equations for both sides are listed separately.

The gravitational force from Earth on the near-Earth lunar hemisphere is denoted as F_1

$$F_1 = \iiint_{\Omega_1} \frac{4Gm_e\rho_m(d-x)}{((d-x)^2+y^2+z^2)^{\frac{3}{2}}} dxdydz \tag{7}$$

$$\Omega_1: 0 \le x \le r_m, 0 \le y \le \sqrt{r_m^2 - x^2}, 0 \le z \le \sqrt{r_m^2 - x^2 - y^2}$$

The centrifugal force experienced by the hemisphere of the Moon closest to the Earth due to the rotation of the Earth-Moon system is denoted as F_2

$$F_2 = \iiint_{\Omega_1} \rho_m \omega^2 (r-x)\, dxdydz \tag{8}$$

$$\Omega_1: 0 \le x \le r_m, 0 \le y \le \sqrt{r_m^2 - x^2}, 0 \le z \le \sqrt{r_m^2 - x^2 - y^2}$$

The gravitational force exerted by the Earth on the hemisphere of the Moon farthest from the Earth is denoted as F_3

$$F_3 = \iiint_{\Omega_2} \frac{4Gm_e\rho_m(d+r_m-x)}{((d+r_m-x)^2+y^2+z^2)^{\frac{3}{2}}} dxdydz \tag{9}$$

$$\Omega_2: -r_m \le x \le 0, 0 \le y \le \sqrt{r_m^2 - x^2}, 0 \le z \le \sqrt{r_m^2 - x^2 - y^2}$$

The centrifugal force experienced by the hemisphere of the Moon farthest from the Earth due to the Earth-Moon system's rotation is denoted as F_4

$$F_4 = \iiint_{\Omega_2} \rho_m \omega^2 (d+r_m-x)\, dxdydz \tag{10}$$

$$\Omega_2: -r_m \le x \le 0, 0 \le y \le \sqrt{r_m^2 - x^2}, 0 \le z \le \sqrt{r_m^2 - x^2 - y^2}$$

Therefore, the total shearing force exerted by the Earth on the lunar central cross-section is denoted as F

$$F = \frac{F_1 + F_4 - F_2 - F_3}{2} \tag{11}$$

This equation possesses universality and is applicable for calculating the forces exerted on moving celestial bodies due to tidal forces under various conditions across the universe. Taking the Earth-Moon system as an illustrative example, there exists a net force at the lunar central cross-section, which is directed towards and away from the Earth, respectively. This force exerts a significant stretching effect on the Moon, as illustrated in Figure 5,This process will have an impact on the internal structure of the Moon [9], which will be analyzed in the following chapter.

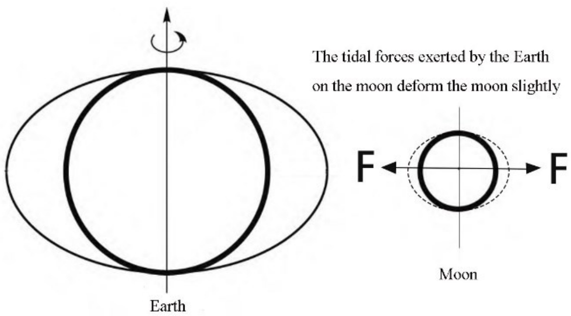

Figure 5. The tidal forces of the Earth on the Moon

This analytical approach is also applicable to estimating the tidal forces exerted by Jupiter on its moon Io. The tidal gravitational variations induce significant compressional and tensile stresses that maintain Io's interior in a state of intense heat and molten activity, thereby precipitating the moon's vigorous volcanic phenomena [10].

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5. Dynamic Simulation of the Tidal Forces Exerted by Earth on the Moon

To achieve higher algebraic accuracy in the integration functions of SciPy‘s integrate library, the computational program employs Simpson‘s rule. By dividing the definite integral range into n subintervals and treating each as a parabolic segment for definite integration, the exact value of the definite integral can be approximated by summing these values.

$$\int_a^b f_{(x)}\,dx \approx \frac{b-a}{3n}\left[(y_0+y_n)+2\sum_{i=2}^{n-2}y_i+4\sum_{i=1}^{n-1}y_i\right] \tag{12}$$

Algorithm 2: Calculation of the Tidal Forces Exerted by Earth on the Moon(Focus solely on presenting the core concepts and implementation methods)

Input: Dataset of physical quantities of the Earth-Moon system

Output: The tidal force exerted on the central cross-section of the Moon

```
def function_gravition_lunar_neartoearth(z,y,x):
    f_total1=(m_earth*p_lunar*G/((r_earthtolunarcycle+r_earth-x)**2+y**2+z**2))
    f_vector1=(r_earthtolunarcycle+r_earth-
x)/np.sqrt((r_earthtolunarcycle+r_earthx)**2+y**2+z**2)*f_total1
   return (f_vector1)
f1,f11=integrate.tplquad(function_gravition_lunar_neartoearth, 0,r_lunar, lambda
x: 0, lambda x: np.sqrt(r_lunar**2-x**2),lambda x, y: 0, lambda x, y:
np.sqrt(r_lunar**2-x**2-y**2))
def function_gravition_lunar_fartoearth(z,y,x):
f_total2=(m_earth*p_lunar*G/((r_earthtolunarcycle+r_earth+r_lunar-
x)**2+y**2+z**2))
f_vector2=(r_earthtolunarcycle+r_earth+r_lunar-
x)/np.sqrt((r_earthtolunarcycle+r_earth+r_lunar-x)**2+y**2+z**2)*f_total2
   return (f_vector2)
f2,f21=integrate.tplquad(function_gravition_lunar_fartoearth, -r_lunar,0 , lambda
x: 0, lambda x: np.sqrt(r_lunar**2-x**2),lambda x, y: 0, lambda x, y:
np.sqrt(r_lunar**2-x**2-y**2))
def function_centrifugal_closetoearth(z,y,x):
    f__centrifugal1=p_lunar*w_spinoflunarencircleearth**2*(r_earthtolunarcycle+r_e
arth-x)
   return (f__centrifugal1)
f3,f31=integrate.tplquad(function_centrifugal_closetoearth, 0,r_lunar , lambda x:
0, lambda x: np.sqrt(r_lunar**2-x**2),lambda x, y: 0, lambda x, y:
np.sqrt(r_lunar**2-x**2-y**2))
def function_centrifugal_fartoearth(z,y,x):
    f__centrifugal1=p_lunar*w_spinoflunarencircleearth**2*(r_earthtolunarcycle+r_e
arth+r_lunar-x)
    return (f__centrifugal1)
f4,f41=integrate.tplquad(function_centrifugal_fartoearth, -r_lunar,0 , lambda x:
0, lambda x: np.sqrt(r_lunar**2-x**2),lambda x, y: 0, lambda x, y:
np.sqrt(r_lunar**2-x**2-y**2))
F=(f1*4-f2*4+f4*4-f3*4)/2
```

The average distance separating the Earth and the Moon is recorded at 384,403.9 kilometers. At this distance, the Earth‘s tidal force on the Moon‘s central cross-section is substantial, amounting to 1.138×10^{18} Newtons. This formidable force exerts a stretching effect on both lunar hemispheres. While tidal forces are insufficient to disintegrate the Moon, they are capable of inducing deformation to a certain degree. The actual orbital trajectory of the Earth-Moon system follows an elliptical path with major and minor axes. The proximity at perigee, the point of closest approach, is 357,431 kilometers, whereas the distance at apogee, the point of farthest separation, extends to 406,222 kilometers. Consequently, as the Moon undergoes its periodic transition between perigee and apogee, the variance in the tidal force exerted by Earth induces periodic deformation of the lunar body.

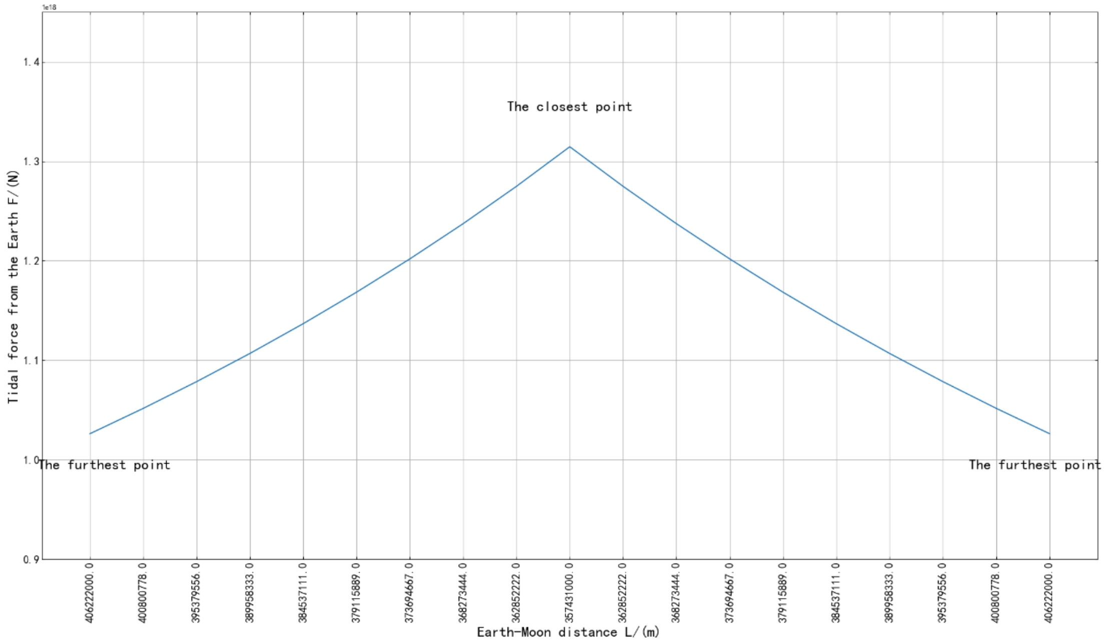

Figure 6. Tidal forces on the Moon's central cross-section vary with Earth-Moon distance changes

To ascertain the extent of tidal force variation, the cyclical distance (a complete cycle from the Earth-Moon farthest distance to the closest approach and back to the farthest distance) between Earth and the Moon is segmented into 19 discrete nodes. Each node's distance is sequentially input into the computational model delineated by Algorithm 2 to calculate the corresponding tidal forces. These calculations are aggregated into a dataset, which is subsequently fitted and graphically represented to illustrate the relationship between tidal force and the Earth-Moon distance. The resulting plot is presented in Figure 6. When the Moon attains a perigee distance of 357,431 kilometers, the Earth's peak tidal force exerted on the lunar body culminates at 1.315×10^{18} Newtons. Conversely, as the Moon recedes to an apogee distance of 406,222 kilometers, the tidal force diminishes to a nadir of 1.026×10^{18} Newtons. Similar to the case of Io, the incessant cyclical variations in tidal forces give rise to continuous compression and extension within the Moon's interior, consequently elevating its internal temperature [11]. Figure 6 clearly elucidates the physical principles of tidal heating.

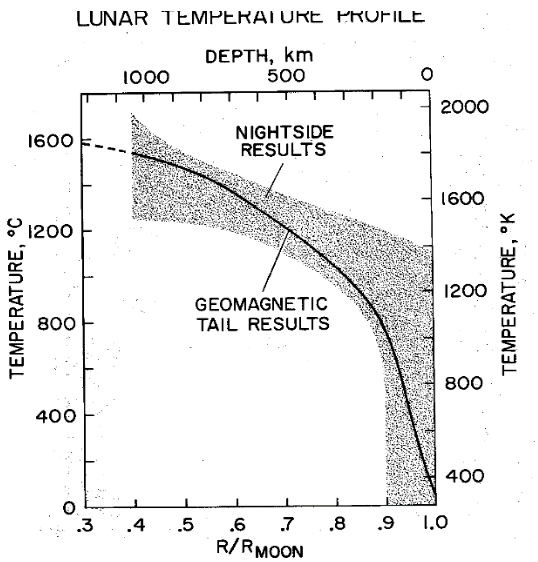

Figure 7. Lunar temperature profile [12]

6. Conclusion

This study presents a comprehensive approach for the high-precision computation, simulation, and visualization of tidal forces within the Earth-Moon system. Key findings and contributions are delineated as follows:

1.Utilizing the algorithmic model outlined in this manuscript, it is feasible to compute the magnitude of tidal forces across all areas of the Earth and to delineate a distribution map of tidal forces on the Earth‘s surface. Furthermore, the model serves as a predictive tool for tidal intensity at diverse global locations and throughout various time periods.

2.Utilizing the computational model detailed in this manuscript, we have determined the numerical values of the tidal forces exerted by the Earth on the central cross-section of the Moon and have constructed a corresponding graphical representation. This visualization effectively demonstrates the relationship between variations in the Earth-Moon distance and the magnitude of tidal forces. The calculated results and the associated graphics elucidate the physical principles underlying tidal heating. Furthermore, this conclusion enhances our comprehension of the substantial tidal forces exerted by Jupiter on its moon Io, which contribute to the Io‘s significant seismic and volcanic activities.

3.This manuscript presents a computational methodology for determining tidal forces, utilizing third-party Python libraries. This method provides a robust framework for efficiently and accurately calculating numerical values and trends for celestial bodies under diverse gravitational conditions. This framework lays the foundation for further analytical and research endeavors in this domain.

Acknowledgments

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